Chapter Five

Nuclear Reactions

General Remarks

A *nuclear reaction* is an interaction between two particles, a fast bombarding particle, called the *projectile*, and a slower or stationary *target*. The products of the reaction may be two or more particles. For the energies considered here ($< 20\ MeV$), the products are also two particles (with the exception of fission)

If x_1, X_2 are the colliding particles and x_3, X_4 are the products, the reaction is indicated as

$$^{A_1}_{Z_1}x_1 + ^{A_2}_{Z_2}X_2 \rightarrow ^{A_3}_{Z_3}x_3 + ^{A_4}_{Z_4}X_4$$

or

$$X_2(x_1, x_3)X_4$$

The particles in parentheses are the light particles, x_1 being the projectile. Another representation for the reason is based on the light particles only, in which case the reaction shown above is indicated as an (x_1, x_3) reaction. For example, the reaction

$$^{1}_{0}n + \ ^{10}_{5}B \ \rightarrow \ ^{4}_{2}He + \ ^{7}_{3}Li$$

may be indicated as $^{10}_{5}B(n, \alpha)^{7}_{3}Li$ or simply as an (n, α) reaction.

The reaction ${}_{2}^{4}He + {}_{4}^{9}Be \rightarrow {}_{0}^{1}n + {}_{6}^{12}C$

may be indicated as ${}_{4}^{9}Be(\alpha,n){}_{6}^{12}C$ or simply as an (α,n) reaction

Certain quantities are conserved when a nuclear reaction takes place. Four are considered here. For the reaction shown above, the following quantities are conserved

Charge (Atomic number) :

$$Z_1 + Z_2 = Z_3 + Z_4$$

Mass number:

$$A_1 + A_2 = A_3 + A_4$$

Total energy:

$$E_1 + E_2 = E_3 + E_4 \text{ (rest mass plus kinetic energy)}$$

Linear momentum:

$$\boldsymbol{P}_1 + \boldsymbol{P}_2 = \boldsymbol{P}_3 + \boldsymbol{P}_4$$

Therefore for the above reaction ${}_{5}^{10}B(n,\alpha){}_{3}^{7}Li$:

Atomic number: 0+5=2+3=5

Mass number: 1+10=4+7=11

and the conservation law is correct for total energy and linear momentum in the same manner of atomic numbers and mass number.

Many nuclear reactions proceed through the formation of a *compound nucleus*. The compound nucleus, formed after particle x_1 collides with X_2, is highly excited and lives for a time of the order of 10^{-12} to 10^{-14}s before it decays to x_3 and X_4. A compound nucleus may be formed in more than one way and may decay by more than one mode that does not depend on the mode of formation. Consider the example of the compound nucleus ${}^{14}_{7}N^*$ as shown in figure 5.1

