Chapter Five: Linear Momentum

The linear momentum of an object is defined as the product of its mass and its velocity. It is measured in units of Kg.m/sec. The relation for linear momentum is:

$$\vec{P} = m\vec{v}$$

Where P and v are vector quantities. So an object will have have large momentum due to large mass, large velocity, or both.

Change object's Momentum:

The momentum of an object changes it its mass or velocity changes, or both. So, we can obtain a relation for the amount of change by re-writing newton's second law, as follow:

$$(\vec{F}_{Net} = m \times \vec{a})$$

$$\vec{F}_{Net} = m\vec{a} \longrightarrow \vec{F}_{Net} = \frac{\Delta(m\vec{v})}{\Delta t}$$

Thus the general form of newton's second law says that the net force is equal to the rate of change of momentum.

In order to change the momentum of an object, a force must be applied.

If we now multiply both sides of this equation by the time interval Δt , we get an equation that tells us how to produce a change in momentum.

$$\vec{F}_{Net}\Delta t = \Delta(m \times \vec{v})$$

This relationship tell us that the change in momentum is the net force multiplied by the certain time interval.. The change in momentum is called **Impulse**. Impulse is a vector quantity, it has the same direction as the applied force. The unit od impulse is N.s and is equivalent to the change of momentum Kg.m/sec .

Impulse is the product of two things, so there are many ways to change the momentum to the same value.
For example, if we want impulse of 10Ns., we can:
Exert a force of 5 N on the object for 2 sec. Or
Exert 100N for 0.1 sec
Each will produce the same impulse.
Example: Suppose you had to jump from a window. Would you prefer to jump onto a wooden surface or onto a concrete surface? Why?
So , in brief, the impulse is

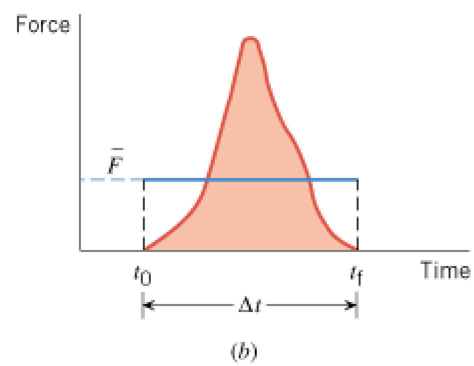
Impulse = average force x time of contact. Or the impulse is the change in momentum P.

I= FΔ t = Δ P

$$\vec{F}\Delta t = \Delta\vec{P}$$

$$\vec{F}dt = d\vec{P}$$

$$\vec{I} = \int_{t_i}^{t_f} d\vec{p} = \int_{t_1}^{t_2} \vec{F}(t)dt$$



Therefore the impulse is area and the graph.

Example 1: What is the momentum of a ball with mass 5kg and velocity 10m/s?

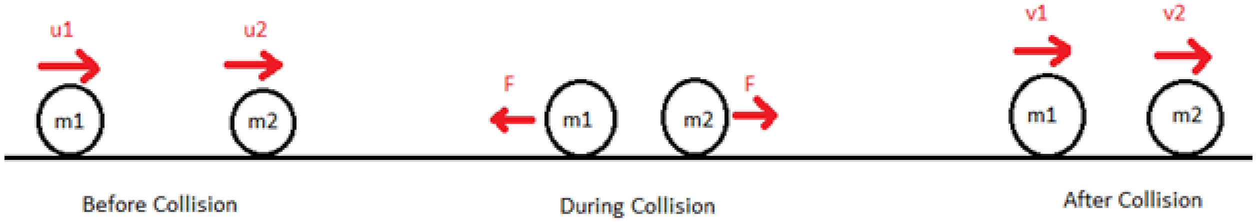
Momentum = mass x velocity
Momentum = 5Kgx 10 m/s = 50 Kg.m/s

Example 2; What will be the change in momentum caused by a net force of 120N acting on an object for 2 seconds?

Change in momentum = net force x time interval
Change in momentum= 120N x 2 s = 240 N.

Conservation of linear momentum:

Newton's Second law relates force with the rate of change of momentum. According to the law, force is directly proportional to the rate of change in momentum. $F \propto \Delta P$



We will use this to state the law of conservation of momentum. According to this, if the net force acting on the system is zero, then the system's momentum remains conserved.

In other words, the change in momentum of the system is zero. According to the second law, we can see as F = 0, so it will also be zero.

