University of Salahaddin - Hawler
College of Engineering
Department of Mechanical Engineering

Problems on Probability and Statistics

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Senior Students (3rd Year)

Lecturer: Mahde A. Molan

Subject: Probability and statistics, Class: Third Year, Lecturer: Mahde A. Molan

Chapter One

<table>
<tr><td>Pr.1</td><td>A and B are two candidates seeking admission in a college. The probabilitythat A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.</td></tr>
<tr><td>Pr.2</td><td>The probability of simultaneous occurrence of at least one of two eventsA and B is p. If the probability that exactly one of A, B occurs is q, then prove thatP (A′) + P (B′) = 2 – 2p + q.</td></tr>
<tr><td>Pr.3</td><td>10% of the bulbs produced in a factory are of red colour and 2% are redand defective. If one bulb is picked up at random, determine the probability of its beingdefective if it is red.</td></tr>
<tr><td>Pr.4</td><td>Two dice are thrown together. Let A be the event ‘getting 6 on the first die’ and B be the event ‘getting 2 on the second die’. Are the events A and B independent?</td></tr>
<tr><td>Pr.5</td><td>A committee of 4 students is selected at random from a group consisting 8boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.</td></tr>
<tr><td>Pr.6</td><td>Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of thetubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day’s production,calculate the probability that it is defective.</td></tr>
<tr><td>Pr.7</td><td>Find the probability that in 10 throws of a fair die a score which is amultiple of 3 will be obtained in at least 8 of the throws.</td></tr>
<tr><td>Pr.8</td><td>A discrete random variable X has the following probability distribution:
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P (X)</td><td>C</td><td>2C</td><td>2C</td><td>3C</td><td>C^2</td><td>$2C^2$</td><td>$7C^2 + C$</td></tr>
</table>
Find the value of C. Also find the mean of the distribution.</td></tr>
<tr><td>Pr.9</td><td>Four balls are to be drawn without replacement from a box containing8 red and 4 white balls. If X denotes the number of red ball drawn, find the probability distribution of X.</td></tr>
<tr><td>Pr.10</td><td>Determine variance and standard deviation of the number of heads inthree tosses of a coin.</td></tr>
<tr><td>Pr.11</td><td>Refer to Example 6. Calculate the probability that the defective tube was produced on machine E_1.</td></tr>
<tr><td>Pr.12</td><td>A car manufacturing factory has two plants, X and Y. Plant X manufactures70% of cars and plant Y manufactures 30%. 80% of the cars at plant X and 90% of thecars at plant Y are rated of standard quality. A car is chosen at random and is found to</td></tr>
</table>

	be of standard quality. What is the probability that it has come from plant X?
Pr.13	Let A and B be two events. If P (A) = 0.2, P (B) = 0.4, P (A□B) = 0.6, then P (A | B) is equal to (A) 0.8 (B) 0.5 (C) 0.3 (D) 0
Pr.14	If A and B are independent events such that 0 < P (A) < 1 and0 < P (B) < 1, then which of the following is not correct? (A) A and B are mutually exclusive (B) A and B□ are independent (C) A□ and B are independent
Pr.15	Let X be a discrete random variable assuming values $x_1, x_2, ..., x_n$ with probabilities $p_1, p_2, ..., p_n$, respectively. Then variance of X is given by (A) E (X^2) (B) E (X^2) + E (X) (C) E $(X^2) - [E(X)]^2$ (D) $\sqrt{E(X^2) - [E(X)]^2}$
Pr.16	Three events A, B and C are said to be independent if P (A□B□C) =P (A) P (B) P (C).
Pr.17	A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn oneby one without replacement. What is the probability that at least one of the threemarbles drawn be black, if the first marble is red?
Pr.18	The probability that at least one of the two events A and B occurs is 0.6. If A andB occur simultaneously with probability 0.3, evaluate P(A) + P(B).

Chapter Two

Pr.19	Refer to Exercise 1 above. If the die were fair, determine whether or not the events A and B are independent
Pr.20	For a loaded die, the probabilities of outcomes are given as under:P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3. The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or notA and B are independent.
Pr.21	Two dice are thrown together and the total score is noted. The events E, F and G are 'a total of 4', 'a total of 9 or more', and 'a total divisible by 5', respectively.Calculate P(E), P(F) and P(G) and decide which pairs of events, if any, are independent.
Pr.22	Explain why the experiment of tossing a coin three times is said to have binomialdistribution.1
Pr.23	If X is the number of tails in three tosses of a coin, determine the standarddeviation of X.

