CHAPTER 5:
CABLES AND ARCHES

5

Chapter Outline

5.1

CABLES

5.1

Cables

- Assumptions when deriving the relations between force in cable & its slope
- Cable is perfectly flexible & inextensible
- Due to its flexibility, cable offers no resistance to shear or bending
- The force acting the cable is always tangent to the cable at points along its length

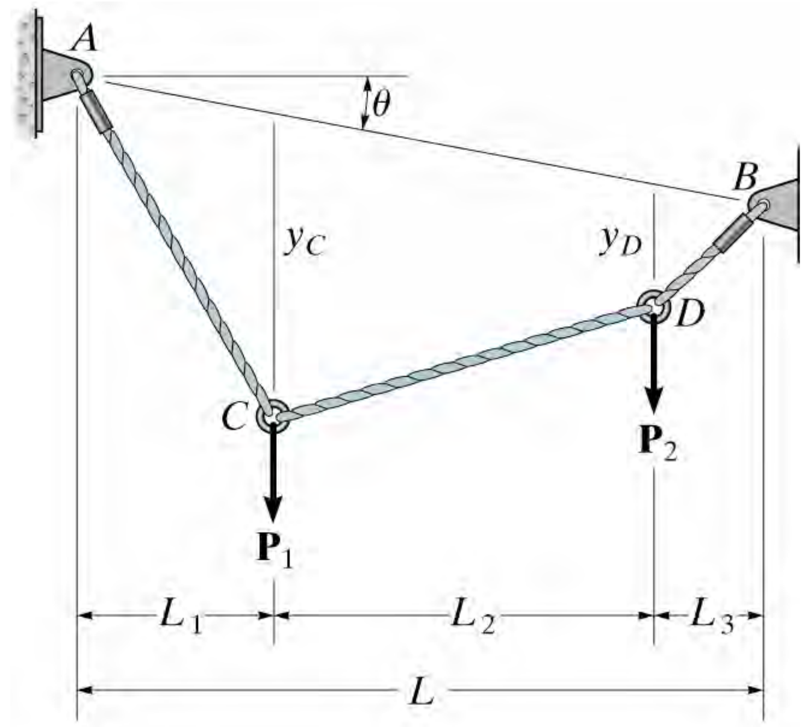
5.2

CABLES SUBJECTED TO CONCENTRATED LOADS

5.2

Cable Subjected to Concentrated Loads

- When a cable of negligible weight supports several concentrated loads, the cable takes the form of several straight line segments
- Each of the segment is subjected to a constant tensile force
- θ specifies the angle of the cord *AB*
- *L* = cable length



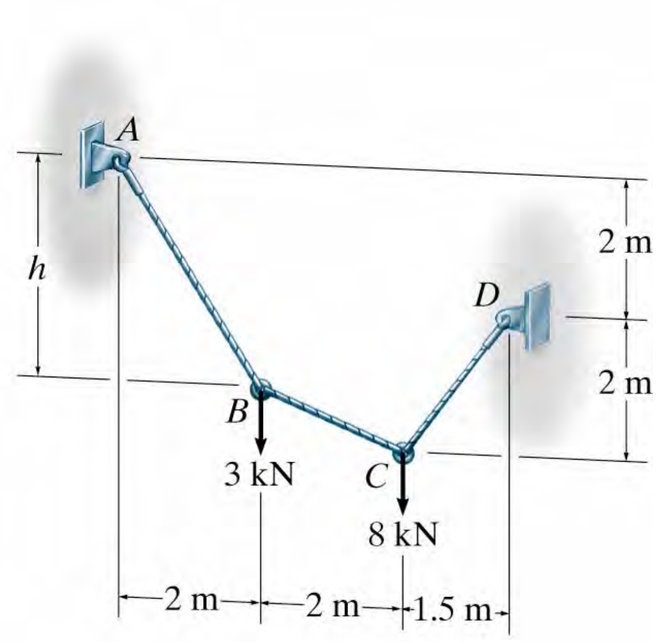
Cable Subjected to Concentrated Loads

- If L_1, L_2 & L_3 and loads P_1 & P_2 are known, determine the 9 unknowns consisting of the tension in each of the 3 segments, the 4 components of reactions at *A* & *B* and the sags y_C & y_D
- For solutions, we write 2 eqns of equilibrium at each of 4 points *A*, *B*, *C* & *D*
- Total 8 eqns
- The last eqn comes from the geometry of the cable

Cable Subjected to Concentrated Loads

Example 5.1

The building roof shown in the photo has a weight of and is 1.5 kN/m^2 supported on 8-m long simply supported beams that are spaced 1 m apart. Each beam as shown *transmits its loading to two* girders, located at the front and back of the building. Determine the internal shear and moment in the front girder at point *C.* Neglect the weight of the members.



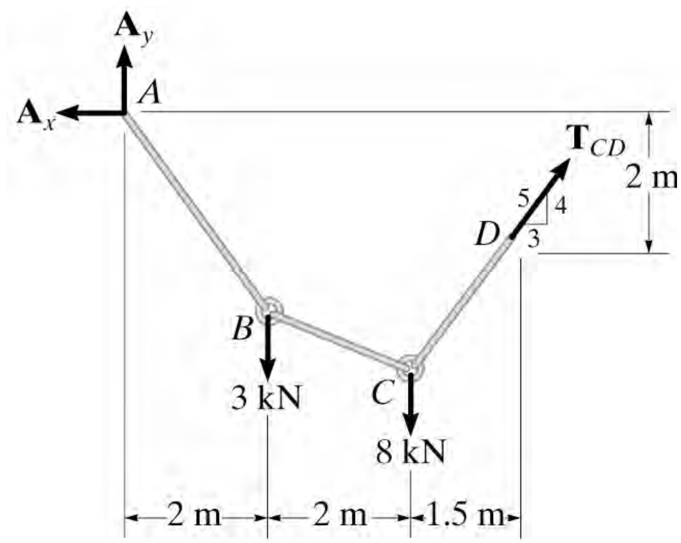
Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

By inspection, there are
➔ 4 unknown external reactions (A_x, A_y, D_x and D_y)
➔ 3 unknown cable tensions

These unknowns and sag, *h* can be determined from available equilibrium eqns applied to points *A* through *D*.