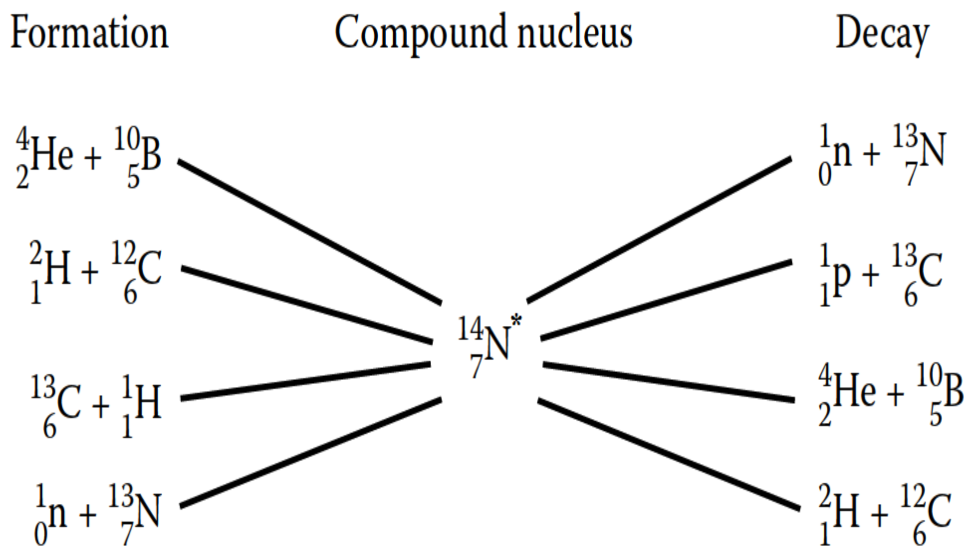


Fig. 5.1: Various ways by which the compound nucleus ${}^{14}_{7}N^*$ formed and decays

From the figure 5.1 we can write more than one way to form compound nucleus, for examples,

$${}^{4}_{2}He + {}^{10}_{5}B \rightarrow {}^{14}_{7}N^{*} \rightarrow {}^{2}_{1}H + {}^{12}_{6}C$$

can be written as ${}^{10}_{5}B(\alpha, d){}^{12}_{6}C$ and known as (α, d) reaction

$${}^{2}_{1}H + {}^{12}_{6}C \rightarrow {}^{14}_{7}N^{*} \rightarrow {}^{4}_{2}He + {}^{10}_{5}B$$

can be written as ${}^{12}_{6}C(d, \alpha){}^{10}_{5}B$ and known as (d, α) reaction

Nuclear Reaction in the Laboratory System

For bombarding energies below 100 MeV, nuclear reactions usually produce two products, i.e., they are of the type

$$a + X \rightarrow b + Y \tag{5.1}$$

where $a = \text{bombarding particle}$
$X = \text{target (at rest in the lab. system)}$
$b = \text{light reaction product}$
$Y = \text{heavy reaction product}$

Note:
-In lab. system (L-system) the target is stationary (not movable)
-In center of mass system (C-system) the target is movable

To shorten the notation a reaction of the type (5.1) is designated by

$$X(a, b)Y \tag{5.2}$$

Since the number of protons remains unchanged in a reaction, all masses can be written as *atomic* masses if electron binding-energy differences of a few eV are ignored

Conservation of energy, therefore, gives for the reaction (5.1))

$$M_a c^2 + T_a + M_X c^2 = M_b c^2 + T_b + M_Y c^2 + T_Y \qquad (5.3)$$

where T represents the (lab.) kinetic energy of each particle. The masses of a and X are ground-state masses. On the other hand, many reactions leave Y in excited states; in that case, M_Y represents the total mass energy of that state.

The Q value of the reaction is defined as the difference between the final and initial kinetic energies

$$Q = T_b + T_Y - T_a \qquad (5.4)$$

$$Q = [M_a + M_X - (M_b + M_Y)]c^2 \qquad (5.5)$$

or

$$Q = [M_a + M_X - (M_b + M_Y)]931.5\ MeV$$

If Q is positive, the reaction is said to be *exoergic*; if Q is negative, it is *endoergic*.

A reaction cannot take place unless particles b and Y emerge with positive kinetic energy, that is, $T_b + T_Y \geq 0$ or

$$Q + T_a \geq 0 \tag{5.6}$$

Although this condition is necessary, it is not sufficient.

The Q value is an important quantity in a nuclear reaction. It can be determined from mass spectroscopy [Eq. (5.5)] or by measuring kinetic energies [Eq. (5.4)]

We can show, as a result of **linear momentum conservation**, only T_b and the angle θ of b with respect to the direction of a (Fig. 5.2) need to be determined

