بانكی پرسیارەكان (Question Bank) كورسی دووەم – ماستەر
لە بابەتی (Distribution Theory) ساڵی خوێندنی 2023-2024

Q1// Let X be a r.v. has Geometric distribution.
1) State when we use it in real life (with example)
2) What do represent a random variable and a parameter in a Geometric distribution?
3) Explain what it means by memory less property for the Geometric distribution, and express it mathematically, then prove this property for this distribution.
4) Find the mode of the Geometric distribution for the two forms.

Q2// **a)** Is the Negative Binomial distribution N.Bin(r, p) belongs to exponential family? Then find the Fisher information in a rssn by using exponential family method.
b) Let X~Exp(1/θ). Is $\hat{\theta} = \frac{n-1}{\sum_{i=1}^{n} X_i}$ unbiased estimator for θ?

Q3// Let X be a r.v. has Poisson distribution.
1) State when we use it in real life (with example)
2) What do represent a random variable and a parameter in a Poisson distribution?
3) What would be the distribution of X in the time interval (0, T)? Write the probability mass function of it, and the mean and the variance of it (only).
4) From (3): How the exponential distribution is derived from the Poisson distribution in this period?

Q4// **a)** What do the parameters (α, β) describe in a Gamma distribution? (with example)
b) Let X ~ Gamma(2, 1/θ) and $T = \frac{2n-1}{\sum_{i=1}^{n} X_i}$ an estimator for θ. Find (M.V.B) for the variance of unbiased estimator of θ, then show that if T is unbiased estimator for θ. Then find the variance of T and compared it with (M.V.B).

Q5// **a)** What is/are the purpose(s) of using beta distribution? State when we use it in real life (with example).
b) In a rss*n* from Beta(α, β). Estimate the parameters α and β using moment method.

Q6// **a)** What are the inflection points in the normal distribution? What do they represent? Explain them with a graph.
b) Let $X_1, X_2, \ldots, X_n$ be a rss*n* from normal distribution N(μ, τ). If μ is known, and only τ is unknown, and $= \frac{1}{\sigma^2}$. Find Bayes estimator for parameter (τ), using informative prior probability.

Q7// for the following distribution:

1) Categorical distribution. **2)** Binomial distribution. **3)** Negative Binomial distribution.

a) Define random variables (Notation).

b) Write the complete p.m.fs.

c) What do the random variables and parameters mean for each distribution?

d) State when we use them in real life (with example).

e) Show how they relates to the other distributions.

f) Find the moment generating function (m.g.f.) of Binomial and Negative Binomial distribution.

Q8// Let $X_1, X_2, \dots, X_r$ are independent random variables each having Geo(θ). Show that $\sum_{i=1}^{r} X_i$ is a complete estimator for θ.

Q9// Let X_i ~Ber(θ), and $X_1, X_2, \dots, X_n$ be a r.s. of size (n) generated from X, can we apply Rao Black well theorem to $E(X_i | \sum_{i=1}^{n} X_i)$, find the distribution of $p(X_1 | \sum_{i=1}^{n} X_i)$ and then find $E(X_1 | \sum_{i=1}^{n} X_i)$ and $Var\ E(X_i | \sum_{i=1}^{n} X_i) < Var(X_i)$.

Q10// Show and drive C.D.F of the Gamma distribution may be obtained from the C.D.F. of the Poisson distribution.

Q11// Prove the following m.l.e. property. If there is an efficient estimator then the m.l.e. method will produce it.

Q12// Use the minimum variance method to obtain an estimator for a parameter θ of binomial population Bin(m, θ).

Q13// Define the exponential-family ad its pdf, then show that if the Poisson distribution belongs to this family. Given a r.v. X has Poisson dist. Write the pdf of it, and the mean, the variance and mgf of it. Find the mode of this dist

Q14// In a random sample of size (n) from normal distn N(θ, σ2). Is $S^2 = \frac{\sum(X_i - \bar{X})^2}{n}$ unbiased estimator for the parameter (σ^2).

Q15// Let X be a r.v. has the Binomial distribution:

a) Define random variables (Notation).

b) Write the complete p.m.f of X.

c) What do the random variable and parameters represent in this distribution?

d) What are the properties of this distribution? Give an example for this distribution).

e) Find the mode of this distribution.

f) Find Bayes estimator for θ by using informative prior.

Q16//

a) Find the moment generating function (m.g.f.) of X for the **second form** of the negative Binomial distribution.

b) Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Negative Binomial dist^n N.Bin(r, p), find the m.l.e for p of the **first form**.

c) Let (r -1) is number of success before the last success from ($x + r - 1$) in Negative Binomial distribution, then show that; $\hat{p} = \frac{r-1}{x+r-1}$ unbiased estimator for p.

Q17// In a rss(n) from uniform distribution C.U(0, θ). Let $Y_1, Y_2, \ldots, Y_n$ are an order statistics of this sample. Is an estimator Y_1 unbiased estimator for θ?

Q18//

a) What are the uses of the Beta distribution? (Give two examples of this distribution).

b) Find the mode of the beta distribution

Q19// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Normal distribution N(θ, σ^2), Find Bayes estimator for parameter (σ^2), if you know (θ known), using informative prior probability.

Q20// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Weibull distribution Wei(α, β).

a) Find the moment generating function (m.g.f.) of X.

b) Find m.l.e for parameter α (if β known).