A more direct approach to the solution is to recognize that the slope of cable *CD* is specified.



Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

With anti-clockwise moment as +ve

$$\Sigma M_A = 0$$

$$T_{CD}(3/5)(2\,\text{m}) + T_{CD}(4/5)(5.5\,\text{m}) - 3\,\text{kN}(2\,\text{m}) - 8\,\text{kN}(4\,\text{m}) = 0$$

$$T_{CD} = 6.79\,\text{kN}$$

Now we can analyze the equilibrium of points C and B in sequence.

Point C (Fig 5.2c)

$$\underset{\rightarrow}{+}\Sigma F_x = 0$$

$$6.79\,\text{kN}(3/5) - T_{BC}\cos\theta_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0$$

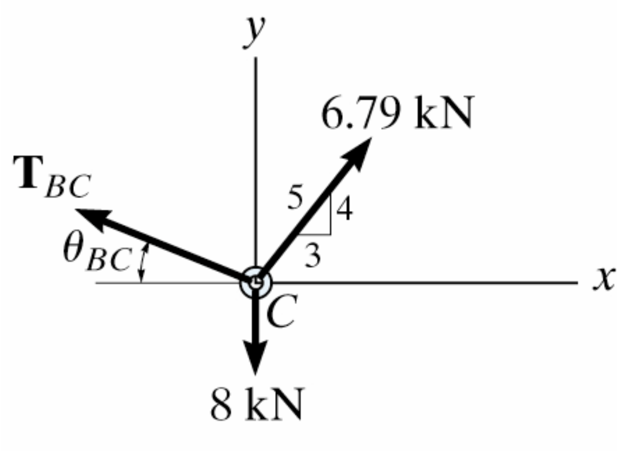
$$6.79\,\text{kN}(4/5) - 8\,\text{kN} + T_{BC}\sin\theta_{BC} = 0$$

$$\theta_{BC} = 32.3^o \quad \text{and} \quad T_{BC} = 4.82\,\text{kN}$$

Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

Point B (Fig 5.2d)



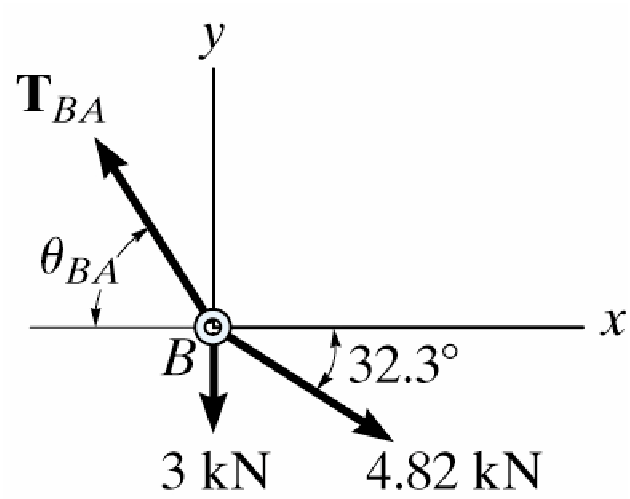
$\overset{+}{\rightarrow}\Sigma F_x = 0$

$-T_{BA}\cos\theta_{BA} + 4.82\,\text{kN}\cos 32.3^o = 0$

$+\uparrow \Sigma F_y = 0$

$T_{BA}\sin\theta_{BA} - 4.82\,\text{kN}\sin 32.3^o - 3\,\text{kN} = 0$

$\theta_{BA} = 53.8^o$ and $T_{BA} = 6.90\,\text{kN}$



Hence from Fig 5.2(a)

$h = (2\,\text{m})\tan 53.8^o = 2.74\,\text{m}$

5.3
CABLES SUBJECTED TO A UNIFORM DISTRIBUTED LOAD

5.3

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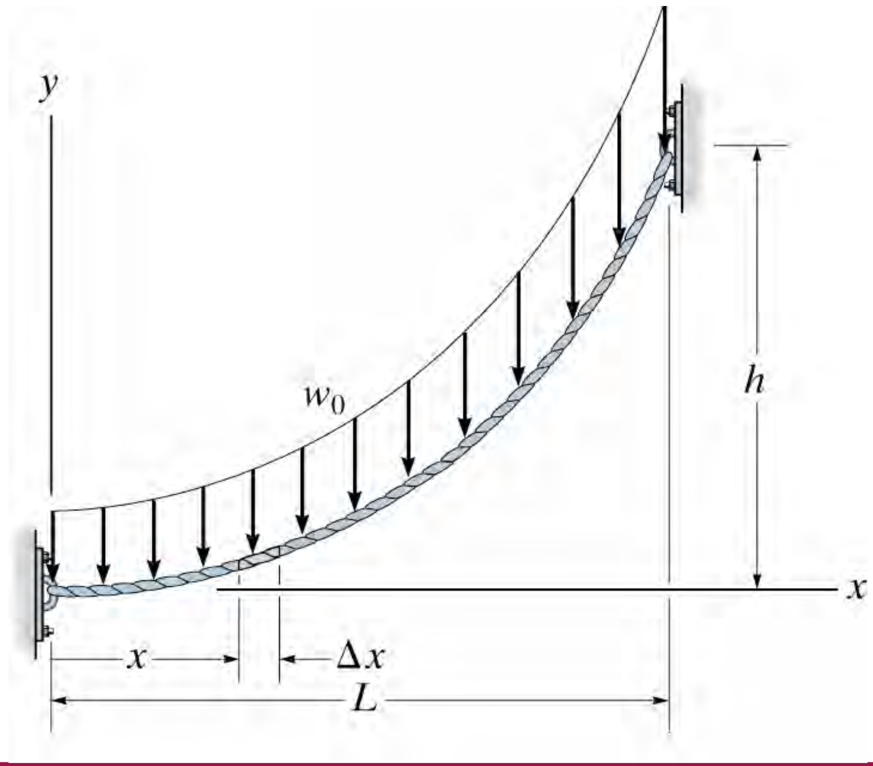
Cable Subjected to a Uniform Distributed Load

