**The Randomized Complete Block Design (RCBD)**

Randomized complete block designs differ from the completely randomized designs in that the experimental units are grouped into blocks according to known or suspected variation which is isolated by the blocks. Variation such as fertility, sand, and wind gradients, or age and litter of animals can be isolated by appropriate blocking. Therefore, within each block, the conditions are as homogeneous as possible, but between blocks, large differences may exist

**Advantages of The Randomized Complete Block Design**

1. Generally more precise than the completely randomized design (CRD).
2. No restriction on the number of treatments or replicates.
3. Some treatments may be replicated more times than others.
4. Missing values are easily estimated
5. **Disadvantages of The Randomized Complete Block Design**
6. Error degrees of freedom is smaller than that for the CRD (problem with a small number of treatments).
7. Large variation between experimental units within a block may result in a large error term If there are missing data, a RCBD experiment may be less efficient than a CRD

NOTE: The most important item to consider when choosing a design is the uniformity of the experimental units. The Layout of the Experiment.

* **Symbolical representation of data**

|  |  |  |  |
| --- | --- | --- | --- |
| Treatments (ti) | Blocksrj | Treatment.TotalYi. | Treatment.Means y ̅. |
| t1 | y11 y12 ….y1j ….y1r | y1. | y ̅1. |
| t2 | y21 y22 ….y2j ….y2r | y2. | y ̅2. |
|  |  |  |  |
| t3 | Y31 y32 ….y3j ….y3r | y3. | y ̅3. |
| t | Yt1 yt2 ….ytj ….ytr | yi. | y ̅i. |
| Y.j | Y.1 Y.2…..Y.j….Y.r | y..total |
| y ̅.jBlock Means | ˉY.1 ˉY.2…..ˉY.j….ˉY.r |  | y ̅..overall |

1. **Linear Model**

Yij = µ + ti + bj+eij i= 1 , 2 ,.....,t

 j = 1 , 2, …,b

Yij : observation value

µ : overall mean

ti : effect of( i) treatment .

bj : effect of (j) block

eij : experiment of error assumed to be normally independent distribution (NID)with zero mean and identical variance IϬ²

1. Test of Hypotheses :

Ho : µ1 = µ2

 Ha: µ1 ≠ µ2

1. Anova Table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sources of VariationS.O.V. |  degree of freedom d.f ا |  Sum Square  S.S. | Mean SquareM.S. | F. Calculate |
| Treatments |  t-1 |  ∑Yi.² SSt = ------ - CF  b  |  SStMSt = ----- b-1 |  MSt  F = ---------  MSe  |
| Blocks |  b-1 |  ∑Y.j² SSb = ------ - CF  t  |  SSbMSt = ----- t-1 |  MSt  F = ---------  MSe  |
| Error |  (t-1)(b-1) | SSe = SST – SSt–SSb |  SSeMSe = -------- (t-1)(b-1) |  |
| Total |  tb-1 | SST = ∑Yij² - CF |  |  |

Methods for the sum of the squares of the deviations can be summarized in the following steps :

**a- correction factor**

 **(y..) ²**

CF = --------

 tb

1. **Total Sum of Squares (SST) :**

SST = ∑yij² - CF

a- Blocks Sum of Squares (SSt)

 ∑Y.j²

SSb= ------------ - CF

 t

1. **Treatments Sum of Squares (SSt)**

 ∑Yi².

