Department of Statistics and Informative

College of Administration and Economics

University of Salahaddin

Subject: Estimation Theory

Course Book – Fourth Stage (Second Semester)

Lecturer's name:

Asst. Prof. Dr. Luceen Immanuel Kework

Academic Year: 2022-2023

Course Book for the First Semester

1. Course name	Estimation Theory
2. Lecturer in charge	Dr. Luceen Immanuel Kework
3. Department/ College	Statistics/ Administration and Economics
4. Contact	e-mail: luceen2015@gmail.com
5. Time (in hours) per week	(3) hours
6. Office hours	(3 hours) during the week
7. Course code	STE401
8. Teacher's academic profile	I got a BSc degree from the college of Administration and Economics, department of Statistics in 1992, ranked very good. I designated (Research Assistant) at the same college in 19/3/1994. In 1999 I accepted in higher education - Masters, and I got an MSc degree in 2002. I worked as an assistant lecturer at the department of Statistics, and I taught the following subjects: Econometrics, Multivariate analysis, Statistical Inference, Mathematical statistics, Operation researches Regression analysis, Probabilities, Linear algebra, Basic programming and Windows and Word software. In 2008 I accepted in higher education - PhD, and I obtained a doctorate in mathematical statistics in 2012. Then I taught Statistical Inference for the fourth stage department of Statistics, and the Econometrics for students master / Statistics dep.. During periods of teaching I supervised the researches of graduate students' fourth stage. After I got my PhD I published four researches.
9. Keywords	Fisher Information , Maximum Likelihood Estimation , Moment Estimation Method, Minimum Variance Method , Bayesian Estimation Method, Interval Estimation

	Neyman -Pearson Theorem, Testing of Statistical Hypotheses.

10. Course overview:

Statistical Inference is considered a topic in department of statistics, because at the beginning the student will get familiar with statistical distribution most of the researches are depending on these distributions for analysing data.

-Via statistics students will learn proving any rules and how they formed, we will make students learn them especially according to their distributions.

-How distribution of functions is found in different researches.

-How proved the properties of best estimators to discrete and continuous distributions.

-How is estimate the parameters of population by traditional method or by Bayesian method.

- How testing of Hypotheses for parameters of population.

The most important things that the students should keep the subject under control, we should consider this point.

1. **The important of the subjects in mathematical statistics in the third stage, students should review the basic rules.**
2. **Memorizing or recognizing statistical rules which are (24) basic rules that we always consider them.**
3. **Students should make a connection between the previous subject and current one.**
4. **While displaying important points students should write them down because these notes are crucial for solving the questions.**
5. **Following up those questions that are left unsolved students should do their best to solve them.**

11. Course objective:

1. **Know what is Inference?**
2. **Know what is the estimation of parameter?**

3. Understand hypothesis testing & the "types of errors" in decision making.

4. Know what the α-level means.

5. Learn how to use test statistics to examine hypothesis about population mean, proportion.

This course is divided into two parts. The first part deals with estimation (point estimation and confidence intervals), properties of an estimator, methods for finding estimators, and the second part deals with hypothesis tests.

Statistical inference is a formal process of using sample data to answer questions or to draw conclusions about a population (Estimating population parameters and testing hypotheses). Confidence intervals provide a method for using sample data to construct estimates of population characteristics, whereas hypothesis tests allow us to use sample data to decide between two competing claims, called hypotheses, about a population characteristic. Although confidence intervals and hypothesis tests are generally used for different purposes, they share a common goal of generalizing from a sample to a population.

12. Student's obligation

The attendance and completion of all tests, exams, assignments, reports.

13. Forms of teaching

Different forms of teaching will be use to reach the objectives of the course: data show PowerPoint presentations for the head titles and summary of conclusion, classification of material and any other illustrations. There will be classroom discussions and the lecture will give enough background to translate, solve, analyze, derive, and evaluate problems by using white board.

14. Assessment scheme

Grading: Grades will be assigned on a curve, using the following percentages: 5% Quizzes and the presence and absence of students, 35% Exams, 60% Final and Pass: 50%.

15. Student learning outcome:

The clarity of the basic objectives of subject for students, namely;

They Learned how to find distribution of random variables of functions by using transformation technique, and order statistics function (discrete or continuous) in univariate and bivariate cases, and how to apply it in real life.

They knew the properties of best estimators for the population parameters, They knew how to estimates the population parameters.
Content article is appropriate to the requirements of the outside world and the labour market because it deals with all types of data in the outside world and the labour market.
The new things that the student learn through this article are: Learned how to test the hypotheses. Learned all the details about the common continuous and discrete distributions in the population and how to deal with it.