- The *x*,*y* axes have their origin located at the lowest point on the cable such that the slope is zero at this point
- Since the tensile force in the cable changes continuously in both magnitude & direction along the cable's length, this change is denoted by ΔT



Cable Subjected to a Uniform Distributed Load

- The distributed load is represented by its resultant force $w_o \Delta x$ which acts at $\Delta x/2$ from point *O*

- Applying eqns of equilibrium yields:

$$\underset{\rightarrow}{+}\Sigma F_x = 0$$

$$-T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$$

$$+\uparrow \Sigma F_y = 0$$

$$-T\sin\theta - w_o(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$$

$$\text{With anti-clockwise moment as +ve}$$

$$\Sigma M_O = 0$$

$$w_o(\Delta x)(\Delta x/2) - T\cos\theta\Delta y + T\sin\theta\Delta x = 0$$

 PEARSON

Cable Subjected to a Uniform Distributed Load

- Dividing each of these eqn by Δx and taking the limit as $\Delta x \rightarrow 0$, hence, $\Delta y \rightarrow 0$, $\Delta\theta \rightarrow 0$ and $\Delta T \rightarrow 0$, we obtain:

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad \text{eqn 1}$$

$$\frac{d(T\sin\theta)}{dx} = w_o \qquad \text{eqn 2}$$

$$\frac{dy}{dx} = \tan\theta \qquad \text{eqn 3}$$

Cable Subjected to a Uniform Distributed Load

- Integrating Eqn 1 where $T = F_H$ at $x = 0$, we have:

$$T \cos\theta = F_H \quad \text{eqn } 4$$

- Which indicates the horizontal component of force at any point along the cable remains constant

- Integrating Eqn 2 realizing that $T \sin\theta = 0$ at $x = 0$, we have:

$$T \sin\theta = w_o x \quad \text{eqn } 5$$

Cable Subjected to a Uniform Distributed Load

- Dividing Eqn 5 by Eqn 5.4 eliminates *T*

- Then using Eqn 3, we can obtain the slope at any point

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \quad \text{eqn 6}$$

- Performing a second integration with *y* = 0 at *x* = 0 yields

$$y = \frac{w_o}{2F_H} x^2 \quad \text{eqn 7}$$

Cable Subjected to a Uniform Distributed Load

- This is the eqn of a parabola
- The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L$
- Thus

$$F_H = \frac{w_o L^2}{2h} \quad \text{eqn 8}$$

- Substituting into Eqn 7

$$y = \frac{h}{L^2} x^2 \quad \text{eqn 9}$$

Cable Subjected to a Uniform Distributed Load

- From Eqn 4, the max tension in the cable occurs when θ is max, i.e. at $x=L$
- From Eqn 4 and 5

$$T_{\max} = \sqrt{F^2{}_H + (w_o L)^2} \qquad \text{eqn 10}$$

- Using Eqn 8 we can express T_{max} in terms of w_o

$$T_{\max} = w_o L \sqrt{1 + (L/2h)^2} \qquad \text{eqn 11}$$

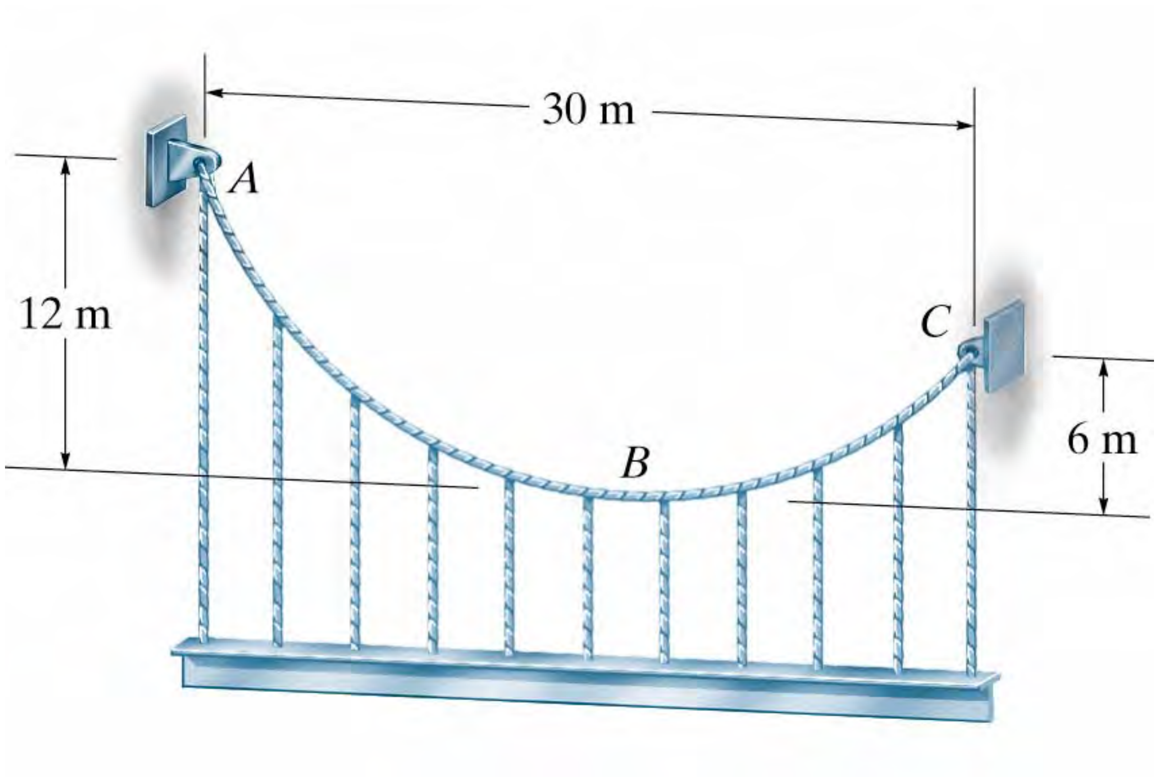
Cable Subjected to a Uniform Distributed Load