Pr.24	In a dice game, a player pays a stake of Re1 for each throw of a die. She receives Rs 5 if the die shows a 3, Rs 2 if the die shows a 1 or 6, and nothing
Pr.25	Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.
Pr.26	Suppose 10,000 tickets are sold in a lottery each for Re 1. First prize is ofRs 3000 and the second prize is of Rs. 2000. There are three third prizes of Rs.500 each. If you buy one ticket, what is your expectation?
Pr.27	A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ballis drawn at random from the second bag. Find the probability that the ball drawnis white.
Pr.28	Bag I contains 3 black and 2 white balls, Bag II contains 2 black and 4 white balls. A bag and a ball is selected at random. Determine the probability of selectinga black ball.
Pr.29	A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue?
Pr.30	A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.
Pr.31	Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all the four cards are kings?
Pr.32	Ten coins are tossed. What is the probability of getting at least 8 heads?
Pr.33	The probability of a man hitting a target is 0.25. He shoots 7 times. What is the probability of his hitting at least twice?
Pr.34	A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?
Pr.35	A die is thrown three times. Let X be 'the number of twos seen'. Find the expectation of X.
Pr.36	Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed. If you take one out, toss it and get a head,what is the probability that it was a fair coin?
Pr.37	Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?
Pr.38	Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the

	distribution.
Pr.39	The random variable X can take only the values 0, 1, 2. Given that P(X = 0) = P (X = 1) = *p* and that $E(X^2) = E[X]$, find the value of *p*.
Pr.40	A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

Chapter Three

Pr.41	A college president wants to find out whichcourses are popular with students. What procedure would be most appropriate for obtaining an unbiased sample of students? A. Survey a random sample of students fromthe English department. B. Survey the first hundred students from analphabetical listing. C. Survey a random sample of students from alist of the entire student body. Have students voluntarily mail in theirpreference.
Pr.42	Describe the sample space and all 16 events for a trial in which two coins are thrown and each showseither a *head* or a *tail*.
Pr.43	A fair coin is tossed, and a fair die is thrown. Write down sample spaces for (a) the toss of the coin; (b) the throw of the die; (c) the combination of these experiments. Let A be the event that a head is tossed, and B be the event that an odd number is thrown. Directlyfrom the sample space, calculate $P(A \cap B)$ and $P(A \cup B)$.
Pr.44	A bag contains fifteen balls distinguishable only by their colours; ten are blue and five are red. I reachinto the bag with both hands and pull out two balls (one with each hand) and record their colours. (a) What is the *random phenomenon*? (b) What is the *sample space*? (c) Express the *event* that the ball in my left hand is red as a subset of the sample space.

Pr.45	M&M sweets are of varying colours and the different colours occur in different proportions. The table below gives the probability that a randomly chosen M&M has each colour, but the value for tan candies is missing. Colour: Brown, Red, Yellow, Green, Orange, Tan Probability: 0.3, 0.2, 0.2, 0.1, 0.1, ? (a) What value must the missing probability be? (b) You draw an M&M at random from a packet. What is the probability of each of the following events? i. You get a brown one or a red one. ii. You don't get a yellow one. iii. You don't get either an orange one or a tan one. iv. You get one that is brown or red or yellow or green or orange or tan.
Pr.46	You consult Joe the bookie as to the form in the 2.30 at Ayr. He tells you that, of 16 runners, the favourite has probability 0.3 of winning, two other horses each have probability 0.20 of winning, and the remainder each have probability 0.05 of winning, excepting Desert Pansy, which has a worse than no chance of winning. What do you think of Joe's advice?
Pr.47	Suppose that for three dice of the standard type all 216 outcomes of a throw are equally likely. Denote the scores obtained by X_1, X_2 and X_3. By counting outcomes in the events find (a) $P(X_1 + X_2 + X_3 \leq 5)$; (b) $P(\min(X_1, X_2, X_3) \geq i)$ for $i = 1, 2, \ldots 6$; (c) $P(X_1 + X_2 < (X_3)^2)$.
Pr.48	You play draughts against an opponent who is your equal. Which of the following is more likely: (a) winning three games out of four or winning five out of eight; (b) winning at least three out of four or at least five out of eight?
Pr.49	A lucky dip at a school fête contains 100 packages of which 40 contain tickets for prizes. Let X denote the number of prizes you win when you draw out three of the packages. Find the probability density of X i.e. $P(X = i)$ for each appropriate i.
Pr.50	Two sisters maintain that they can communicate telepathically. To test this assertion, you place the sisters in separate rooms and show sister A a series of cards. Each card is equally likely to depict either a circle or a star or a square. For each card presented to sister A, sister B writes down 'circle', or 'star' or 'square', depending on what she believes sister A to be looking at. If ten cards are shown, what is the probability that sister B correctly matches at least one?