16. Course Reading List and References:

1. Introduction to Mathematical Statistics, 5th edition; By Robert V. Hogg and Craig,1995.

2. Introduction to Probability Theory and Statistical Inference, 3rd edition; By Harold J. Larson, 1982.

3. Statistical inference / George Casella, Roger L. Berger.-2nd edition 2002.

4. Principles of Statistical Inference, D.R. Cox, 2006.

5. An introduction to Probability and Mathematical Statistics, Rohatgi, V.K. , 1976.

6. Theory of Point Estimation, E.L. Lehmann George Casella 2nd edition 1998.

7. Statistical Distributions. Merran Evans, Nicholas Hastings, Brian Peacock, 3rd Edition, 2000.

8. Mathematical Statistics. Ferguson, T.S. 1968.

9. Statistical inference. Silvey 1973.

10. Bayesian Inference in Statistical Analysis. Box and Tiro 1973.

11. The Theory of Statistical Inference. Zacks, S.

12. Introduction to Probability and Statistical Inference. George Roussas 2003.

13. Probability and Mathematical Statistics. Prasanna Sahoo 2013.

17. The Topics: Contents	Lecturer's name
Methods of Point Estimation First: Maximum Likelihood Method (MLE)	First week 3 hrs 2023 / 1 / 8
Examples	Second week 3 hrs 2023 / 1 / 15
Second: Moments Estimation Method (M.E.M)	Third week 3 hrs 2023 / 1 / 22
Examples	Fourth week 3 hrs 2023 / 1 / 29
Third: Least Square Method (Minimum Variance Method)(M.V.M)	Fifth week 3 hrs 2023 / 2 / 5
First Midterm Exam for the Second Semester	Sixth week 3 hrs 2023 / 2 / 12
Fourth: Bayesian Estimation Method (B.E.M) a) Non Informative prior probability (Jeffery's rule)	Seventh week 3 hrs 2023 / 2 / 19
Examples	Eighth week 3 hrs 2023 / 2 / 26
Fourth: Bayesian Estimation Method (B.E.M) b) Informative prior probability	Ninth week 3 hrs 2023 / 3 / 5
Examples	Tenth week 3 hrs 2023 / 3 / 12
Interval Estimation (General Concepts and Definitions)	Eleventh week 3 hrs 2023 / 3 / 26
Second Midterm Exam for the Second Semester	Twelfth week 3 hrs 2023 / 4 / 2

1) Confidence Interval for Means when the Variance is Known	**Thirteenth week 3 hrs 2023 / 4 / 9**
2) Confidence Interval for Means when the Variance is Unknown	**Fourteenth week 3 hrs 2023 / 4 / 16**
3) Confidence Interval For Difference Between Two Means	**Fifteenth week 3 hrs 2023 / 4 / 23**
4) Confidence Interval For The Variance	**Sixteenth week 3 hrs 2023 / 4 / 30**

18. Practical Topics (If there is any)	
There isn't any Practical Topics	

19. Examinations:

Q1: Let $X_1, X_2, \ldots, X_n$ denote a random sample from Bernoulli dist^n Ber(θ), find the m.l.e for θ.

Sol:

$\because$ X ~ Ber(θ)

$$f(x;\theta) = \theta^x(1-\theta)^{1-x} \quad , \quad x = 0\,,1$$

$$\because X's\ are\ indep.$$

$$L(\theta) = f(x_1, x_1, \ldots, x_1;\ \theta) = \Pi\, f(x_i;\theta)$$

$$= \theta^{\Sigma x_i}(1-\theta)^{n-\Sigma x_i}$$

$$\ln L(\theta) = \Sigma x_i \ln(\theta) + (n - \Sigma x_i)\ln(1-\theta)$$

$$\frac{\partial \ln L(\theta)}{\partial\theta} = \frac{\Sigma x_i}{\theta} - \frac{n-\Sigma x_i}{1-\theta} \quad , \quad \frac{\partial \ln L(\theta)}{\partial\theta} = 0$$

$$\frac{\Sigma x_i}{\theta} - \frac{n-\Sigma x_i}{1-\theta} = 0$$

$$\frac{(1-\theta)\Sigma x_i - \theta(n-\Sigma x_i)}{\theta^\wedge(1-\theta^\wedge)} = 0$$

$$\Sigma x_i - \theta^\wedge \Sigma x_i - n\theta^\wedge + \theta^\wedge \Sigma x_{i_i} = 0$$

$$\Sigma x_i - n\theta^\wedge = 0$$

$$\Sigma x_i = n\theta^\wedge \qquad \theta^\wedge{}_{m.l.e} = \frac{\Sigma X_i}{n} = \overline{X}$$

$$\frac{\partial^2 \ln L(\theta)}{\partial\theta^2} = -\frac{\Sigma x_i}{\theta^2} - \frac{n-\Sigma x_i}{(1-\theta)^2} < 0$$

$\therefore \theta^\wedge = \overline{X}$ is m.l.e for θ .