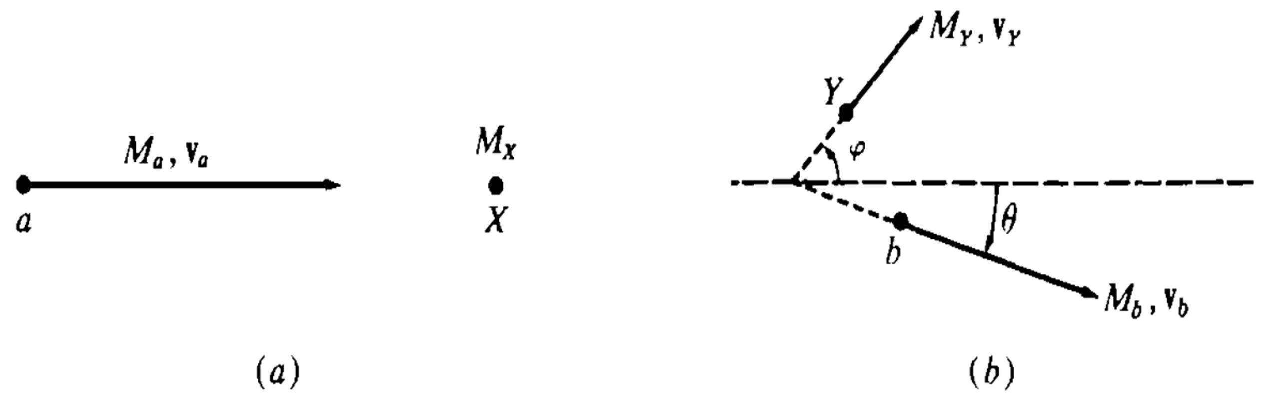
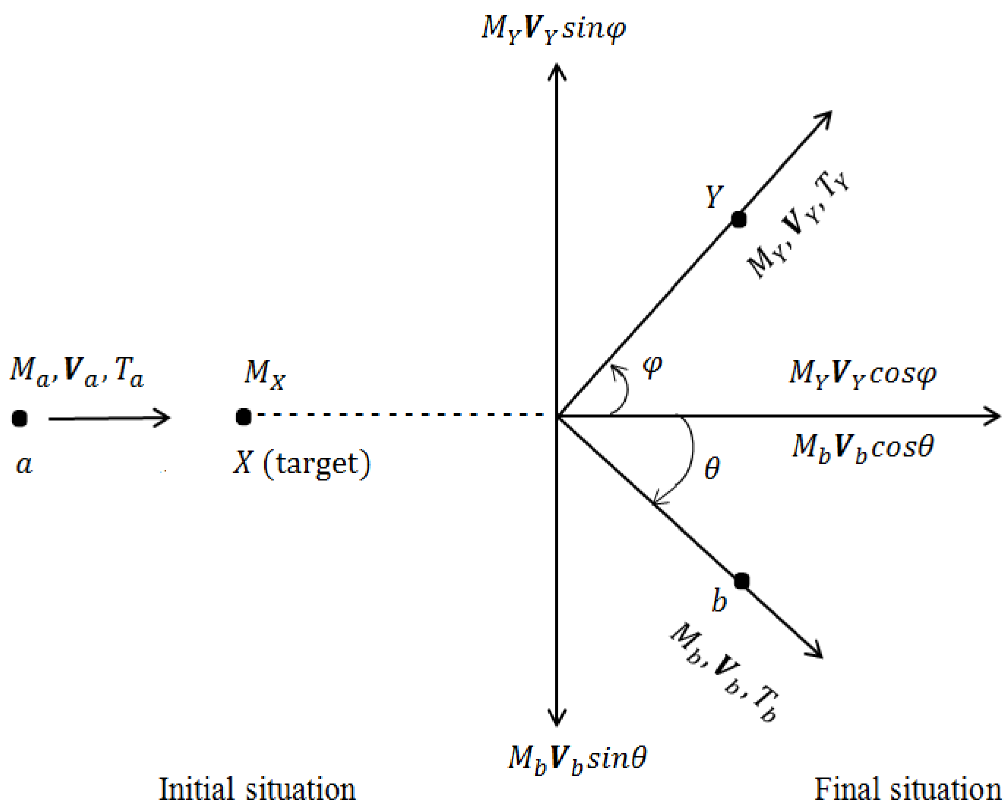


Fig. 5.2: Nuclear reaction in the lab. system. (a) Initial situation. (b) Final situation

Figure 5.2 can be plotted in terms of momentum components.



