

Chapter Four

Application in Matlab

1) Solution of linear systems

i) Back slash (inverse) method

Example 1:- Solve the following matrix equation (the system) in Matlab by using back slash (inverse) method

$$5x - 3y - 10 - 2z = 0$$

$$4y = -8y + 3x + 20$$

$$2x + 4y - 4z = 9$$

Solution: - 1) rearrange the equation

$$5x - 3y - 2z = 10$$

$$-3x + 8y + 4z = 20$$

$$2x + 4y - 9z = 9$$

2) Write the equation in matrix, $Ax=B$:-

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

3) Solve the matrix equation in Matlab by using back slash (\) or inverse (*inv*) method.

```

Command Window
>> A=[5 -3 2; -3 2 4 ; 2 4 -9];
>> B=[10;20;9];
>> x=inv(A)*B

x =

    5.9586
    9.6828
    4.6276

```

ii) Gauss – Elimination Method (*rref*) :-

Example 2:- Solve the following system using Gauss - Elimination Method

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 8 & 4 \\ 2 & 4 & -9 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 10 \\ 20 \\ 9 \end{bmatrix}$$

```

>> A=[5 -3 2; -3 8 4;2 4 -9];
>> B=[10;20;9];
>> c=rref([A,B])

c =

    1.0000         0         0     3.4442
         0     1.0000         0     3.1982
         0         0     1.0000     1.1868

```

iii)Gramer Method

Example: - Solve the following system using Gramer method

$$5x - 3y - 2z = 10$$

$$-3x + 8y + 4z = 20$$

$$2x + 4y - 9z = 9$$

Solution

Command Window

```
>> A=[5 -3 2; -3 2 4 ; 2 4 -9];  
>> B=[10;20;9];  
>> dA=det(A);  
>> A1=A;  
>> A1(:,1)=B;  
>> dA1=det(A1);  
>> A2=A;  
>> A2(:,2)=B;  
>> dA2=det(A2);  
>> A3=A;  
>> A3(:,3)=B;  
>> dA3=det(A3);
```

```
>> x=dA1/dA
```

```
x =
```

```
5.9586
```

```
>> y=dA2/dA
```

```
y =
```

```
9.6828
```

```
>> z=dA3/dA
```

```
z =
```

```
4.6276
```

2) Eigen Value & Eigen Vectors

A) Eigen Value

$$\text{Let } A = \begin{bmatrix} 9 & 4 & 4 \\ 2 & 8 & 0 \\ 6 & 7 & 8 \end{bmatrix}$$

```
>> A=[9 4 4;2 8 0; 6 7 8];
```

```
>> E=eig(A)
```

```
E =
```

```
14.8574
```

```
4.4303
```

```
5.7123
```

B) Eigen Vectors

```
>> [V,E]=eig(A)
```

```
V =
```

```
    0.6375    0.7778    0.7523  
    0.1859   -0.4358   -0.6577  
    0.7476   -0.4528    0.0394
```

```
E =
```

```
   14.8574         0         0  
         0    4.4303         0  
         0         0    5.7123
```

2) Polynomials**1) Roots(p)**

Example: - $p(x) = x^3 - 6x^2 + 11x - 6$

```
>> p=[1 -6 11 -6];
>> E=roots(p)

E =

    3.0000
    2.0000
    1.0000

>> p=poly(E)

p =

    1.0000   -6.0000   11.0000   -6.0000

>> polyval(p,3)
```

```
ans =
```

```
-3.5527e-15
```

```
>> q=[1 0 -1];
>> p=[1 -6 11 -6];
>> s=conv(p,q)
```

```
s =
```

```
    1    -6    10     0   -11     6
```

```
>> [s v]=deconv(p,q)
```

```
s =
```

```
    1    -6
```

```
v =
```

```
    0     0    12   -12
```