- We have neglected the weight of the cable which is uniform along the length
- A cable subjected to its own weight will take the form of a catenary curve
- If the sag-to-span ratio is small, this curve closely approximates a parabolic shape

Cable Subjected to a Uniform Distributed Load

Example 5.2

The cable supports a girder which weighs 12 kN/m. Determine the tension in the cable at points *A*, *B* & *C*.

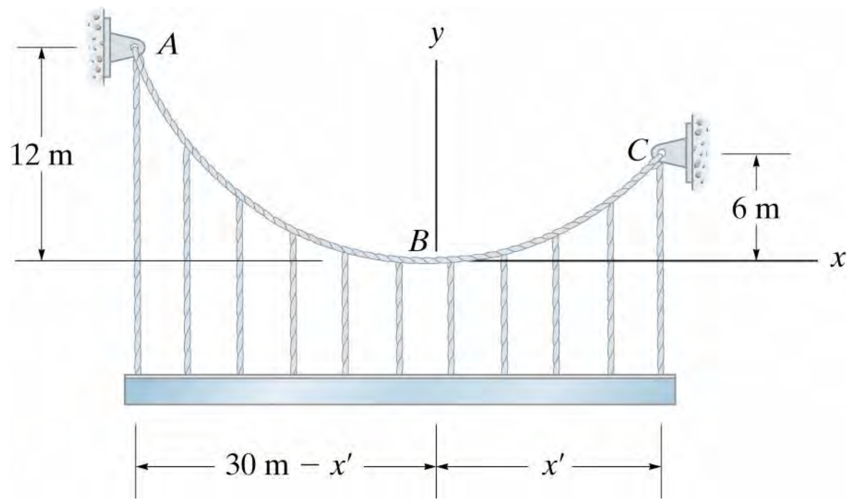


Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

The origin of the coordinate axes is established at point *B*, the lowest point on the cable where slope is zero,

$$y = \frac{w_o}{2F_H}x^2 = \frac{12\,\text{kN/m}}{2F_H}x^2 = \frac{6}{F_H}x^2 \quad (1)$$



Assuming point *C* is located *x*' from *B* we have:

$$6 = \frac{6}{F_H}x'^2 \Rightarrow F_H = 1.0x'^2 \quad (2)$$

PEARSON

Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

For point *A*,

$$12 = \frac{6}{F_H}[-(30 - x')]^2$$

$$12 = \frac{6}{1.0x'^2}[-(30 - x')]^2$$

$$x'^2 + 60x' - 900 = 0 \Rightarrow x' = 12.43\,\text{m}$$

Thus from eqn 2 and 1, we have:

$$F_H = 1.0(12.43)^2 = 154.4\,\text{kN}$$

$$\frac{dy}{dx} = \frac{12}{154.4}x = 0.7772x \quad (3)$$

Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

At point *A*,

$$x = -(30 - 12.43) = -17.57\,\text{m}$$

$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{x=-17.57} = 0.7772(-17.57) = -1.366$$

$$\theta_A = -53.79^o$$

We have,

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{154.4}{\cos(-53.79^o)} = 261.4\,\text{kN}$$

Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

At point *B*, *x* = 0

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=0} = 0 \Rightarrow \theta_B = 0^o$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{154.4}{\cos 0^o} = 154.4\,\text{kN}$$

At point *C*, *x* = 12.43 m

$$\tan\theta_C = \left.\frac{dy}{dx}\right|_{x=12.43} = 0.7772(12.43) = 0.9660$$

$$\theta_C = 44.0^o$$

$$T_C = \frac{F_H}{\cos\theta_C} = \frac{154.4}{\cos 44.0^o} = 214.6\,\text{kN}$$

