Chapter Three: Equations of Motion----Mechanics

Motion in a straight line with constant acceleration

Suppose a particle moving in a straight line with a constant or uniform acceleration(a)for a time interval t.

During this time its velocity will change from u to v.

The particle travels a distance s.

If you know any three of the quantities (a, t, u, v ,s) then you can calculate the remaining two.

The initial speed/velocity is	u.
The final speed/velocity is	v.
The acceleration is	a.
The distance/displacement is	s.
The time take taken is	t.

The equations: $v = u + at$. (no s)

$s = ut + \frac{1}{2} at^2$. (no v)

$s = vt - \frac{1}{2} at^2$. (no u)

$s = \frac{1}{2} (u + v) t$. (no a)

$v^2 = u^2 + 2as$. (no t)

Example 1 (units: metres and seconds)

(i) Find v when u = 10, a = 6 and t = 2.

(ii) Find s when u = 10, a = 8 and t = 2.

(iii) Find s when v = 30, a = 4 and t = 5.

(iv) Find s when u = 10, v = 8 and t = 2.

(v) Find v when u = 10, a = 5 and s = 6.

Solutions

(i) $v = u + at$ □ $v = 10 + 6 \times 2 = 10 + 12 = 22 \text{ ms}^{-1}$

(ii) $s = ut + \frac{1}{2} at^2$ □ $s = 10 \times 2 + 0.5 \times 8 \times 2^2 = 20 + 16 = 36$ m.

(iii) $s = vt - \frac{1}{2} at^2$ □ $s = 30 \times 5 - 0.5 \times 4 \times 5^2 = 150 - 50 = 100$ m.

(iv) $s = \frac{1}{2} (u + v) t$ □ $\frac{1}{2} (10 + 8)\ 2\ 2 = 18$ m.

(v) $v^2 = u^2 + 2as$ □ $v^2 = 10^2 + 2 \times 5 \times 6 = 100 + 60 = 160$

$v = (160)^{1/2} = 12.6 \text{ ms}^{-1}$.

Example 2

Decide which equations to use in each of these situations.

(i) Given u, s, a ; find v.	(ii) Given u, t, a ; find v.
(iii) Given u, t, a ; find s.	(iv) Given u, s, v ; find t.
(v) Given u, s, v ; find a.	(vi) Given u, s, t ; find a.
(vii) Given u, a, v ; find s.	(viii) Given a, s, t ; find v.

Solutions

(i) $v^2 = u^2 + 2as$	(ii) $v = u + at$
(iii) $s = ut + \frac{1}{2} at^2$	(iv) $s = \frac{1}{2} (u + v) t$
(v) $v^2 = u^2 + 2as$	(vi) $s = ut + \frac{1}{2} at^2$
(vii) $v^2 = u^2 + 2as$	(viii) $s = vt - \frac{1}{2} at^2$

Example 3

A particle is moving in a straight line from O to A with a constant acceleration of 2 ms $^{-2}$. Its velocity at A is 30 ms^{-1} and it takes 15 seconds to travel from O to A. Find

(a) Particle's velocity at O,

(b) Distance OA.

Solution

(a) Given a, v, t; needed u: Equation: v = u + at

$$30 = u + 2 \times 15 \;\square\; u = 0$$

The velocity at O is 0 ms $^{-1}$

(b) Known a, u, v, t; needed s: equations: $s = ut + ½ at^2$ or $s = ½ (u + v) t$

$$s = 0 \times 15 + 0.5 \times 2 \times 15^2 = 225$$

The distance OA is 225 m.

Example 4

A boy on a skateboard is travelling up a hill. He experienced a constant deceleration of magnitude 2 ms $^{-2}$. Given that his speed at the bottom of the hill was 10 ms^{-1}, determine how far he will travel before he comes to rest.

Solution

Known quantities are a, u, v; needed s:

Required equation $v^2 = u^2 + 2as$

$0 = 10^2 + 2 \times -2 \times s = 100 - 4s$ then $\square$ $s = 100/4 = 25$

The boy travels a distance of 25 m before coming to rest.

Exercise

1	A particle moves in a straight line with uniform acceleration 5 ms $^{-2}$. It starts from rest when t = 0. Find its velocity when t = 3 s.
2	A particle moves in straight line. When t = 0 its velocity is 3 ms $^{-1}$. When t = 4 its velocity is 12 ms $^{-1}$. Find its acceleration.
3	A particle moves in straight line with constant retardation 4 ms $^{-2}$. When t = 3 s its velocity is 5 ms $^{-1}$. Find its initial velocity.
4	A particle moves in a straight line with uniform acceleration 5 ms $^{-2}$. How long will it take for the particle's velocity to increase from 2 ms $^{-1}$to 24 ms $^{-1}$?
5	A particle starts from rest and moves in a straight line with constant acceleration 4

	ms^{-2} for 3 s. How far does it travel ?
6	A particle moves along a line PQ with constant acceleration 3 ms^{-2} . If PQ is 2 m and the particle takes 0.5 s to travel from P to Q , what was its velocity at P?
7	A particle moves in a straight line with uniform acceleration 8 ms^{-2} . If its initial velocity is 2 ms^{-1} how long will it take to travel 6 m ?
8	A particle moving in a straight line experience a constant retardation of 6 ms^{-2} . How long will it take to decrease its speed from 20 ms^{-1} to 8 ms^{-1} and how far will it travel while doing so ?
9	A particle is moving with uniform acceleration. If it starts from rest and 5 s later has a speed of 18 ms^{-1} find the distance it has travelled.
10	A particle is moving in a straight line with uniform acceleration . If it travels 120 m while increasing speed from 5 ms^{-1} to 25 ms^{-1} find its acceleration.
11	A car is accelerating uniformly while travelling along a straight road. Its speed increases from 8 ms^{-1} to 22 ms^{-1} in 10 s. Find the distance travelled during this time and the acceleration of the car.