Pr.51	An examination consists of multiple-choice questions, each having five possible answers. Suppose you are a student taking the exam. and that you reckon you have probability 0.75 of knowing the answer to any question that may be asked and that, if you do not know, you intend to guess an answer with probability 1/5 of being correct. What is the probability you will give the correct answer to a question?
Pr.52	I have in my pocket ten coins. Nine of them are ordinary coins with equal chances of coming up head and tail when tossed and the tenth has two heads. (a) If I take one of the coins at random from my pocket, what is the probability that it is the coinwith two heads ? (b) If I toss the coin and it comes up heads, what is the probability that it is the coin with two heads? (c) If I toss the coin one further time and it comes up tails, what is the probability that it is one ofthe nine ordinary coins ?
Pr.53	A certain person considers that he can drink and drive: usually he believes he has a negligible chance of being involved in an accident, whereas he believes that if he drinks two pints of beer, his chance of being involved in an accident on the way home is only one in five hundred. Assuming that he drives home from the same pub every night, having drunk two pints of beer, what is the chance that he is involved in at least one accident in one year? Are there any assumptions that you make in answering the question?
Pr.54	An urn contains r red balls and b blue balls, $r \geq 1$, $b \geq 3$. Three balls are selected, without replacement, from the urn. Using the notion of conditional probability to simplify the problem, find the probability of the sequence Blue, Red, Blue.
Pr.55	Three babies are given a weekly health check at a clinic, and then returned randomly to their mothers.What is the probability that at least one baby goes to the right mother?
Pr.56	In a certain town, 30% of the people are Conservatives; 50% Socialists; and 20% Liberals. In this town at the last election, 65% of Conservatives voted, as did 82% of the Socialists and 50% of the Liberals.A person from the town is selected at random, and states that she voted at the last election. What is the probability that she is a Socialist?
Pr.57	A six-sided die has four green and two red faces and is balanced so that each face is equally likely to come up. The die will be rolled several times. Suppose that we score 4 if the die is rolled and comes up green, and 1 if it comes up red. Define the random variable X to be this score. Write down the distribution of probability for X and calculate the expectation and variance for X.

Pr.58	For two standard dice all 36 outcomes of a throw are equally likely. Find $P(X_1 + X_2 = j)$ for all j and calculate $E(X_1 + X_2)$. Confirm that $E(X_1) + E(X_2) = E(X_1 + X_2)$.
Pr.59	X takes values 1, 2, 3, 4 each with probability 1/4 and Y takes values 1, 2, 4, 8 with probabilities 1/2, 1/4, 1/8 and 1/8 respectively. Write out a table of probabilities for the 16 paired outcomes which is consistent with the distributions of X and Y. From this find the possible values and matching probabilities for the total $X + Y$ and confirm that $E(X + Y) = E(X) + E(Y)$.
Pr.60	Calculation practice for the binomial distribution. Find $P(X = 2)$, $P(X < 2)$, $P(X > 2)$ when (a) $n = 4$, $p = 0.2$; (b) $n = 8$, $p = 0.1$; (d) $n = 64$, $p = 0.0125$. (c) $n = 16$, $p = 0.05$;

Chapter Four

Pr.61	A wholesaler supplies products to 10 retail stores, each of which will independently make an order on a given day with chance 0.35. What is the probability of getting exactly 2 orders? Find the most probable number of orders per day and the probability of this number of orders. Find the expected number of orders per day.
Pr.62	A machine produces items of which 1% at random are defective. How many items can be packed in a box while keeping the chance of one or more defectives in the box to be no more than 0.5? What are the expected value and standard deviation of the number of defectives in a box of that size?
Pr.63	Suppose that 0.3% of bolts made by a machine are defective, the defectives occurring at random during production. If the bolts are packaged in boxes of 100, what is the Poisson approximation that a given box will contain x defectives? Suppose you buy 8 boxes of bolts. What is the distribution of the number of boxes with no defective bolts? What is the expected number of boxes with no defective bolts?
Pr.64	Events which occur randomly at rate r are counted over a time period of length s so the event count X is Poisson. Find $P(X = 2)$, $P(X < 2)$ and $P(X > 2)$ when (a) $r = 0.8$, $s = 1$; (b) $r = 0.1$, $s = 8$; (c) $r = 0.01$, $s = 200$; (d) $r = 0.05$, $s = 200$
Pr.65	Given that 0.04% of vehicles break down when driving through a certain tunnel find the probability of (a) no (b) at least two breakdowns in an hour when 2,000 vehicles enter the tunnel.