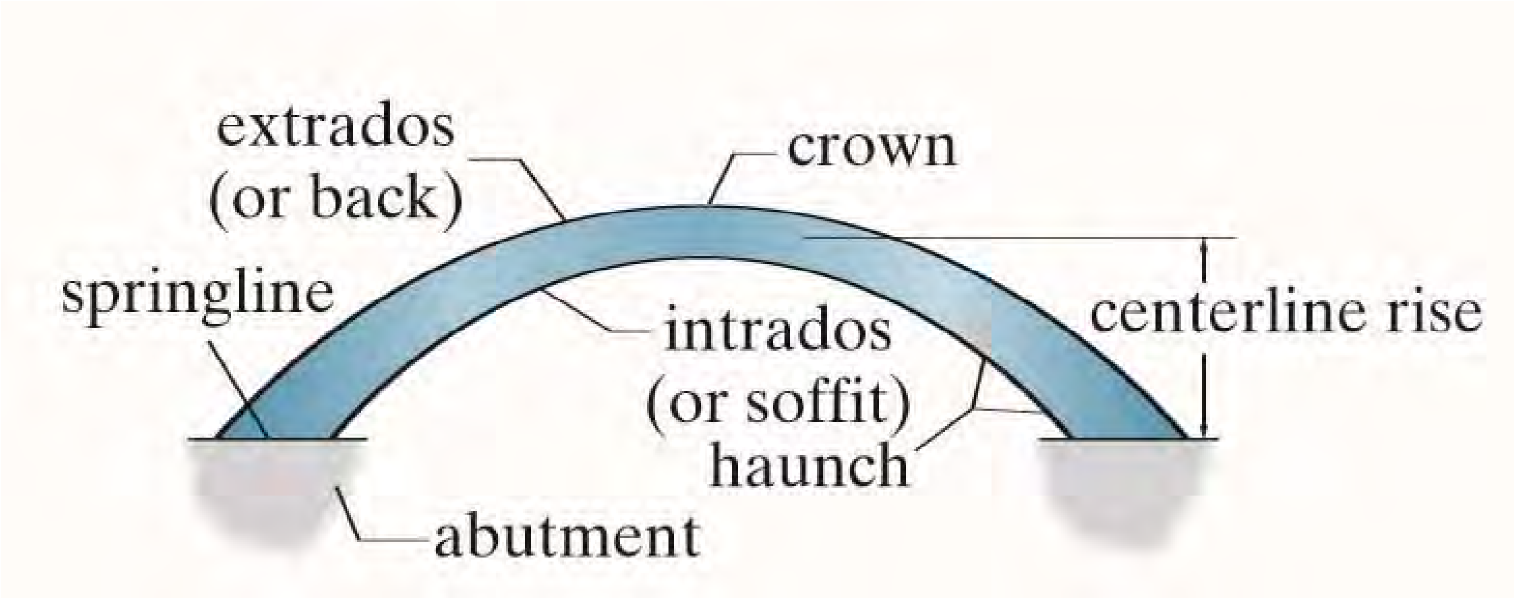
5.4

ARCHES

5.4

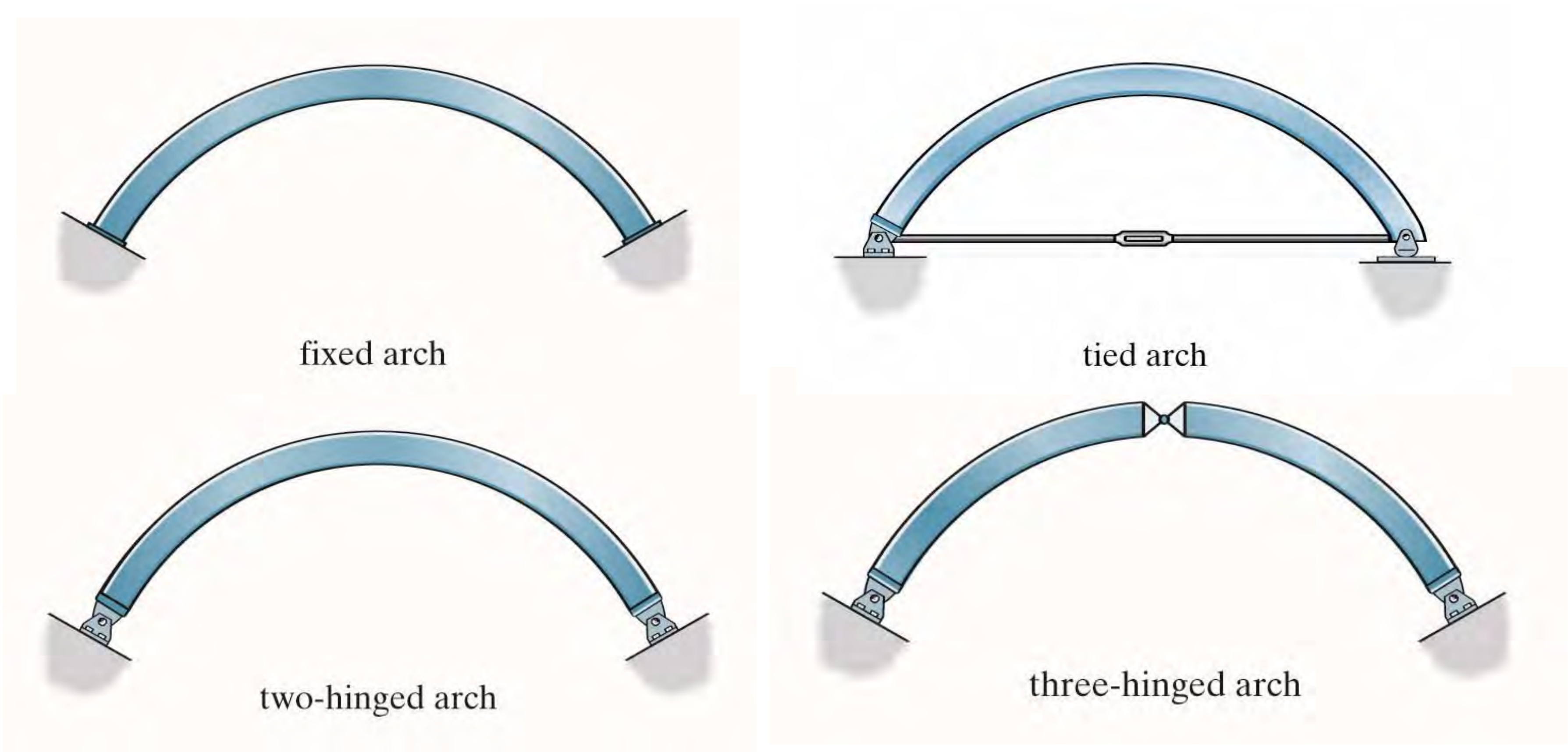
Arches

- An arch acts as inverted cable so it receives loading in compression
- Because of its rigidity, it must also resist some bending and shear depending upon how it is loaded & shaped



Arches

- Depending on its uses, several types of arches can be selected to support a loading