3) Derivative of polynomial

To find the derivative of P

i) $\text{polyder}(p) = p'$ return to the derivative of P

Ex:- $p(x) = x^4 + 5x^3 - 2x + 7$

$$p = [1 \ 5 \ 0 \ -2 \ 7]$$

$$d = p' = \text{polyder}(p)$$

```
>> p=[1 5 0 -2 7];
>> polyder(p)

ans =

     4     15     0     -2
```

ii) To find the derivative of product of two polys, $p(x), q(x)$

$$C = \text{polyder}(p, q); \quad (p \cdot q)'$$

$$d = \text{polyder}(\text{compose}(p, q)); \quad (p \circ q)'$$

$\text{Ployint}(p)$ Integral of polynomial

$$w = \text{polyder}(\text{polyder}(p)); \quad p''$$

Example:- $p(x) = x^3 + 5x - 3, \quad q(x) = x^2 + 6x + 9$

Command Window

```
>> syms x
>> p = x^3 + 5*x - 3; q = x^2 + 6*x + 9;
>> p1=[1 0 5 -3];q1=[1 6 9];
>> C=polyder(p1,q1)
```

```
C =
```

```
    5    24    42    54    27
```

```
>> polyint(p1)
```

```
ans =
```

```
    0.2500         0    2.5000   -3.0000         0
```

```
>> polyder(polyder(p1))
```

```
ans =
```

```
    6     0
```


4) Symbolic Calculation

Creating symbolic Expression

Variable x, y, z, a, b, c, \dots can be declared as symbolic variables with command

```
>> syms x y z a b c  
>> A=[a b 1; 0 1 c ;x 0 0]
```

```
A =  
  
[ a, b, 1]  
[ 0, 1, c]  
[ x, 0, 0]  
  
>> d=det(A)  
  
d =  
  
b*c*x - x
```

5) Differentiation in Matlab

First create a symbolic expression the command $\text{diff}(f)$ the differentiation of f with respect to x or $\text{diff}(f, x)$

Example:- find the Differentiation of function $f(x) = 7x^4 - 5x^3 + 2x^2 - 10x + 11$

```

>> syms x
>> f=7*x^4-5*x^3+2*x^2-10*x+11;
>> df=diff(f)

df =

28*x^3 - 15*x^2 + 4*x - 10

>> d2f=diff(f,2)

d2f =

84*x^2 - 30*x + 4

>> f=12;df=diff(f)

df =

fx      []

```

6) Partial derivative

Example :- $f(x) = x^2y + 3y^2 + x$

$$1-\frac{\partial f}{\partial x} = f_x$$

$$2-\frac{\partial^2 f}{\partial x^2} = f_{xx}$$

$$3-\frac{\partial f}{\partial y} = f_y$$

$$4-\frac{\partial^2 f}{\partial y^2} = f_{yy}$$

$$5-\frac{\partial}{\partial x} \frac{\partial f}{\partial y} = f_{xy}$$

$$6-\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = f_{yx}$$

```

>> syms x y
>> f=x^2+3*y^2+x;
>> dfx=diff(f,x)

dfx =

2*x + 1

>> dfy=diff(f,y)

dfy =

6*y

>> df2x=diff(f,x,2)

df2x =

2

>> df2y=diff(f,y,2)

df2y =

6

```

```

>> dyxf=diff(diff(f,x),y)

dyxf =

0

>> dxyf=diff(diff(f,y),x)

dxyf =

0

```

8- limits

Limit of an expression:-

1) $\text{limit}(f, x, a)$ take the limit of the symbolic expression f as $x \rightarrow a$ ie $\lim_{x \rightarrow 0} f(x)$

2) $\text{limit}(f)$ uses $a = 0$ ie $\lim_{x \rightarrow 0} f(x)$

3) $\text{limit}(f, x, a, 'left')$ ie $\lim_{x \rightarrow a^-} f(x)$

$\text{limit}(f, x, a, 'right')$ ie $\lim_{x \rightarrow a^+} f(x)$

Example 1:-

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

```
>> syms x
>> f=sin(x)/x;
>> limit(f)
```

```
ans =
```

```
1
```