Let's take the following example:
We consider m_1 and m_2 as our system. So during the collision, the net force on the system is zero, and hence we can conserve the system's momentum. The equation for momentum will be:

Initial momentum = $m_1u_1 + m_2u_2$

Final momentum = $m_1v_1 + m_2v_2$

So, according to the conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

But one thing to take care is that conservation is only true for a system and not one body because if we consider only a single body m_1 or m_2, then the net force will be acting on it, so we cannot write

$m_1u_1 \neq m_1v_1$ or $m_2u_2 \neq m_2v_2$.

Discuss the law of conservation of momentum. State its unit.

The law of conservation of momentum states that when two objects collide in an isolated system, the total momentum before and after the collision remains equal. This is because the momentum lost by one object is equal to the momentum gained by the other.
In other words, if no external force is acting on a system, its net momentum gets conserved.
The unit of momentum in the S.I system is kgm/s or simply Newton Second(Ns).

Conservation of Linear Momentum Example

Two bodies of mass m1 and m2 are moving in opposite directions with the velocities v1 and v2. If they collide and move together after the collision, we have to find the velocity of the system Vfinal.

Since there is no external force acting on the system of two bodies, momentum will be conserved.

Initial momentum = Final momentum

$(m1v1 – m2v2) = (m1+m2)V_{Final}$

Thus Vfinal = (m1v1-m2v2)/(m1+m2)

From this equation, we can easily find the final velocity of the system.

Definition of conservation of momentum
For two or more bodies in an isolated system acting upon each other, their total momentum remains constant unless an external force is applied. Therefore, momentum can neither be created nor destroyed.

The principle of conservation of momentum is a direct consequence of Newton’s third law of motion.

Derivation of Conservation of Momentum

Newton’s third law states that for a force applied by an object A on object B, object B exerts back an equal force in magnitude, but opposite in direction.

This idea was used by Newton to derive the law of conservation of momentum.

Consider two colliding particles A and B whose masses are m_1 and m_2 with initial and final velocities as u_1 and v_1 of A and u_2 and v_2 of B.

The time of contact between two particles is given as t.

A=m1(v1−u1) (change in momentum of particle A)

B=m2(v2−u2) (change in momentum of particle B)

FBA=−FAB (from third law of motion)
Then

FBA=m2∗a2=m2(v2−u2)t
and
FAB=m1∗a1=m1(v1−u1)t
Then
m2(v2−u2)t=−m1(v1−u1)t

Therefore
$m1u1+m2u2 = m1v1+m2v2$

Therefore, above is the equation of law of conservation of momentum where

m1u1+m2u2 is the representation of total momentum of particles A and B before the collision and

m1v1+m2v2 is the representation of total momentum of particles A and B after the collision.

Solved Problems on Law of Conservation of Momentum

Q1. There are cars with masses 4 kg and 10 kg respectively that are at rest. The car having the mass 10 kg moves towards the east with a velocity of 5 $m.s^{-1}$. Find the velocity of the car with mass 4 kg with respect to ground.
Ans: Given,

$m_1 = 4$ kg

$m_2 = 10$ kg

$v_1 = ?$

$v_2 = 5\ m.s^{-1}$

$P_{initial} = 0$, as the cars are at rest

$P_{final} = p_1 + p_2$

$P_{final} = m_1.v_1 + m_2.v_2$

$= (4\ kg).(v_1) + (10\ kg).(5\ m.s^{-1})$

$= 4kg \times V1 + 50\ kg.m.s^{-1}$

We know from the law of conservation of momentum that,

$P_{initial} = P_{final}$

$0=4\ kg.v_1+50\ kg.m.s^{-1}$

Then

$V1= - 50/4$

$v_1 = 12.5\ m.s^{-1}$

The negative sign means the opposite direction

Q2. Find the velocity of a bullet of mass 5 grams which is fired from a pistol of mass 1.5 kg. The recoil velocity of the pistol is 1.5 $m.s^{-1}$.

Ans: Given,

Mass of bullet, m_1 = 5 gram = 0.005 kg

Mass of pistol, m_2 = 1.5 kg

The velocity of a bullet, v_1 = ?

Recoil velocity of pistol, v_2 = 1.5 $m.s^{-1}$

Using law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Here, Initial velocity of the bullet, $u_1 = 0$

Initial recoil velocity of a pistol, $u_2 = 0$

$\therefore$ (0.005 kg)(0) + (1.5 kg)(0) = (0.005 kg)(v_1) + (1.5 kg)(1.5 $m.s^{-1}$)

0 = (0.005 kg)(v_1)+(2.25 $kg.m.s^{-1}$)

Then

V1= - 2.25/0.005

v_1= -450 $m.s^{-1}$

Hence, the recoil velocity of the pistol is 450 $m.s^{-1}$.

The negative sign means the recoil velocity is in the opposite direction.