SSt= ------------ - CF

 b

1. **Error Sum of Squares (SSe)**

SSe = SST –SSt– SSb

Example : an experiment was conducted to study the effect of seven levels of glutamine supplementation (0.0, 0.25, 0.50, 0.75, 0.10, 0.125 and 0.150) %. on daily body weight of lamb and the experiment was repeated in four blocks in the fields. the data as follows, find ANOVA table , discuss the result and comparison among treatments use L.S.D and Duncan?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Mean | Yi. | b4 | b3 | b2 | b1 | Treats |
| 2.03 | 8.1 | 1.9 | 2.1 | 2.3 | 1.8 | t1 |
| 2.15 | 8.6 | 2.8 | 2.1 | 1.9 | 1.8 | t2 |
| 2.33 | 9.3 | 2.4 | 2.5 | 2.3 | 2.1 | t3 |
| 2.48 | 9.9 | 2.3 | 2.7 | 2.5 | 2.4 | t4 |
| 2.65 | 10.6 | 2.7 | 2.6 | 2.8 | 2.5 | t5 |
| 2.58 | 10.3 | 2.7 | 2.7 | 2.4 | 2.5 | t6 |
| 2.75 | 11 | 2.8 | 2.9 | 2.7 | 2.6 | t7 |
|  | 67.8 | 17.6 | 17.6 | 16.9 | 15.7 | y.j |

Solution :

* Linear Model

Yij = µ + ti + bj + eij

Yij : observation value

µ : overall mean

ti : effect of( i) treatment .

bj : effect of (j) blocks

eij : experiment of error assumed to be normally independent distribution (NID)with zero mean and identical variance IϬ²

* Test of Hypotheses :

Ho : µ1 = µ2

 Ha: µ1 ≠ µ2

* Anova Table

 (y..) ²

CF = -------- = $\frac{\left(67.8\right)^{2}}{4×7}$ = 164.17

 tb

 ∑Yi².

SSt= ------------ - CF $=\frac{(8.1)^{2}+ … + (11)^{2} }{4}-164.17=\frac{663.52 }{4}-164.17= 1.71 $

 b

$SS\_{b}=\frac{\sum\_{}^{}y.j^{2}}{t}-C.F =\frac{\left(15.7\right)^{2}+ … + \left(17.6\right)^{2} }{7}-164.17 =\frac{1151.16 }{7}-164.17= 0.35 $

 SST = ∑yij² - CF = (24)2+(52)2+ ... +(162)2–164.17 =166.98–164.17= 2.81

 = 2.81 – 1.71 – 0.35 = 0.75

$SS\_{e}= SS\_{T}- SS\_{t}- SS\_{b}$

 = 2.81 – 1.71 – 0.35 = 0.75

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| F tab. | F cal. | MS | SS | df | S.O.V |
|  |  | 0.12 | $$0.35$$ | 3 | Blocks |
| 2.66 | 7.25 | 0.29 | $$1.71$$ | 6 | treats |
|  | 0.04 | 0.75 | 18 | Error |
|  | 2.81 | 27 | Total |

Due to F.cal (7.25) greater than F.table (α0.05 , df 6,18) (2.66) we accept Ho and Reject Ha this meaning significant differences among treatments .

a- L.S.D test

$L.S.D=1.414\*2.101\left(\sqrt{\frac{0.04}{4} }\right)$ = 0.3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| L.S.D = 0.3 | t12.03 | t22.15 | t32.33 | t42.48 | t62.58 | t72.65 |
| treats | Mean |
| t7 | 2.75 | 0.72\* | 0.6\* | 0.42\* | 0.27 | 0.17 | 0.1 |
| t5 | 2.65 | 0.62\* | 0.5\* | 0.32\* | 0.17 | 0.07 | 0.0 |
| t6 | 2.58 | 0.55\* | 0.43\* | 0.25 | 0.1 | 0.0 |  |
| t4 | 2.48 | 0.45\* | 0.33\* | 0.15 | 0.0 |  |  |
| t3 | 2.33 | 0.3\* | 0.18 | 0.0 |  |  |  |
| t2 | 2.15 | 0.12 | 0.0 |  |  |  |  |