Example 2:-

$$\lim_{x \rightarrow 0} \frac{5}{x}$$

```
>> syms x
>> f=x/5;
>> y1=limit(f,x,0,'right')
```

```
y1 =
```

```
0
```

```
>> y2=limit(f,x,0,'left')
```

```
y2 =
```

```
0
```

9- integration

The symbolic of integration is $int(f)$

We have two type of integration 1- indefinite integral 2- definite intgral

1- indefinite integral

i- $int(f)$

ii- $int(f, x)$

2- definite intgral

i- $\text{int}(f, a, b)$

ii- $\text{int}(f, x, a, b)$

example 1:- $\int \frac{dx}{\sqrt{x}}$ 2- $\int_0^1 x \log(x + 5) dx$

```
>> syms x
>> f=1/sqrt(x);
>> int(f)

ans =

2*x^(1/2)

>> syms x
>> f=x*log(x+5);
>> int(f,0,1)

ans =

(25*log(5))/2 - 12*log(6) + 9/4
```

10- Pretty

Pretty(f) : print the symbolic expression f in type set method

Example :- $f(x) = \frac{\log(x^2+7)}{x^4-5x^2+1}$

Command Window

```
>> syms x
>> f(x)=(log(x^2+7))/(x^4-5*x^2+1);
>> pretty(f)
```

$$\frac{\log(x^2 + 7)}{x^4 - 5x^2 + 1}$$

fx >> |

11-simplify

Example 1:- $r\sin^2\theta + r\cos^2\theta = r$

Command Window

```
>> syms r x
>> f=r*sin(x)^2+r*cos(x)^2;
>> simplify(f)
```

ans =

r

Example 2:-

Jacobian matrix of f is

$$x = r \cos \varphi;$$

$$y = r \sin \varphi.$$

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -r \sin \varphi \\ \sin \varphi & r \cos \varphi \end{bmatrix}$$

```
Command Window
>> syms r u
>> x=r*cos(u);
>> y=r*sin(u);
>> A=jacobian([x y]);
>> B=det(A)

B =

r*cos(u)^2 + r*sin(u)^2
```

12- factor

factor(n) return a vector containing the prime factor of n

Example 1:- factor of (27)

```
Command Window
>> factor(27)

ans =

     3     3     3
```

Example 2:- factor of (x^2-1)

```
>> syms x
>> f=x^2-1;
>> factor(f)

ans =

[ x - 1, x + 1]
```

12- Collect

$\text{Collect}(f, x)$ regard each element of the symbolic f as a polynomial in x and rewrite f in terms of the power of x

Example:- $1 - f(x) = (x + 1)(x - 1)(x^2 + 1)$

$$2 - f(x) = x^2y + yx - x^2 - 2x$$

Command Window

```
>> syms x
>> f=(x+1)*(x-1)*(x^2+1);
>> collect(f)

ans =

x^4 - 1

>> syms x y
>> f=x^2*y+y*x-x^2-2*x;
>> collect(f)

ans =

(y - 1)*x^2 + (y - 2)*x
```


13- Expand

Expand (f): –distributes product over summation

Command Window

```
>> % Example 1:- f=a*(b+c)
>> syms a b c
>> f=a*(b+c);
>> expand(f)

ans =

a*b + a*c

>> % Example 2:- f=cos(x+y)
>> syms x y
>> f=cos(x+y);
>> expand(f)

ans =

cos(x)*cos(y) - sin(x)*sin(y)
```

14- substitution

Subs(f, variable, point)

Subs(f, [x, y, z], [x₀, y₀, z₀])

Command Window

```
>> % Example:- f(x,y,z)=x*y^2-z*x*y+z^2*x where x0=1, y0=3, z0=7
>> syms x y z
>> f=x*y^2-z*x*y+z^2*x;
>> subs(f, [x, y, z], [1, 3, 7])

ans =

37
```