Nuclear reaction in the lab. system. (a) Initial situation. (b) Final situation in terms of momentum components

In the lab. system

$X-$ direction $$M_a v_a = M_Y v_Y cos\varphi + M_b v_b cos\theta$$

$Y-$ direction $$0 = M_Y v_Y sin\varphi - M_b v_b sin\theta \qquad (5.7)$$

rearranging Eq. (5.7) and squaring both sides and adding we obtain

$$(M_a v_a - M_b v_b cos\theta)^2 = (M_Y v_Y cos\varphi)^2$$

$$(M_b v_b sin\theta)^2 = (M_Y v_Y sin\varphi)^2$$

$$(M_a v_a)^2 - 2M_a v_a M_b v_b cos\theta + (M_b v_b cos\theta)^2 = (M_Y v_Y cos\varphi)^2$$

$$(M_b v_b sin\theta)^2 = (M_Y v_Y sin\varphi)^2$$

since $\quad sin^2\theta + cos^2\theta = 1$ and $sin^2\varphi + cos^2\varphi = 1$

therefore $\quad (M_a v_a)^2 - 2M_a v_a M_b v_b cos\theta + (M_b v_b)^2 = (M_Y v_Y)^2$

$$T = \frac{1}{2} M v^2 \implies 2MT = M^2 v^2$$

therefore

$$2M_aT_a - 2(2M_aT_a2M_bT_b\ )^{1/2}cos\theta + 2M_bT_b = 2M_YT_Y$$

divide by 2

$$M_aT_a - 2\ (M_aT_aM_bT_b\ )^{\frac{1}{2}}cos\theta + M_bT_b = M_YT_Y$$

divide by M_Y to get

$$\frac{M_a}{M_Y}\ T_a - \frac{2}{M_Y}\ (M_aT_aM_bT_b\ )^{\frac{1}{2}}cos\theta + \frac{M_b}{M_Y}T_b = T_Y$$

by adding the term $T_b - T_a$ of both sides of the equation above we obtain

$$T_b - T_a + \frac{M_a}{M_Y}\ T_a - \frac{2}{M_Y}\ (M_aT_aM_bT_b\ )^{\frac{1}{2}}cos\theta + \frac{M_b}{M_Y}T_b = T_b - T_a + T_Y$$

rearrange the equation above

$$T_b + \frac{M_b}{M_Y} T_b - T_a + \frac{M_a}{M_Y}\, T_a - \frac{2}{M_Y}\, (M_a T_a M_b T_b\ )^{\frac{1}{2}} cos\theta = \ T_b + T_Y - T_a$$

we have $Q = \ T_b + T_Y - T_a$

therefore

$$T_b + \ \frac{M_b}{M_Y} T_b - T_a + \frac{M_a}{M_Y}\, T_a - \frac{2}{M_Y}\, (M_a T_a M_b T_b\ )^{\frac{1}{2}} cos\theta = \ Q$$

therefore

$$Q \ = T_b \left(1 + \ \frac{M_b}{M_Y}\right) - T_a \left(1 - \frac{M_a}{M_Y}\ \right) - \frac{2}{M_Y}\, (M_a T_a M_b T_b\ )^{\frac{1}{2}} cos\theta \qquad (5.8)$$

This is called the Q *equation*.

