

Q1/ Find  $\frac{dy}{dx}$  for the following functions :

1)  $y = (x - 3)(1 - x)$

(ans.:  $4 - 2x$ )

2)  $y = \frac{ax + b}{x}$

(ans.:  $-\frac{b}{x^2}$ )

3)  $y = \frac{3x + 4}{2x + 3}$

(ans.:  $\frac{1}{(2x + 3)^2}$ )

4)  $y = 3x^3 - 2\sqrt{x} + \frac{5}{x^2}$

(ans.:  $9x^2 - \frac{1}{\sqrt{x}} - \frac{10}{x^3}$ )

5)  $y = \left( \sqrt{x^3} - \frac{1}{\sqrt{x^3}} \right)^2$

(ans.:  $\frac{3(x^6 - 1)}{x^4}$ )

6)  $y = (2x - 1)^2(3x + 2)^3 + \frac{1}{(x - 2)^2}$

(ans.:  $(2x - 1)(3x + 2)^2(30x - 1) - \frac{2}{(x - 2)^3}$ )

7)  $y = \ln(\ln x)$

(ans.:  $\frac{1}{x \ln x}$ )

8)  $y = \ln(\cos x)$

(ans.:  $-\tan x$ )

9)  $y = \sin x^3$

(ans.:  $3x^2 \cdot \cos x^3$ )

10)  $y = \cos^{-3}(5x^2 + 2)$

(ans.:  $\frac{30x \cdot \sin(5x^2 + 4)}{\cos^4(5x^2 + 4)}$ )

11)  $y = \sin^{-1}(5x^2)$

(ans.:  $\frac{10x}{\sqrt{1 - 25x^4}}$ )

12)  $y = \tan^{-1}(\ln x)$

(ans.:  $\frac{1}{x(1 + (\ln x)^2)}$ )

13)  $y^{\frac{4}{3}} = \frac{\sqrt{\sin x \cdot \cos x}}{1 + 2 \ln x}$

(ans.:  $\frac{3y}{4} \left( \frac{\cot x}{2} - \frac{\tan x}{2} - \frac{2}{x(1 + 2 \ln x)} \right)$ )

14)  $\sqrt{y} = \frac{x^5 \cdot \tan^{-1} x}{(3 - 2x) \cdot \sqrt[3]{x}}$

(ans.:  $2y \left( \frac{14}{3x} + \frac{1}{(1 + x^2) \cdot \tan^{-1} x} + \frac{2}{3 - 2x} \right)$ )

15)  $y = \sec^{-1} e^{2x}$

(ans.:  $\frac{2}{\sqrt{e^{4x} - 1}}$ )

16)  $y = (\cos x)^{\sqrt{x}}$

(ans.:  $\frac{y}{2\sqrt{x}} (\ln \cos x - 2x \cdot \tan x)$ )

17)  $\sqrt{xy} + 1 = y$

Q2. Show for  $y = u \cdot v$  that  $y''' = uv''' + 3u'v'' + 3u''v' + u'''v$ .

Q3/ Prove that:  $\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

**Q4/ Find the limits for the following functions by using L'Hopital's rule**

- |  |   |
|--|---|
| 1) $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x}{7x^2 + 1}$            | 2) $\lim_{t \rightarrow 0} \frac{\sin t^2}{t}$                                    |
| 3) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{2x - \pi}{\cos x}$        | 4) $\lim_{t \rightarrow 0} \frac{\cos t - 1}{t^2}$                                |
| 5) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$ | 6) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$ |
| 7) $\lim_{x \rightarrow 1} \frac{2x^2 - (3x + 1)\sqrt{x} + 2}{x - 1}$  | 8) $\lim_{x \rightarrow 0} \frac{x(\cos x - 1)}{\sin x - x}$                      |
| 9) $\lim_{x \rightarrow 0} x \cdot \csc^2 \sqrt{2x}$                   | 10) $\lim_{x \rightarrow 0} \frac{\sin x^2}{x \cdot \sin x}$                      |

**Q5/ Find any local maximum and local minimum values , then sketch each curve by using first derivative :**

- |   |                                 |
|---|---------------------------------|
| 1) $f(x) = x^3 - 4x^2 + 4x + 5$               | (ans.: max.(0.7,6.2);min.(2,5)) |
| 2) $f(x) = \frac{x^2 - 1}{x^2 + 1}$           | (ans.: min.(0,-1))              |
| 3) $f(x) = x^5 - 5x - 6$                      | (ans.: max.(-1,-2);min.(1,-10)) |
| 4) $f(x) = x^{\frac{4}{3}} - x^{\frac{1}{3}}$ | (ans.: min.(0.25,-0.47))        |