More Examples

A car starts from rest at a point O and moves in a straight line.

The car moves with constant acceleration of 4 ms^{-2} until it passes the point A when it is moving with the speed 10 ms^{-1}. It then moves with constant acceleration of 2 ms^{-2} for 5 seconds until it reaches the point B.

Find (a) The speed of the car at B,

(b) The distance OB.

Solution

(a) A to B: $u = 10$, $t = 5$, $a = 2$, v? Required equation $v = u + at$

$v = 10 + 2 = 5 = 20$ ms^{-1} .

(b) Distance OA: $v^2 = u^2 + 2as$: $10^2 = 0 + 2 \times 4 \times s$

$100 = 8s$ gives $s = 12.5$

Distance AB: $u = 10$, $v = 20$, $t = 5$, s? : $s = 75$

Distance OB = 12.5 + 75 = 87.5 m.

Figure below shows the graph of displacement versus time for a body moving with constant velocity. It can be seen that the graph consists of a *straight-line*. This line can be represented algebraically as

$$x = x_0 + v\,t.$$

Here, x_o is the displacement at time $t = 0$: this quantity can be determined from the graph as the *intercept* of the straight-line with the x-axis.

Likewise, $v = dx/dt$ is the constant velocity of the body: this quantity can be determined from the graph as the *gradient* of the straight-line (*i.e.*, the ratio $\Delta x/\Delta t$, as shown). Note that $a = d^2x/dt^2 = 0$, as expected.

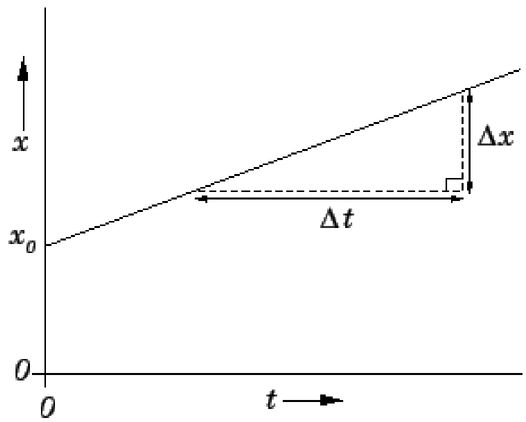

Figure 6: *Graph of displacement versus time for a body moving with constant velocity*

Free-fall under gravity

Galileo Galilei was the first scientist to appreciate that, *neglecting the effect of air resistance*, all bodies in free-fall close to the Earth's surface accelerate vertically downwards with the same acceleration: namely, $g = 9.81\,\mathrm{m\,s^{-2}}$.[1] The neglect of air resistance is a fairly good approximation for large objects which travel relatively slowly (*e.g.*, a shot-putt, or a basketball), but becomes a poor approximation for small objects which travel relatively rapidly (*e.g.*, a golf-ball, or a bullet fired from a pistol).

Equations above can easily be modified to deal with the special case of an object free-falling under gravity:

$$\begin{aligned} s &= v_0\,t - \frac{1}{2}\,g\,t^2, \\ v &= v_0 - g\,t, \\ v^2 &= v_0^2 - 2\,g\,s. \end{aligned}$$

Here, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the downward acceleration due to gravity, s is the distance the object has moved vertically between times $t = 0$ and t (if $s > 0$ then the object has risen s meters, else if $s < 0$ then the object has fallen $|s|$ meters), and v_0 is the object's instantaneous velocity at $t = 0$. Finally, v is the object's instantaneous velocity at time t.

Let us illustrate the use of Eqs. (24)-(26). Suppose that a ball is released from rest and allowed to fall under the influence of gravity. How long does it take the ball to fall h meters? Well, according to Eq. (24) [with $v_0 = 0$ (since the ball is released from rest), and $s = -h$ (since we wish the ball to *fall* h meters)], $h = g\,t^2/2$, so the time of fall is

$$t = \sqrt{\frac{2\,h}{g}}.$$

Suppose that a ball is thrown vertically upwards from ground level with velocity u. To what height does the ball rise, how long does it remain in the air, and with what velocity does it strike the ground? The ball attains its maximum height when it is momentarily at rest (*i.e.*, when $v = 0$). According to Eq. (25) (with $v_0 = u$), this occurs at time $t = u/g$. It follows that the maximum height of the ball is given by

$$h = \frac{u^2}{2\,g}.$$

When the ball strikes the ground it has traveled zero net meters vertically, so $s = 0$. It follows that $v = -u$. In other words, the ball hits the ground with an equal and opposite velocity to that with which it was thrown into the air. Since the ascent and decent phases of the ball's trajectory are clearly symmetric, the ball's time of flight is simply twice the time required for the ball to attain its maximum height: *i.e.*,

$$t = \frac{2\,u}{g}.$$

Equations of Motion

English	Math / Symbols
Kinematic Equations: Find Velocity	$v = v_0 + at$
Kinematic Equations: Find Position	$x = x_0 + v_0t + ½at^2$
Kinematic Equations: No time given, Find Velocity	$v^2 = v_0^2 + 2a(x - x_0)$
(Kinematic Equations) No time given, Find Position	$(x - x_0) = (v^2 - v_0^2) / 2a$
Find Average Velocity (Using Velocity)	$V_{ave} = (v + v_0) / 2$
(Kinematic Equations) Find Acceleration (Without Time)	$a = (v^2 - v_0^2) / 2(x - x_0)$
Find Average Velocity (Using Position/Time)	$V_{ave} = \Delta x/\Delta t$