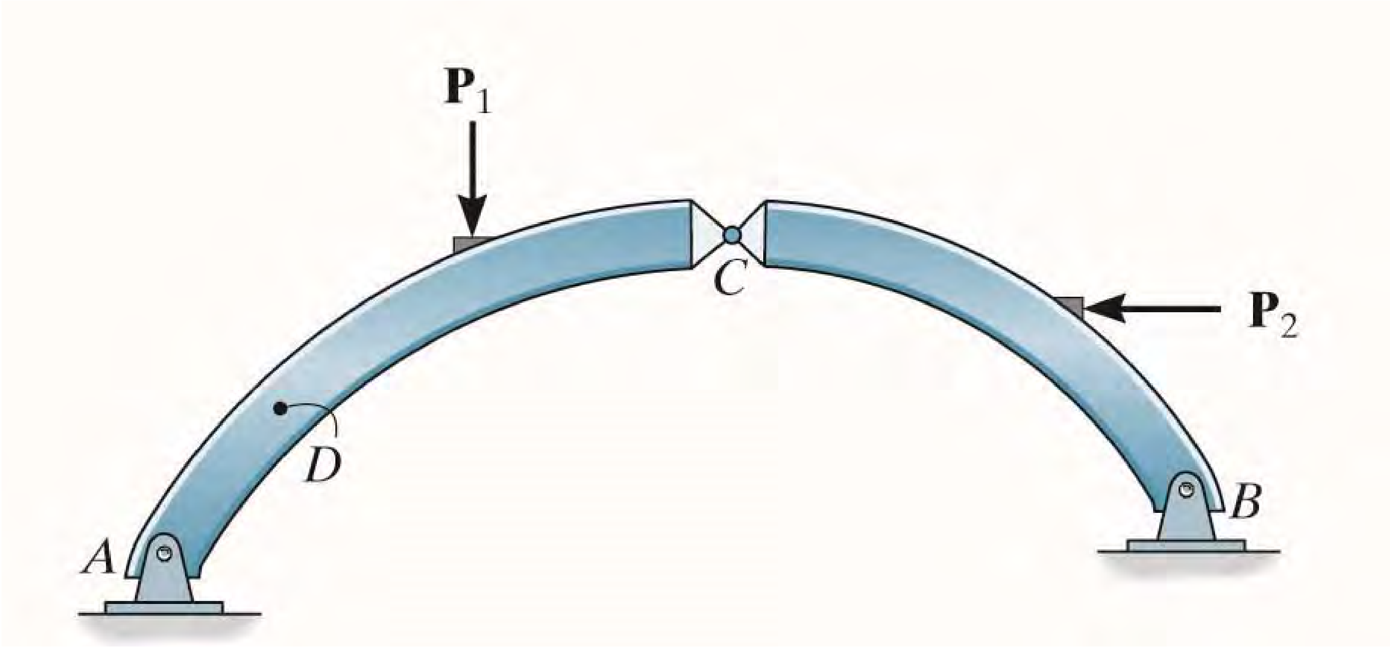


5.5
THREE-HINGED ARCH

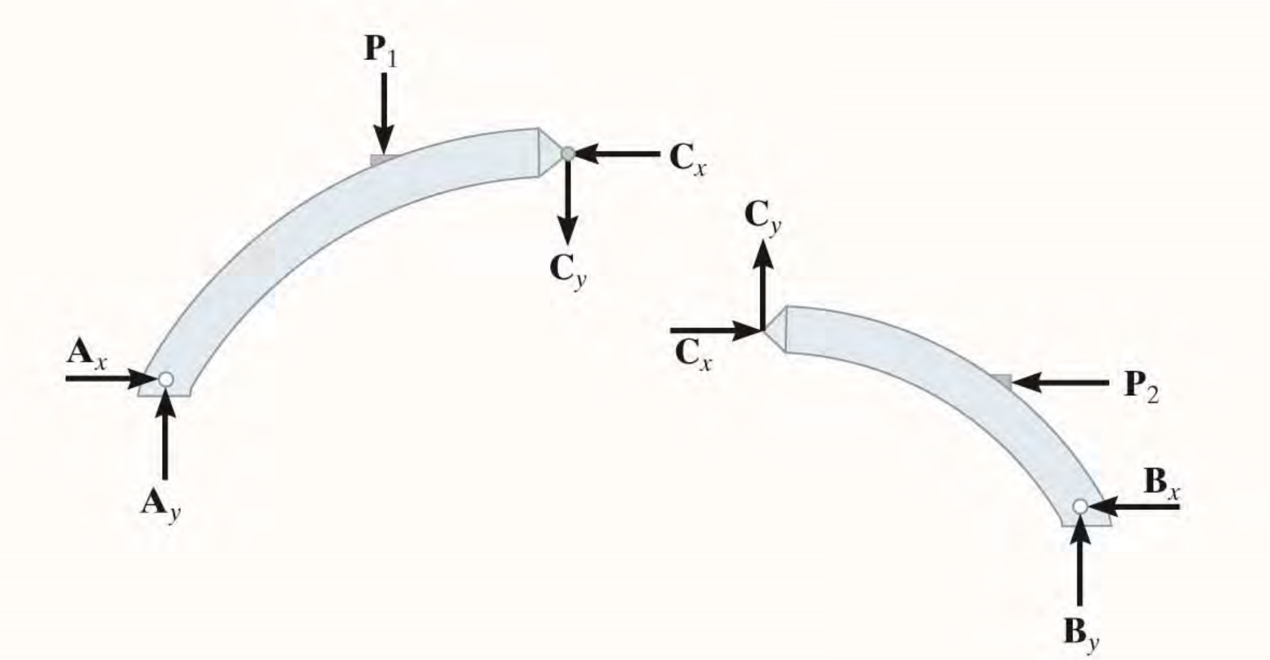
5.5

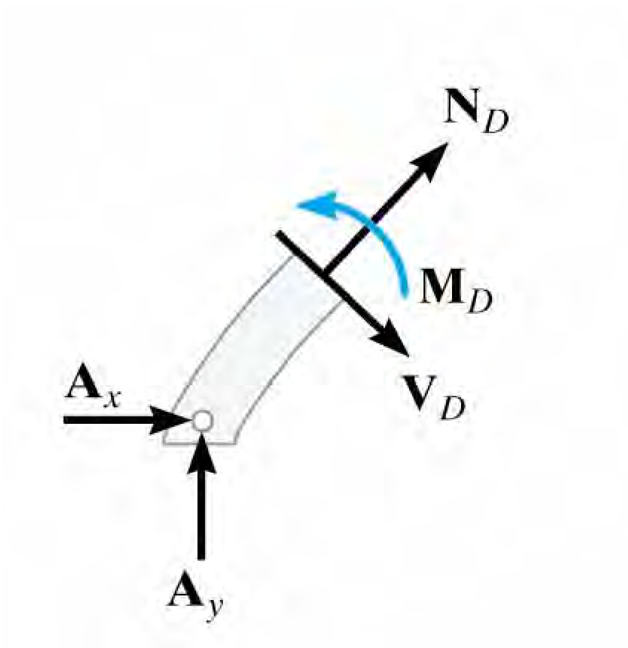
Three-Hinged Arch

- The third hinge is located at the crown & the supports are located at different elevations
- To determine the reactions at the supports, the arch is disassembled



Three-Hinged Arch

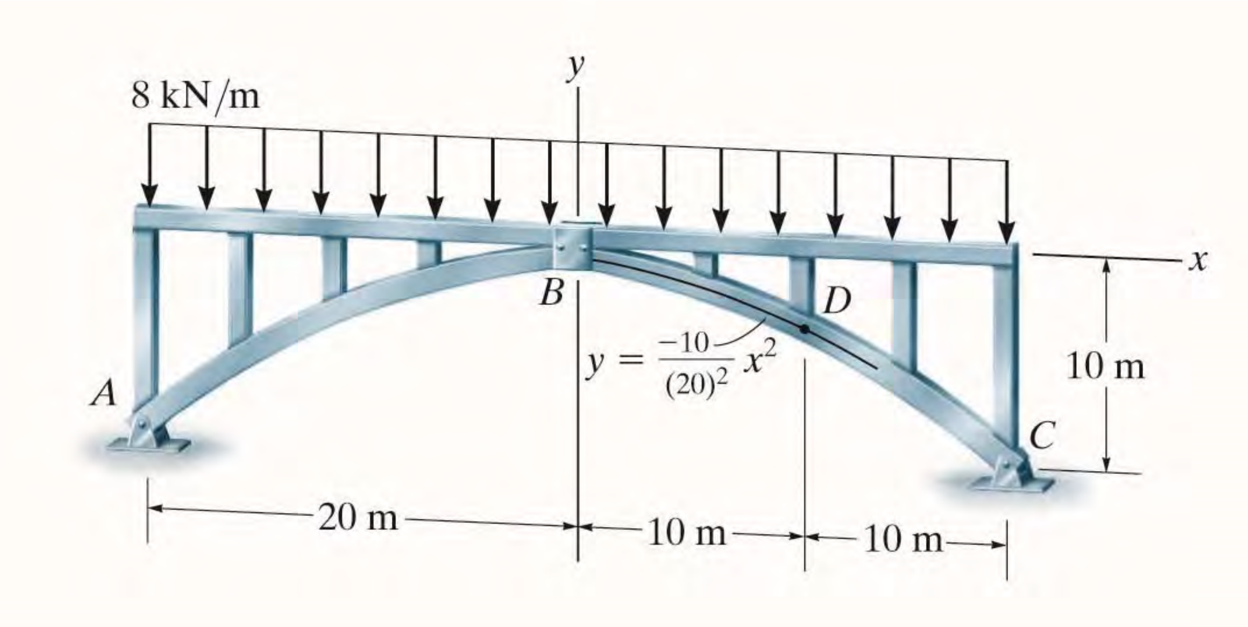




Three-Hinged Arch