**Q6/ Evaluate the following integrals:**

- |  |   |
|--|---|
| 1) $\int (x^2 - 1) \cdot (4 - x^2) dx$         | (ans.: $\frac{5}{3}x^3 - \frac{1}{5}x^5 - 4x + c$ ) |
| 2) $\int e^x \cdot \sin e^x dx$                | (ans.: $-\cos e^x + c$ )                            |
| 3) $\int \tan(3x + 5) dx$                      | (ans.: $-\frac{1}{3} \ln \cos(3x + 5)  + c$ )       |
| 4) $\int \frac{\cot(\ln x)}{x} dx$             | (ans.: $\ln \sin(\ln x)  + c$ )                     |
| 5) $\int \frac{\sin x + \cos x}{\cos x} dx$    | (ans.: $-\ln \cos x  + x + c$ )                     |
| 6) $\int \frac{dx}{1 + \cos x}$                | (ans.: $-\cot x + \csc x + c$ )                     |
| 7) $\int \cot(2x + 1) \cdot \csc^2(2x + 1) dx$ | (ans.: $-\frac{1}{4} \cot^2(2x + 1) + c$ )          |

- 8)  $\int \frac{dx}{\sqrt{1-9x^2}}$  (ans.:  $\frac{1}{3} \sin^{-1}(3x) + c$ )
- 9)  $\int \frac{dx}{\sqrt{2-x^2}}$  (ans.:  $\sin^{-1} \frac{x}{\sqrt{2}} + c$ )
- 10)  $\int e^{2x} \cdot \cos e^{2x} dx$  (ans.:  $\frac{1}{2} \sin e^{2x} + c$ )
- 11)  $\int e^{\sin x} \cdot \cos x dx$  (ans.:  $e^{\sin x} + c$ )
- 12)  $\int \frac{dx}{e^{3x}}$  (ans.:  $-\frac{1}{3} e^{-3x} + c$ )
- 13)  $\int \frac{x^3}{x-1} dx$  (ans.:  $\frac{1}{3} x^3 - \frac{1}{2} x^2 + x + \ln(x-1) + c$ )
- 14)  $\int \frac{3x+2}{3x-1} dx$  (ans.:  $x + \ln(3x-1) + c$ )
- 15)  $\int x^2 \cdot e^{-x} dx$  (ans.:  $-e^{-x}(x^2 + 2x + 2) + c$ )
- 16)  $\int x \cdot \sin x^2 dx$  (ans.:  $-\frac{1}{2} \cos x^2 + c$ )
- 17)  $\int \sqrt{x^2-1} dx$  (ans.:  $\frac{x}{2} \sqrt{x^2-1} - \frac{1}{2} \ln|x + \sqrt{x^2+1}| + c$ )
- 18)  $\int \frac{3x+13}{(5x-1)(7x+2)} dx$  (ans.:  $\frac{4}{5} \ln|5x-1| - \frac{5}{7} \ln|7x+2| + c$ )
- 19)  $\int \frac{2x-3}{(x-1)(x-2)(x+3)} dx$  (ans.:  $\frac{1}{4} \ln|x-1| + \frac{1}{5} \ln|x-2| - \frac{9}{20} \ln|x+3| + c$ )
- 20)  $\int \frac{dx}{x^4-1}$  (ans.:  $\frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + c$ )
- 21)  $\int \ln x dx$  (ans.:  $x \cdot \ln x - x + c$ )
- 22)  $\int \tan^{-1} x dx$  (ans.:  $x \cdot \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + c$ )
- 23)  $\int x \cdot \ln x dx$  (ans.:  $\frac{x^2}{2} \ln x - \frac{x^2}{4} + c$ )
- 24)  $\int x \cdot \tan^{-1} x dx$  (ans.:  $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$ )

$$25) \int x^2 \cdot \cos ax \, dx$$

$$(\text{ans.: } \frac{x^2}{a^2} \sin ax + \frac{2x}{a^2} \cos ax - \frac{2}{a^3} \sin ax + c)$$

$$26) \int \sin(\ln x) \, dx$$

$$(\text{ans.: } \frac{x}{2} (\sin(\ln x) - \cos(\ln x)) + c)$$