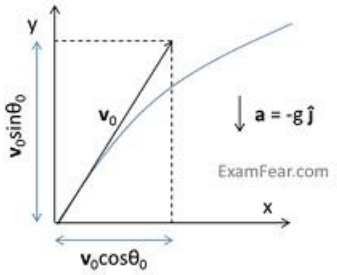
Chapter Four: Projectile Motion

An object that becomes airborne after it is thrown or projected is called **projectile**. Example, football..

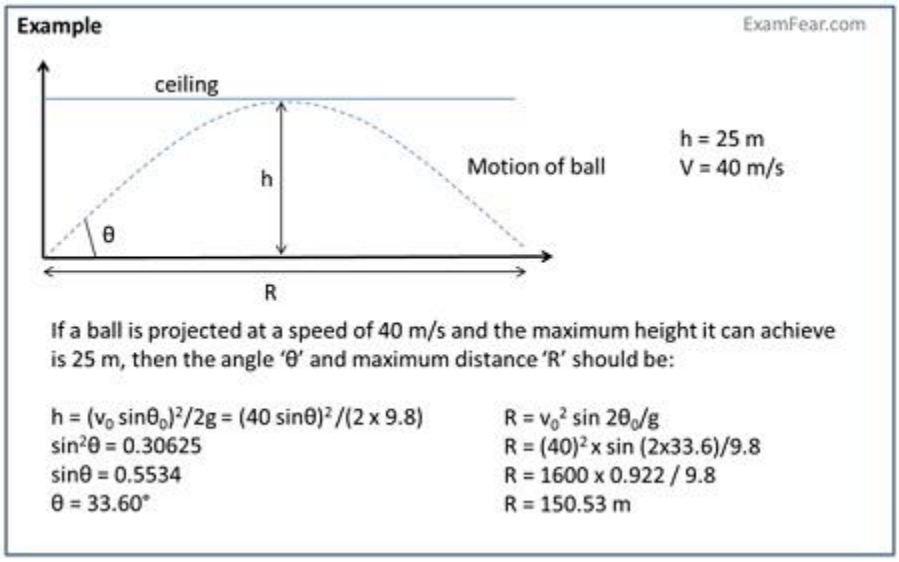
Projectile Motion

- Projectile motion consist of two parts.
    1- Horizontal motion of no acceleration;
    2- 2- Vertical motion of constant acceleration due to gravity.
- Projectile motion is in the form of a parabola, $y = ax + bx^2$.



Motion of an object projected with velocity $\mathbf{v}_0$ at an angle θ_0

Quantity	**Value**
Components of velocity at time t	$v_x = v_0 \cos\theta_0$ $v_y = v_0 \sin\theta_0 – gt$
Position at time t	$x = (v_0 \cos\theta_0)t$ $y = (v_0 \sin\theta_0)t – ½ gt^2$
Equation of path of projectile motion	$y = (\tan\theta_0)x – gx^2/2(v_0 \cos\theta_0)^2$
Time of maximum height	$t_m = v_0 \sin\theta_0 /g$
Time of flight	$2\ t_m = 2\ (v_0 \sin\theta_0 /g)$
Maximum height of projectile	$h_m = (v_0 \sin\theta_0)^2/2g$
Horizontal range of projectile	$R = v_0^2 \sin 2\theta_0/g$
Maximum horizontal range (θ_0=45°)	$R_m = v_0^2/g$



Problem 1

An object is launched at a velocity of 20 m/s in a direction making an angle of 25° upward with the horizontal.

a) What is the maximum height reached by the object?

b) What is the total flight time (between launch and touching the ground) of the object?

c) What is the horizontal range (maximum x above ground) of the object?

d) What is the magnitude of the velocity of the object just before it hits the ground?

Solution to Problem 1:

a) The formulas for the components V_x and V_y of the velocity and components x and y of the displacement are given by :

$Vx=V_0 \cos(\theta)$ $V_y = V_0 \sin(\theta) - g\,t$

$x = V_0 \cos(\theta)\,t$ $y = V_0 \sin(\theta)\,t - (1/2)\,g\,t^2$

In the problem $V_0 = 20$ m/s, $\theta = 25°$ and $g = 9.8$ m/s^2.

The height of the projectile is given by the component y, and it reaches its maximum value when the component V_y is equal to zero.

$V_y = V_0 \sin(\theta) - g\,t = 0$

solve for t

$t = V_0 \sin(\theta) / g = 20 \sin(25°) / 9.8 = 0.86$ seconds

Maximum height is:

$y = 20 \sin(25°)(0.86) - (1/2)\,(9.8)\,(0.86)^2 = 3.64$ meters

b) The time of flight is:

$V_0 \sin(\theta)\,t - (1/2)\,g\,t^2 = 0$

Solve for t

$t(V_0 \sin(\theta) - (1/2)\,g\,t) = 0$

either $t = 0$ or $t = 2\,V_0 \sin(\theta) / g$

Time of flight $= 2\,(20) \sin(\theta) / g = 1.72$ seconds.

c) The time of flight $t = 2\,V_0 \sin(\theta) / g$. The horizontal range is the horizontal distance given by x at t .