If we write Eq. (5.1) for $\alpha -$ particle $({}_{2}^{4}He)$ instead of bombarding particle a then Eq. (5.1) becomes

$$ {}_{2}^{4}He + {}_{Z}^{A}X \rightarrow {}_{0}^{1}n + {}_{Z+2}^{A+3}Y $$

and for this case $M_a = M_\alpha \approx 4$; $M_b = M_n \approx 1$; $M_Y \approx A + 3$

$$ T_b = T_n \ and\ T_a = T_\alpha $$

therefore equation (5.8) becomes

$$ Q = T_n\left(1 + \frac{M_n}{M_Y}\right) - T_\alpha\left(1 - \frac{M_\alpha}{M_Y}\right) - \frac{2}{M_Y}\ (M_\alpha T_\alpha M_n T_n\ )^{\frac{1}{2}} cos\theta $$

$$ = T_n\left(1 + \frac{1}{A+3}\right) - T_\alpha\left(1 - \frac{4}{A+3}\right) - \frac{2}{A+3}\ (4T_\alpha 1T_n\ )^{\frac{1}{2}} cos\theta $$

$$ Q\ = T_n\left(\frac{A+4}{A+3}\right) - T_\alpha\left(\frac{A-1}{A+3}\right) - \frac{4}{A+3}\ (T_\alpha T_n\ )^{\frac{1}{2}} cos\theta \qquad (5.9) $$

If $\theta = 90^0$ then $cos90 = 0$, then equation (5.9) becomes

$$ Q\ = T_n\left(\frac{A+4}{A+3}\right) - T_\alpha\left(\frac{A-1}{A+3}\right) $$

therefore the kinetic energy of neutron at 90^0 is

$$T_n(90^0) = Q\left(\frac{A+3}{A+4}\right) + T_\alpha\left(\frac{A-1}{A+4}\right) \tag{5.10}$$

where A is the mass number of the target.

The smallest value of bombarding energy at which the reaction can take place is called the *threshold energy*, E_{th} .

The threshold energy is then

$$E_{th} = -Q\left(1 + \frac{M_\alpha}{M_X}\right) \tag{5.11}$$

5.1b Nuclear Reaction in the Center of Mass System

Part of the incident energy T_a is used up as *kinetic energy of the center of mass* and is not available for the nuclear reaction itself. Although we can study all the resulting effects by means of Eq. (5.8), more insight is gained if we consider the reaction in the c.m. system, shown in Fig. (5.4)

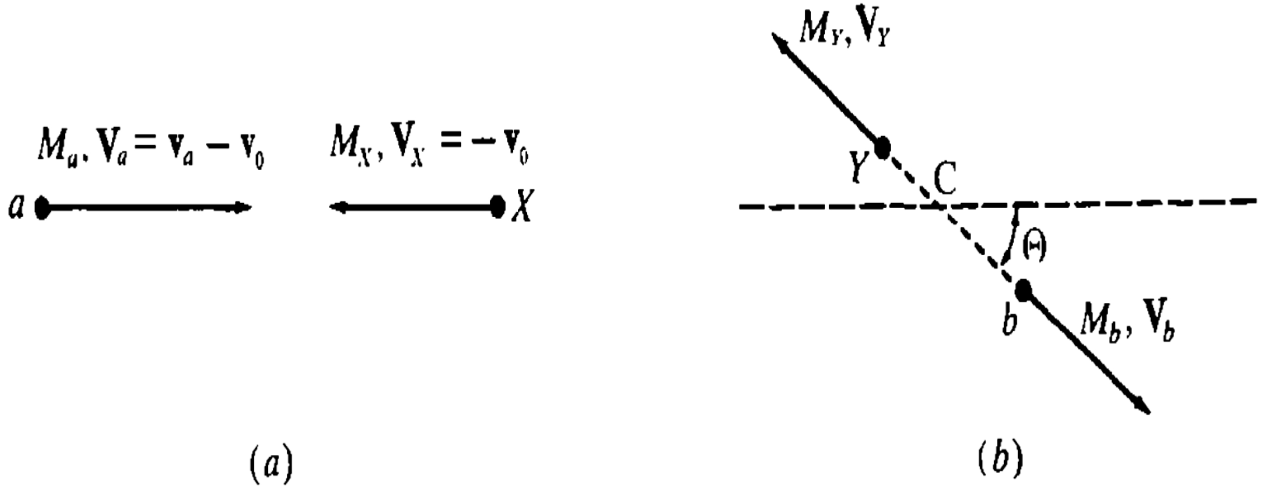


Fig. 5.4: Nuclear reaction in c.m. system. (a) Initial situation. (b) Final situation. The speed of the center of mass is $v_0 = v_a M_a/(M_a + M_X)$. Also $M_b V_b = M_Y V_Y$

