Department of Statistics and Informative

College of Administration and Economics

University of Salahaddin

Subject: Distribution Theory

Course Book – Master Students

Lecturer's name:

Asst. Prof. Dr. Luceen Immanuel Kework

Academic Year: 2023-2024

Course Book

1. Course name	Distribution Theory
2. Lecturer in charge	Dr. Luceen Immanuel Kework
3. Department/ College	Statistics & Informative/ Administration and Economics
4. Contact	e-mail: luceen.kework@su.edu.krd
5. Time (in hours) per week	(3) hours
6. Office hours	(3 hours Theory) during the week
7. Course code	STE401
8. Teacher's academic profile	I got a BSc degree from the college of Administration and Economics, department of Statistics in 1992, ranked very good. I designated (Research Assistant) at the same college in 19/3/1994. In 1999 I accepted in higher education - Masters, and I got an MSc degree in 2002. I worked as an assistant lecturer at the department of Statistics, and I taught the following subjects: Econometrics, Multivariate analysis, Statistical Inference, Mathematical statistics, Operation researches Regression analysis, Probabilities, Linear algebra, Basic programming and Windows and Word software. In 2008 I accepted in higher education - PhD, and I obtained a doctorate in mathematical statistics in 2012. Then I taught Statistical Inference for the fourth stage department of Statistics, and the Econometrics for students master / Statistics dep.. During periods of teaching I supervised the researches of graduate students' fourth stage. After I got my PhD I published four researches.
9. Keywords	Discrete Probability Distributions, Uniform, Bernoulli, Binomial, Geometric, Negative Binomial, Hypergeometric, Poisson. Continuous Probability Distributions , Uniform, Beta, Gamma, Exponential, Chi-Square, Weibull and Normal. All Properties of these distributions. Point and Interval Estimation, Properties of estimators, Unbiasedness, Consistency , Sufficiency Estimator, Completenes, Uniqueness, Efficiency , Fisher Information

	, Maximum Likelihood Estimation . Moments Method, Minimum Variance Method Bayesian Estimation Method, Interval Estimation.

10. Course overview:

Distribution Theory is considered a topic in department of statistics, because at the beginning the student of Master will get familiar with statistical distribution most of the researches are depending on these distributions for analysing data.

-Via statistics Master students will learn the philosophy of distributions in real life, proving any rules and how they formed, we will make students learn them especially according to their distributions.

-How distribution of functions is found in different researches.

- How proved all the properties of these distributions.

-How proved the properties of best estimators to discrete and continuous distributions.

-How is estimate the parameters of population by traditional method or by Bayesian method.

The most important things that the students should keep the subject under control, we should consider this point.

1. **Know the philosophy of each statistical distributions, when and how to use each distribution in real life, and what the parameters of each distribution indicate in real life.**
2. **The importance of the subjects in mathematical statistics in the Postgraduate Studies , Master students should review the basic rules.**
3. **Students should make a connection between the previous subject and current one.**
4. **While displaying important points students should write them down because these notes are crucial for solving the questions.**
5. **Following up those questions that are left unsolved students should do their best to solve them.**

11. Course objective:

1. Know the philosophy of each statistical distributions ?

2. Know all the properties of each distributions.

3. Know all the properties of estimators for parameters for these distributions.

4. Know what is the estimation of parameter?

5. Knowing most of the methods used to estimate the parameters of each statistical distribution.

This course is divided into fifteen weeks. In the first seven weeks we study discrete distributions, we study the distribution theory of the discrete Uniform distribution, which includes when and how to use this distribution in real life, the characteristics of this distribution, methods for estimating the parameters of the distribution, and the properties of the estimator of the distribution.

And so for the distributions (Bernoulli, Binomial, Geometric, Negative Binomial, Hypergeometric and Poisson), then, In the next seven weeks we study continuous distributions (Uniform, Beta, Gamma, Exponential, Chi-Square, Weibull and normal)

12. Student's obligation

The attendance and completion of midterm test, quiz, assignments, reports, presentation and final exam.