Range $= x = V_0 \cos(\theta)\,t = 2\,V_0 \cos(\theta)\,V_0 \sin(\theta) / g = V_0^2 \sin(2\theta) / g = 20^2 \sin(2x25°) / 9.8 =$ 31.26 meters

d) The object hits the ground at t =2 V0 sin(θ) / g
The components of the velocity at t are given by
$V_x = V_0 \cos(\theta)$ $V_y = V_0 \sin(\theta) - g\,t$
The components of the velocity at t = 2 V0 sin(θ) / g are given by
$V_x = V_0 \cos(\theta) = 20 \cos(25°)$ $V_y = V_0 \sin(25°) - g\,(2\,V_0 \sin(25°) / g) = - V_0 \sin(25°)$
The magnitude V of the velocity is given by
$V = \sqrt{[\,V_x^2 + V_y^2\,]} = \sqrt{[\,(20 \cos(25°))^2 + (- V_0 \sin(25°))^2\,]} = V_0 = 20$ m/s

Problem 2

A ball is kicked at an angle of 35° with the ground.
a) What should be the initial velocity of the ball so that it hits a target that is 30 meters away at a height of 1.8 meters?
b) What is the time for the ball to reach the target?
Solution to Problem 2:
a)
$x = V_0 \cos(35°)\,t$
$30 = V_0 \cos(35°)\,t$
$t = 30 / V_0 \cos(35°)$
$1.8 = -(1/2)\,9.8\,(30 / V_0 \cos(35°))^2 + V_0 \sin(35°)(30 / V_0 \cos(35°))$
$V_0 \cos(35°) = 30\,\sqrt{}\,[\,9.8 / 2(30 \tan(35°)-1.8)\,]$
$V_0 = 18.3$ m/s
b)
$t = x / V_0 \cos(35°) = 2.0$ s

Problem 3

A ball kicked from ground level at an initial velocity of 60 m/s and an angle θ with ground reaches a horizontal distance of 200 meters.
a) What is the size of angle θ?
b) What is time of flight of the ball?

Solution to Problem 3:
a)
Let T be the time of flight. Two ways to find the time of flight
1) $T = 200 / V_0 \cos(\theta)$ (range divided by the horizontal component of the velocity)
2) $T = 2\,V_0 \sin(\theta) / g$
equate the two expressions
$200 / V_0 \cos(\theta) = 2\,V_0 \sin(\theta) / g$
which gives
$2\,V_0^2 \cos(\theta)\sin(\theta) = 200\,g$
$V_0^2 \sin(2\theta) = 200\,g$
$\sin(2\theta) = 200\,g / V_0^2 = 200\,(9.8) / 60^2$
Solve for θ to obtain

$\theta = 16.5°$
b)Time of flight = $200 / V_0 \cos(16.5°) = 3.48$ s

Problem 4
A projectile starting from ground hits a target on the ground located at a distance of 1000 meters after 40 seconds.
a) What is the size of the angle θ?
b) At what initial velocity was the projectile launched?
Solution to Problem 4:
a) $Vx = V_0 \cos(\theta) = 1000 / 40 = 25$ m/s
Time of flight = $2 V_0 \sin(\theta) / g = 40$ s
$V_0 \sin(\theta) = 20 g$
Combine the above equation with the equation $V_0 \cos(\theta) = 25$ m/s
$\tan(\theta) = 20 g / 25$
Use calculator to find $\theta = 82.7°$
b)We now use any of the two equations above to find V_0.
$V_0 \cos(\theta) = 25$ m/s
$V_0 = 25 / \cos(82.7°) = 196.8$ m/s

Problem 5
The trajectory of a projectile launched from ground is given by the equation $y = -0.025 x^2 + 0.5 x$, where x and y are the coordinate of the projectile on a rectangular system of axes.
Find the initial velocity and the angle at which the projectile is launched.
Solution to Problem 5:
$y = \tan(\theta) x - (1/2) (g / (V_0 \cos(\theta))^2) x^2$
hence $\tan(\theta) = 0.5$ which gives $\theta = \arctan(0.5) = 26.5°$
$-0.025 = -0.5 (9.8 / (V_0 \cos(26.5°))^2)$
Solve for V_0 to obtain $V_0 = 15.6$ m/s

Problem 6
Two balls A and B of masses 100 grams and 300 grams respectively are pushed horizontally from a table of height 3 meters. Ball has is pushed so that its initial velocity is 10 m/s and ball B is pushed so that its initial velocity is 15 m/s.
a) Find the time it takes each ball to hit the ground.
b) What is the difference in the distance between the points of impact of the two balls on the ground?
Solution to Problem 6:
a) The two balls are subject to the same gravitational acceleration and therefor will hit the ground at the same time t found by solving the equation
$-3 = -(1/2) g t^2$
$t = \sqrt{(3(2)/9.8)} = 0.78$ s
b) Horizontal distance XA of ball A
XA = 10 m/s * 0.78 s = 7.8 m
Horizontal distance XB of ball B
XB = 15 m/s * 0.78 s = 11.7 m
Difference in distance XA and XB is given by
$|XB - XA| = |11.7 - 7.8| = 3.9$ m