Pr.66	Experiments by Rutherford and Geiger in 1910 showed that the number of alpha particles emitted per unit time in a radioactive process is a random variable having a Poisson distribution. Let X denote the count over one second and suppose it has mean 5. What is the probability of observing fewer than two particles during any given second? What is the $P(X \geq 10)$? Let Y denote the count over a separate period of 1.5 seconds. What is $P(Y \geq 10)$? What is $P(X + Y \geq 10)$?
Pr.67	A process for putting chocolate chips into cookies is random and the number of choc chips in a cookie has a Poisson distribution with mean λ. Find an expression for the probability that a cookie contains less than 3 choc chips.
Pr.68	Let X have the density $f(x) = 2x$ if $0 \leq x \leq 1$ and $f(x) = 0$ otherwise. Show that X has the mean 2/3 and the variance 1/18. Find the mean and the variance of the random variable $Y = -2X + 3$.
Pr.69	Let the random variable X have the density $f(x) = kx$ if $0 \leq x \leq 3$. Find k. Find x_1 and x_2 such that $P(X \leq x_1) = 0.1$ and $P(X \leq x_2) = 0.95$. Find $P(\lvert X - 1.8 \rvert < 0.6)$.
Pr.70	A small petrol station is supplied with petrol once a week. Assume that its volume X of potential sales (in units of 10, 000 litres) has the probability density function $f(x) = 6(x - 2)(3 - x)$ for $2 \leq x \leq 3$ and $f(x) = 0$ otherwise. Determine the mean and the variance of this distribution. What capacity must the tank have for the probability that the tank will be emptied in a given week to be 5%?
Pr.71	Find the probability that none of the three bulbs in a set of traffic lights will have to be replaced during the first 1200 hours of operation if the lifetime X of a bulb (in thousands of hours) is a random variable with probability density function $f(x) = 6[0.25 - (x - 1.5)^2]$ when $1 \leq x \leq 2$ and $f(x) = 0$ otherwise. You should assume that the lifetimes of different bulbs are independent.
Pr.72	Suppose X is N(10, 1). Find (i) $P[X > 10.5]$, (ii) $P[9.5 < X < 11]$, (iii) x such that $P[X < x] = 0.95$. *You will need to use Standard Normal tables.*
Pr.73	The height of a randomly selected man from a population is normal with μ = 178cm and σ = 8cm. What proportion of men from this population are over 185cm tall? There are 2.54cm to an inch. What is their height distribution in inches? The heights of the women in this population are normal with μ = 165 cm and σ = 7cm. What proportion of the women are taller than half of the men?

Pr.74	*N* independent trials are to be conducted, each with "success" probability *p*. Let $X_i = 1$ if trial *i* is a success and $X_i = 0$ if it is not. What is the distribution of the random variable $X = X_1+X_2+...+X_N$? Express $P[a \le X \le b]$ as a sum (where $a \le b$ and these are integers between 0 and *N*). Use the central limit theorem to provide an approximation to this probability. Compare your approximation with the limit theorem of De Moivre and Laplace on p1189 of Kreyszig.
Pr.75	There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers ontwo cards drawn. Find the mean and variance of X
Pr.76	An airfreight company has various classes of freight. In one of these classes the average weight of packages is 10kg and the variance of the weight distribution is $9kg^2$. Assuming that the package weights are independent (it is not the case that a single company is sending a large number of identical packages, for instance), estimate the probability that 100 packages will have total weight more than 1020kg.
Pr.77	In one sample of 8 observations the sum of squares of deviations of the sample values from the sample mean was 84.4 and another sample of 10 observations it was 102.6 .test whether there is any significant difference between two sample variances at at 5% level of significance
Pr.78	The means of two random samples of sizes 9,7 are 196.42 and 198.82.the sum of squares of deviations from their respective means are 26.94,18.73.can the samples be considered to have been the same population?
Pr.79	Producer of 'gutkha' claims that the nicotine content in his 'gutkha' on the average is 83 mg. can this claim be accepted if a random sample of 8 'gutkhas' of this type have the nicotine contents of 2.0,1.7,2.1,1.9,2.2,2.1,2.0,1.6 mg.
Pr.80	A factory produces components of which 10% are defective. The components are packed in boxes of 10. A box selected at random. i) Find the probability that the box contains exactly one defective component. ii) Find the probability that there are at most two defective components in the box. iii) Find the probability that there are more than 1 but at most 5 defective components in the box. Estimate the mean value and standard deviation of the binominal distribution