13. Forms of teaching

Different forms of teaching will be use to reach the objectives of the course: data show PowerPoint presentations for the head titles and summary of conclusion, classification of material and any other illustrations. There will be classroom discussions and the lecture will give enough background to translate, solve, analyze, derive, and evaluate problems by using white board.

14. Assessment scheme

Grading: Grades will be assigned on a curve, using the following percentages: 10 Grads for Quizzes and the presence and absence of students, 20 Grads for Midterm Exam, 10 Grads for report, 10 Grads for presentation and 50 Grads for Final Exam and Pass: 60%.

15. Student learning outcome:

The clarity of the basic objectives of subject for students, namely;

They Learned the philosophy of each statistical distribution, and when and how to use each of these distributions in real life, and how to apply it in real life. They knew the properties of these distributions and the properties of best estimators for the population parameters, They knew how to estimates the population parameters.

Content article is appropriate to the requirements of the outside world and the labour market because it deals with all types of data in the outside world and the labour market.

The new things that the student learn through this a lecture are learned how to test the hypotheses. Learned all the details about the common continuous and discrete distributions in the population and how to deal with it.

16. Course Reading List and References:

1) Introduction to Theory of Statistics. By Mood Graybill & Boes, 3rd edition (1974).

2) Introduction to the Theory of Statistics. By Mood Graybill, 2nd edition (1963).

3) Theoretical Statistics. By Cox D.R. & Hinkley D.V. (1974).

4) Introduction to Mathematical Statistics. By Hogg & Craig, 4th edition (1978).

5) Introduction to Mathematical Statistics. By Hogg Mekean & Craig, 6th edition (2006).

6) Statistical Theory. By Lindgren B. (1982).

7) An Introduction to Probability & Statistics. By Rohatgi Vijary & Saleh, 2nd edition (2001).

8) Mathematical Statistics. By Freund Walpole & Meyer, (1979).

9) Introductory to Probability & Statistics Applications. By Paul Meyer, (1978).

10) Probability & Statistics for Engineers & Scientists. By Walpole Myers Myers Ye 7th edition (2002).

11) Modern Probability Theory and its Applications. Emmanuel Parzen.

12) Probability Theory. By Filler.

13) Introduction to Mathematical Statistics, 5th edition; By Robert V. Hogg and Craig,1995.

14) Introduction to Probability Theory and Statistical Inference, 3rd edition; By Harold J. Larson, 1982.

17. The Topics: Contents	Lecturer's name
Discrete Statistical Distributions: Distribution Theory for Discrete Uniform Distribution.	First week 3 hrs 2024 / 2 / 20
Distribution Theory for Bernoulli Distribution.	Second week 3 hrs 2024 / 2 / 27
Distribution Theory for Binomial Distribution.	Third week 3 hrs 2024 / 3 / 5
Distribution Theory for Geometric Distribution.	Fourth week 3 hrs 2024 / 3 / 12
Distribution Theory for Negative Binomial Distribution.	Fifth week 3 hrs 2024 / 3 / 19
Distribution Theory for Hypergeometric Distribution.	Sixth week 3 hrs 2024 / 3 / 26
Distribution Theory for Poisson Distribution.	Seventh week 3 hrs 2024 / 4 / 2
Midterm Exam1 + Complete the Poisson Distribution	Eighth week 3 hrs 2024 / 4 / 9
Continuous Statistical Distributions: Distribution Theory for Continuous Uniform Distribution.	Ninth week 3 hrs 2024 / 4 / 16
Distribution Theory for Gamma Distribution. Report and Presentation (the report)	Tenth week 3 hrs 2024 / 4 / 23
Distribution Theory for Exponential and Chi-Square Distribution.	Eleven week 3 hrs 2024 / 4 / 30
Distribution Theory for Beta Distribution.	Twelve week 3 hrs 2024 / 5 / 7