$$V_a = v_a - v_0 \tag{5.12}$$

where,

V represents the speed of each particle in the c.m. system

$v_0 = v_a M_a/(M_a + M_X)$ is the speed of the center of mass

v represents the speed of each particle in the lab. system

and

$$V_X = v_X - v_0$$

since $v_X = 0$ in lab. system,

Therefore,

$$V_X = -v_0 \tag{5.13}$$

The momentum in the c.m. system before the collision

$$M_a V_a + M_X V_X = 0 \qquad (5.14)$$

The momentum in the c.m. system after the collision

$$M_b V_b - M_Y V_Y = 0 \qquad (5.15)$$

or

$$M_b V_b = M_Y V_Y$$

by substituting Eq. (5.12) and (5.13) in Eq. (5.14) we obtain

$$M_a(v_a - v_0) + M_X(-v_0) = 0$$

$$M_a v_a = M_a v_0 + M_X v_0 \qquad \Longrightarrow$$

therefore the speed of the center of mass v_0 can deduced

$$v_0 = M_a v_a / (M_a + M_X) \qquad (5.16)$$

The kinetic energy of the center of mass is

$$T_{c.m.} = \frac{1}{2}(M_a + M_X) v_0^2 \qquad (5.17)$$

The kinetic energy T_0 of the initial particles in the c. m. system can be calculated in two equivalent ways as

$$T_0 = T_a - T_{c.m.} \tag{5.18}$$

or

$$T_0 = \frac{1}{2} M_a V_a^2 + \frac{1}{2} M_X V_X^2 \tag{5.19}$$

put the $T_{c.m.}$ in equation (5.17) into equation (5.18) to get

$$\begin{aligned} T_0 &= T_a - T_{c.m.} \\ &= T_a - [\tfrac{1}{2}(M_a + M_X)v_0^2] \\ &= T_a - \frac{1}{2} M_a v_0^2 - \frac{1}{2} M_X v_0^2 \\ &= T_a - \frac{1}{2} v_0^2 (M_a + M_X) \end{aligned}$$

Substitute $v_0 = M_a v_a/(M_a + M_X)$ and we get

$$\begin{aligned} T_0 &= T_a - \tfrac{1}{2}(M_a v_a/(M_a+M_X))^2 (M_a + M_X) \\ &= T_a - \frac{1}{2}(M_a v_a)^2/(M_a+M_X) \end{aligned}$$

since $T_a = \frac{1}{2} M_a v_a^2$, then

$$\begin{aligned} T_0 &= T_a - \frac{1}{2} M_a^2 v_a^2/(M_a+M_X) \\ &= T_a - T_a M_a/(M_a+M_X) \\ &= T_a(1 - M_a/(M_a+M_X)) \end{aligned}$$

therefore

$$T_0 = \frac{M_X}{M_a + M_X} T_a \tag{5.20}$$

Where

T_0 is the initial kinetic energy in c.m. system;

T_a is the kinetic energy in lab. system.

The *energy available for the nuclear reaction* is $Q + T_0$ (5.21)

which is equal to the kinetic energy of the reaction product in the c.m. system

$$Q + T_0 = \frac{1}{2} M_b V_b^2 + \frac{1}{2} M_Y V_Y^2 \tag{5.22}$$

A necessary and sufficient condition that the reaction proceed is that the right-hand side of Eq. (5.22) be positive, i.e., $Q + T_0 \geq 0$ (5.23)

Problem (H.W).

5.1 Determine the Q-value and the threshold energy of the following reaction?

$${}_0^1n + {}_{92}^{235}U \rightarrow {}_{92}^{236}U^* \rightarrow {}_0^1n + {}_{36}^{94}Kr + {}_{56}^{141}Ba + Q$$