Q21// Let X be a r.v. has the Negative Binomial distribution: (First Form)

a) Define random variables (Notation).

b) Write the complete p.m.f of X.

c) What do the random variable and parameters represent in this distribution?

d) State when we use them in real life (with example).

e) Show that $\frac{r+X}{r}$ is an unbiased estimator for $1/p$.

f) Find the mode of this distribution.

Q22// Let $X_1, X_2, \ldots, X_n$ are independent random variables each having Poi(θ).

a) Explain how the Poisson distribution relates to other distributions.

b) Show that $\sum_{i=1}^{n} X_i$ is a complete estimator for θ.

c) Find Bayes estimator for parameter (θ), using non-informative prior probability.

Q23// Find Bayes estimator for parameters of; **1)** Exp($1/\theta$), **2)** N(θ, σ^2) , using non informative prior probability.

Q24// In a rssn, find m.v.e. for the parameters of; **1)** Ber(θ). **2)** N(θ, σ^2).

Q25// Estimate the parameter by using moment method for:

1) Ber(θ). **2)** Exp($1/\theta$). **3)** Geo(θ).

Q26// Let $X_1, X_2, \ldots, X_n$ be a rssn from normal distn N(θ, σ 2),

1) find m.l.e for parameters θ and σ^2 . **2)** If S^2 is m.l.e. for σ^2 , then find m.l.e. for σ.

Q27// In a rssn from Geometric distn Geo(θ), with p.d.f ; f(x;θ) = θ(1 – θ) x , x = 0,1,2,..., find the m.l.e for θ:

Q28// Let $X_1, X_2, \ldots, X_n$ denote a random sample from Poisson distn Poi(θ), find the m.l.e for θ.

Q29// let $X_1, X_2, \ldots, X_n$ be a random sample of size (n) rssn taken from C.U(0,1). let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of this sample. **Find** the p.d.f. of Y_1 and Y_n, the j.p.d.f. of Y_1 and Y_n

Q30// In a random sample of size (n) taken from exponential dist^n Exp(θ). Show that;

1. $T_1 = \overline{X}$ is unbiased estimator for the parameter (θ).

2. $T_2 = \frac{n}{n+1}\overline{X}^2$ is unbiased estimator for the parameter (θ^2).

Q31// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n, show that $\hat{\theta} = \overline{X}$ is consistent estimator for θ.

Q32// Show that $\hat{\theta} = Y_n$ is consistent estimator for θ from C.U(0 , θ), (by theorem).

Q33// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Bernoulli dist^n Ber(θ). Show that $\hat{\theta} = \sum X_i$ is sufficient estimator for the parameter θ.

Q34// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from a dist^n with p.d.f.:

$$f(x;\theta) = e^{2\theta - x} \quad , \quad x \geq 2\theta$$

Show that Y_1 is sufficient estimator for the parameter θ.

Q35// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n Poi(θ), show that $\hat{\theta} = \sum X_i$ is sufficient estimator for θ?

Q36// from Exp($1/\theta$). Is $\sum_{i=1}^{n} X_i$ sufficient estimator for θ? (by factorization theorem).

Q37// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Gamma dist^n $\Gamma(\alpha, 1/\theta)$, find the jointly sufficient estimators for the parameters (α, θ).

Q38// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from normal dist^n N(θ, σ^2), show that $\sum X_i$, $\sum X_i^2$ are the jointly sufficient estimators for the parameters (θ, σ^2) respectively.

Q39// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from C.U($\theta_1 - \theta_2$, $\theta_1 + \theta_2$), and $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics, show that Y_1 and Y_n are the jointly sufficient estimators for the parameters (θ_1, θ_2) respectively.

Q40// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n Poi(θ), show that if the dist^n of X can be written in exponential form?

Q41// Let X and Y be two random variables with j.p.d.f.;

$$f(x, y) = \frac{2}{\theta^2} e^{-(x+y)/\theta} \qquad , \quad 0 < x < y < \infty$$

Show that;
1) $E(Y) = E(E(Y|X))$.
2) $Var(Y) \geq Var(E(Y|X))$.

Q42// Let X be a random variable from; **1)** Bernoulli dist^n. **2)** Poisson dist^n. **3)** Normal dist^n.

Show that the family of X is complete.

Q43// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n poi(θ). Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. Find the unique continuous function of Y, which is the best estimator for θ (M.V.U.E).

Q44// Let $X_1, X_2, \ldots, X_n$ denote a random sample of size $n > 2$ from a dist^n with p.d.f. $f(x;\theta) = \theta e^{-\theta x} \quad 0 < x < \infty$, $and \ \theta > 0$. **1)** Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. **2)** Prove that (n – 1)/Y is the unique continuous function of Y which is the best estimator for θ (M.V.U.E).

Q45// Let $X_1, X_2, \ldots, X_n$ denote a random sample from a dist^n which is N(0, θ). Then $Y = \sum X_i^2$ is a sufficient estimator for θ. Find the best estimator for θ^2 (M.V.U.E).

Ex: Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from exponential dist^n Exp(θ);

1) If $T = \overline{X}$ is an efficient estimator for $\phi(\theta) = \theta$.

2) Find RCLB for each of [$\phi(\theta) = \ln\theta, \phi(\theta) = 2\theta$].