Q2: Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from normal dist^n N(θ, σ^2), estimate the parameters θ and σ^2 using moment method.

Sol:

$$m_k = M_k$$

$$m_k = \frac{\sum X_i^k}{n} \quad , \quad M_k = E(X^k)$$

$$m_1 = \frac{\sum X_i}{n} \quad \Rightarrow \quad M_1 = E(X) = \theta$$

$$m_1 = M_1$$

$$\frac{\sum X_i}{n} = \theta \quad \Rightarrow \therefore \hat{\theta} = \overline{X}$$

$$m_2 = \frac{\sum X_i^2}{n} \quad \Rightarrow \quad M_2 = E(X^2)$$

$$M_2 = E(X^2) = V(X) + (E(X))^2 = \sigma^2 + \theta^2$$

$$m_2 = M_2$$

$$\frac{\sum X_i^2}{n} = \sigma^2 + \overline{X}^2$$

$$\therefore \hat{\sigma}^2 = \frac{\sum X_i^2}{n} - \overline{X}^2$$

Q3: Find Bayes estimator for parameter of; 2) Poisson(ϑ), using non informative prior probability.

Sol:

2) $X \sim Poi(\theta)$

$$f(x;\theta) = \frac{e^{-\theta}\theta^{x}}{x!} \quad , x = 0,1,.......$$

$$\ln f(x;\theta) = -\theta + x\ln(\theta) - \ln(x!)$$

$$\frac{\partial \ln f(x;\theta)}{\partial \theta} = -1 + \frac{x}{\theta}$$

$$\frac{\partial^2 \ln f(x;\theta)}{\partial \theta^2} = -\frac{x}{\theta^2}$$

$$-E\left(\frac{\partial^2 \ln f(x;\theta)}{\partial \theta^2}\right) = \frac{E(X)}{\theta^2} = \frac{\theta}{\theta^2} = \frac{1}{\theta}$$

$$p(\theta) \propto (I_s(\theta))^{1/2}$$

$$\propto \left(\frac{1}{\theta}\right)^{1/2} = \theta^{-1/2}$$

$$L(\theta) = \frac{e^{-n\theta}\theta^{\sum x_i}}{(\prod_{i=1}^{n} x_i)!}$$

$$L(\theta) \propto e^{-n\theta}\theta^{\sum x_i}$$

$$p(\theta \mid x_1, x_2, ..., x_n) \propto L(\theta)\, p(\theta)$$

$$\propto e^{-n\theta}\theta^{\sum x_i}\,\theta^{-1/2}$$

$$\propto e^{-n\theta}\theta^{\sum x_i - \frac{1}{2}}$$

$$p(\theta \mid x_1, x_2, \dots, x_n) \sim \Gamma(\alpha = \sum x_i + \tfrac{1}{2} \ , \beta = n)$$

$$when \ \ X \sim \Gamma(\alpha \, , \beta) \ , \quad f(x;\alpha \, , \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \, x^{\alpha - 1} \, e^{-\beta x} \quad , \quad E(X) = \frac{\alpha}{\beta}$$

$$\alpha - 1 = \sum x_i - \tfrac{1}{2} \qquad \Rightarrow \ \alpha = \sum x_i + \tfrac{1}{2}$$

$$\beta = n$$

$$p(\theta \mid x_1, x_2, \dots, x_n) = \frac{n^{\sum x_i + \frac{1}{2}}}{\Gamma\left(\sum x_i + \frac{1}{2}\right)} \, \theta^{\sum x_i - \frac{1}{2}} \, e^{-n\theta}$$

$$\therefore \hat{\theta}_{Bayes} = E(\theta \mid x_1, x_2, \dots, x_n) = \frac{\sum X_i + \frac{1}{2}}{n} = \overline{X} + \frac{1}{2n}$$

20. Extra notes:

There isn’t any extra notes or comments

21. Peer review **پێداچوونەوەی هاوەڵ**