15- Sum Σ

$Symsum(s, x)$ is define summation of S with respect to the symbolic variable determined by $syms$

$$Symsum(s, x) \rightarrow \sum_{x=0}^{\infty} x$$

$$Symsum(s, a, b) \rightarrow \sum_a^b x$$

Command Window

```
>> syms x
>> symsum(x, 0, 10)

ans =

55

>> syms x
>> symsum(x, 0, inf)

ans =

Inf

>> syms x n
>> symsum(x^2, n, 0, inf)

ans =
|
piecewise(x == 0, 0, x ~= 0, Inf*x^2)

>> symsum(x^n, n, 0, inf)

ans =

piecewise(1 <= x, Inf, abs(x) < 1, -1/(x - 1))
```

16- Taylor Expression

1 – $Taylor(f)$ find the fifth order maclorain polu appr. Of f

Example:- $f(x) = e^x$

Command Window

```
>> syms x
>> f=exp(x);
>> taylor(f)

ans =

x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

>> taylor(cos(x))

ans =

x^4/24 - x^2/2 + 1
```

17- Compose

$compose(f, g)$ return $g \circ f(x)$ or $g(f(x))$ where $f = f(x)$ and $g = g(x)$, then x is symbolic variable of f as define by syms and y is symbolic variable of f as define by syms.

```

Command Window
>> syms x y z
>> f=(1/x^2+5);
>> g=cos(y);
>> compose(f,g)

ans =

1/cos(y)^2 + 5

>> compose(f,g,z)

ans =

1/cos(z)^2 + 5

```

Home Work Section

H.W 1: Find the following results by using commands windows.

$$f(x) = x^4 + 4x^3 + 5x + 3, \quad g(x) = x^3 + 6x^2 + 3x + 1$$

$$h(x, y) = x^7y^3 + 2y^2 + e^{yx}$$

- 1- Derivative of product of two polys f & g .
- 2- The roots of product of two polys f & g .
- 3- Differentiation of h with respect to y by using symbolic calculation.
- 4- $\int_2^5 e^x dx$
- 5- If $f(x) = x^4 + 4x^3 + 5x + 3$ then find f at point $x = 4$.

H.W 2:- Find the following results by using commands windows.

- 1- $\lim_{x \rightarrow -2} x^3$.
- 2- Eigen value & Eigen vector of the matrix $H = \begin{bmatrix} 4 & 6 & 3 \\ 0 & -8 & 9 \\ -5 & 7 & 1 \end{bmatrix}$
- 3- If $f = x^3 + x^2 + x$ find $f(3)$ by using direct definition of the function
- 4- Find the factor of 95.

H.W 3:- Find the following results by using commands windows.

$$f = (e^{2x} \cos(y^2) z^2), \quad w = (5x^2 - 1), \quad h(x) = x^4 + 4x^3 + 5x + 3$$

- 1- $E = \sum_{y=7}^{11} \sum_{z=0}^5 (f)$,find the value of E at $x = 2$ by using symbolic of summation in one statement .
- 2- $\int_4^5 \int_1^2 x^2 \cos(y) dx dy$.
- 3- Differentiation of f with respect to y by using symbolic calculation.
- 4- Find the roots of product of two polys h, w .
- 5- Find the value of h at point $x = 4$.

H.W 4: Find the following results by using commands windows.

$$f(x) = \frac{x^3 + 3x^2 + 3x}{xe^{2x}}, \quad g(x, y, z) = (xyz)^3 + e^{xz} \sin(xy)$$

$$h = 1555$$

- 1- Pretty of f & g .
- 2- Differentiation of g with respect to y by using symbolic calculation.
- 3- Simplify f
- 4- Find g at point $x = 4$ & $y = 2, z = 5$.
- 6- Factor of h .