b- Duncan multiple range test

$S\_{\overbar{y}}= \sqrt{\frac{MSe}{b}}$ = $\sqrt{\frac{0.04}{4}}=0.1$

$$LSR= S\_{\overbar{y}} ×SSR$$

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 3.35 | 3.32 | 3.27 | 3.21 | 3.12 | 2.97 | SSR |
| 0.1 | $$S\_{\overbar{x}}$$ |
| 0.335 | 3.32 | 0.327 | 0.321 | 0.312 | 0.297 | LSR |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 0.335t52.65 | 3.32t62.58 | 0.327t42.48 | 0.321t32.33 | 0.312t22.15 | 0.297t1 2.03 | LSR |
| Means | Treats |
| 0.1 | 0.17 | 0.27 | 0.42\* | 0.6\* | 0.75\* | 2.75 | t7 |
| 0.0 | 0.07 | 0.17 | 0.32\* | 0.5\* | 0.62\* | 2.65 | t5 |
|  | 0.0 | 0.1 | 0.25 | 0.43\* | 0.55\* | 2.58 | t6 |
|  |  | 0.0 | 0.15 | 0.33\* | 0.45\* | 2.48 | t4 |
|  |  |  | 0.0 | 0.18 | 0.3\* | 2.33 | t3 |
|  |  |  |  | 0.0 | 0.12 | 2.15 | t2 |

|  |  |
| --- | --- |
| Means | Treats |
| 2.75a | t7 |
| 2.65a | t5 |
| 2.58ab | t6 |
| 2.48ab | t4 |
| 2.33bc | t3 |
| 2.15cd | t2 |
| 2.03d | t1 |

* Relative efficiency of R.C.B.D compared with C.R.D :

 (b-1) MSb + b (t-1) MSe

 R.E. % = ----------------------------------- X 100

 (tb -1) MSe

Example : An experiment was analyzed to compare the effect of four levels of nitrogen on the yield of sunflower, using (RCBD) with five replicates (blocks). The results after the analysis were as shown in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| F. cal. | M.S. | S.S. | d.f . | S.O.V. |
| 20.46\*\* | 5.36 | 21.46 | 4 | Blocks |
| 44.83 | 134.45 | 3 | Treats |
| 2.19 | 26.26 | 12 | Error. |
| -------- | 182.17 | 19 | Total |

Solution :

 (b-1) MSb + b (t-1) MSe

 R.E. % = ----------------------------------- X 100

 (tb -1) MSe

 (5-1)\* 5.36 + 5 (4-1) \*2.19

 R.E. % = ----------------------------------- X 100 = 130%

 (4\*5 -1) \*2.19

Relative Efficiency of RCBD = 130% of C.R.D

This meaning 130 replicates of CRD gives the same result of 100 replicates of RCBD , Therefore, the economic cost for conducting the CRD design is higher and using the RCBD design is better

Missing Value :

When the data of any treatment is missing, the missing value can be extracted using the following law:

$$yij = \frac{tYi.+bY.j-Y..}{(t-1)(b-1)}$$

Where as

t : No.of treatments

b: No. of blocks

Yi. : sum of remaining observations for the treatment which has missing value .

Y.j : sum of remaining observations within block which has missing value

Y.. : overall for existing data .

Example: an experiment was conducted by using RCBD design which an observation was missing for some reason. Find missing value and create ANOVA table?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Yi. | R4 | R3 | R2 | R1 | Treats |
| 26.1 | 8.6 | --- | 8.6 | 8.9 | t1 |
| 35.2 | 8.7 | 9 | 8.8 | 8.7 | t2 |
| 34 | 8.5 | 8.6 | 8.4 | 8.5 | t3 |
| 95.3 | 25.8 | 17.6 | 25.8 | 26.1 | Y.j |

Solution :

$yij = \frac{tYi.+bY.j-Y..}{(t-1)(b-1)}$ = $yij = \frac{3\*26.1.+4\*17.6-95.3}{2\*3}$ = 8.9

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Yi. | R4 | R3 | R2 | R1 | Treats |
| 26.1+8.9=35 | 8.6 | ---8.9 | 8.6 | 8.9 | t1 |
| 35.2 | 8.7 | 9 | 8.8 | 8.7 | t2 |
| 34 | 8.5 | 8.6 | 8.4 | 8.5 | t3 |
| 95.3+8.9=104.2 | 25.8 | 17.6+8.9=26.5 | 25.8 | 26.1 | Y.j |

 CF =904.8

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| F. cal. | M.S. | S.S. | d.f . | S.O.V. |
| 8.58 | 0.036 | 0.110 | 3 | Blocks |
| 0.103 | 0.207 | 2 | Treats |
| 0.012 | 0.06 | 6-1=5 | Error. |
|  | 0.377 | 11-1=10 | Total |