Midterm Exam2	Thirteenth week 3 hrs 2024 / 5 / 14
Distribution Theory for Normal Distribution.	Fourteenth week 3 hrs 2024 / 5 / 21
Distribution Theory for Weibull Distribution.	Fifteenth week 3 hrs 2024 / 5 / 28

18. Practical Topics (If there is any)	
There isn't any Practical Topics	

19. Examinations:

Q1: Define the exponential-family ad its pdf, then show that if the Poisson distribution belongs to this family. Given a r.v. X has Poisson dist. Write the pdf of it, and the mean, the variance and mgf of it. Find the mode of this dist

Sol.:

Let X has a p.d.f. $f(x;\theta)$, then the family of $f(x;\theta)$ is belong to exponential class of distribution if it can be expressed as:

$$f(x;\theta) = Exp(\ln f(x;\theta))$$
$$= Exp\big(p(\theta)\ K(x) + S(x) + q(\theta)\big)$$

Such that: $p(\theta)$ $K(x)$ must have to be for exponential class.

$$f(x;\theta) = \frac{e^{-\theta}\theta^{x}}{x!}$$
$$= \exp\big(\ln\ f(x;\theta)\big) \qquad x = 0,1,2,\ldots$$
$$= \exp\big(-\theta + x\ln(\theta) + \ln(x!)\big)$$
$$= \exp\big(q(\theta) + p(\theta)K(x) + S(x)\big)$$

$\therefore$ *the family of X is brlongs to the* exp. *class of distribution*

Mean = variance = θ , $M_X(t) = e^{\theta(e^t-1)}$ $,\theta > 0$

$$\frac{f(m)}{f(m-1)} = \frac{e^{-\theta}\theta^{m}/m!}{e^{-\theta}\theta^{m-1}/(m-1)!} = \frac{\theta}{m} > 1$$

Which is larger than 1 for $m < \theta$ and smaller than 1 for $m > \theta$.

Q2: In a random sample of size (n) from normal distⁿ $N(\theta, \sigma^2)$. Is

$S^2 = \frac{1}{n}\sum(X_i - \overline{X})^2$ **unbiased estimator for the parameter (σ^2).**

Sol:

$$E(S^2) = \frac{1}{n} E\left(\sum (X_i - \overline{X})^2\right) = \frac{1}{n} E\left(\sum X_i^2 - n\overline{X}^2\right)$$

$$= \frac{1}{n} E\left(n\ E(X^2) - n\ E(\overline{X})^2\right)$$

$$E(X^2) = V(X) + (E(X))^2 = \sigma^2 + \theta^2$$

$$E(\overline{X}^2) = V(\overline{X}) + (E(\overline{X}))^2 = \frac{\sigma^2}{n} + \theta^2$$

$$\therefore E(S^2) = \frac{1}{n} \left(n\sigma^2 + n\theta^2 - \sigma^2 - n\theta^2\right)$$

$$= \frac{1}{n} \left((n-1)\ \sigma^2\right) = \frac{(n-1)}{n} \sigma^2 \neq \sigma^2$$

$$\therefore S^2 = \frac{1}{n} \sum (X_i - \overline{X})^2 \ \textit{is not unbiased estimator for } \sigma^2$$

$$\lim_{n\to\infty} E(S^2) = \lim_{n\to\infty} \frac{(n-1)}{n} \sigma^2 = \lim_{n\to\infty} \left(\frac{n}{n}\sigma^2 - \frac{\sigma^2}{n}\right)$$

$$= \lim_{n\to\infty} \frac{n}{n}\sigma^2 - \lim_{n\to\infty} \frac{\sigma^2}{n}$$

$$= \sigma^2 - 0 = \sigma^2 \quad \to \therefore S^2 \ \textit{is unbiased in limit estimator for } \sigma^2.$$

20. Extra notes:

There isn't any extra notes or comments

21. Peer review **پێداچوونەوەی هاوەڵ**