Example 5.4

The three-hinged open-spandrel arch bridge has a parabolic shape and supports a uniform load. Show that the parabolic arch is subjected *only to axial compression* at an intermediate point such as point *D*. Assume the load is uniformly transmitted to the arch ribs.



Three-Hinged Arch

Example 5.4 (Solution)

Applying the eqns of equilibrium, we have:

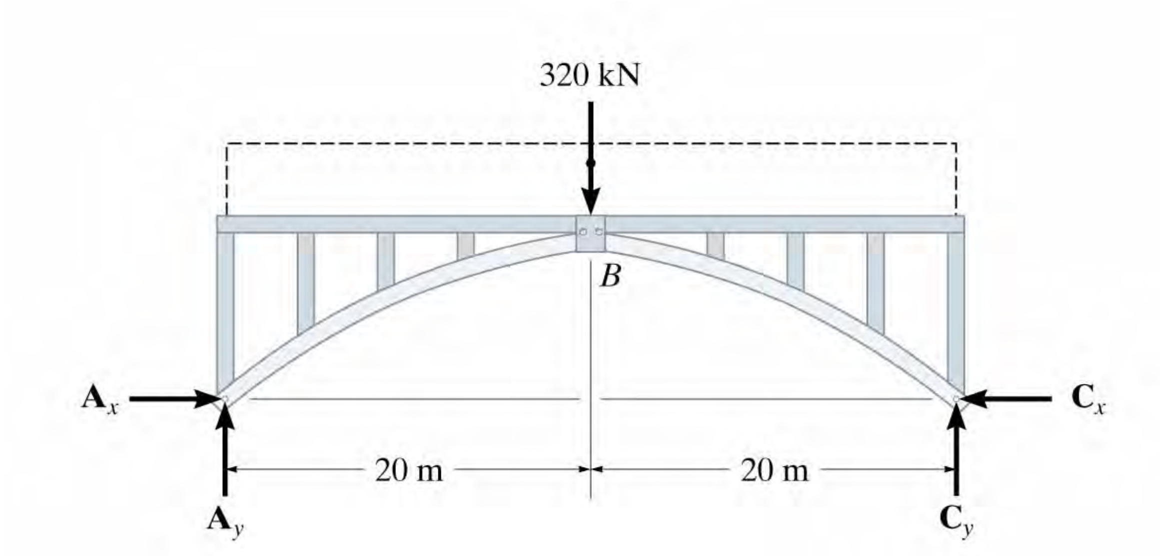
Entire Arch:

With anti-clockwise direction moments as +ve,

$$\sum M_A = 0$$

$$C_y(40\,\text{m}) - 320\,\text{kN}(20\,\text{m}) = 0$$

$$C_y = 160\,\text{kN}$$



Three-Hinged Arch

Example 5.4 (Solution)

Arch segment BC:

With anti-clockwise direction moments as +ve,

$$\sum M_B = 0$$

$$-160\,\text{kN}(10\,\text{m}) + 160\,\text{kN}(20\,\text{m}) - C_x(10\,\text{m}) = 0$$

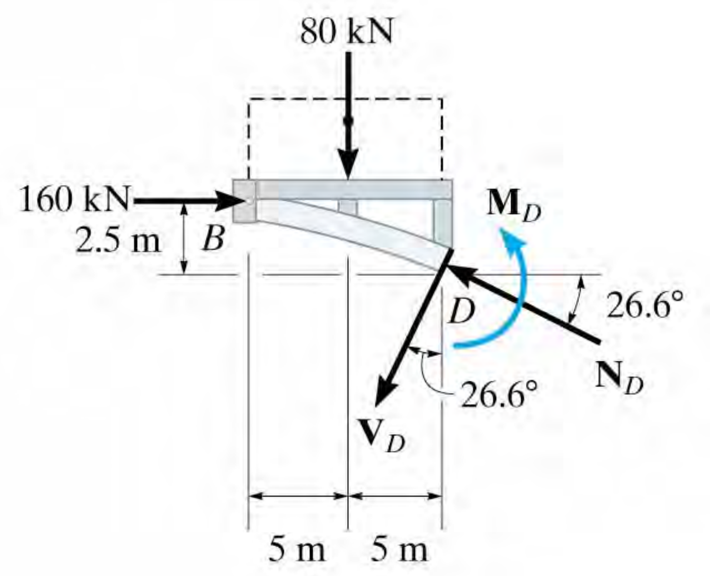
$$C_x = 160\,\text{kN}$$

$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow B_x = 160\,\text{kN}$$

$$+\uparrow \sum F_y = 0$$

$$B_y - 160\,\text{kN} + 160\,\text{kN} = 0$$

$$B_y = 0$$



Three-Hinged Arch

Example 5.4 (Solution)

A section of the arch taken through point *D*

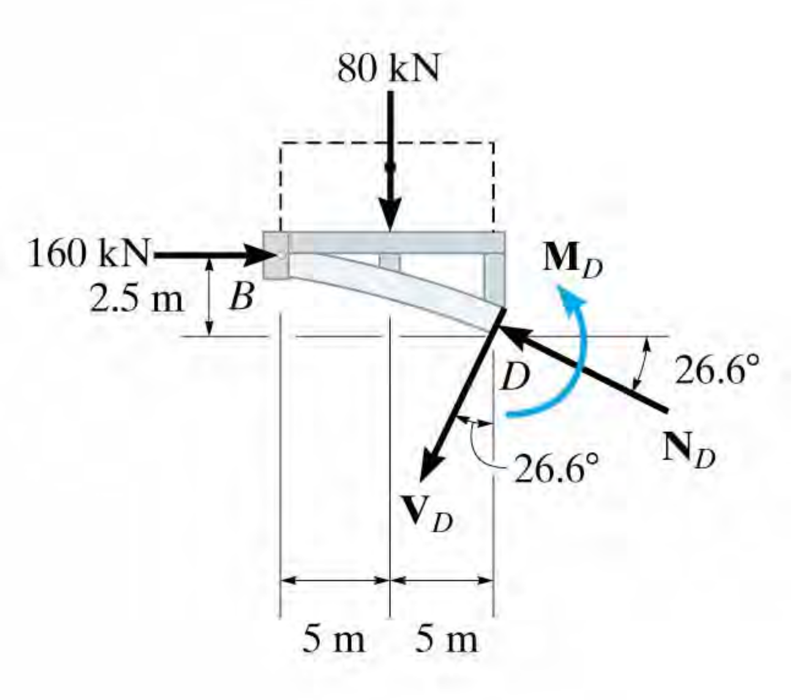
$$x = 10\,\text{m}$$

$$y = -10(10)^2/(20)^2 = -2.5\,\text{m}$$

The slope of the segment at D is:

$$\tan\theta = \frac{dy}{dx} = \frac{-20}{(20)^2}x\Big|_{x=10\,\text{m}} = -0.5$$

$$\theta = 26.6^\circ$$



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Three-Hinged Arch

Example 5.4 (Solution)

Applying the eqn of equlibrium, Fig 5.10(d), we have:

$\underset{\rightarrow}{+} \sum F_x = 0$

$160\,\text{kN} - N_D \cos 26.6^o - V_D \sin 26.6^o = 0$

$+\uparrow \sum F_y = 0$

$\Rightarrow N_D = 178.9 kN$

$-80\,\text{kN} + N_D \sin 26.6^o - V_D \cos 26.6^o = 0$

$\Rightarrow V_D = 0$

With anti-clockwise moments as +ve:

$\sum M_D = 0$

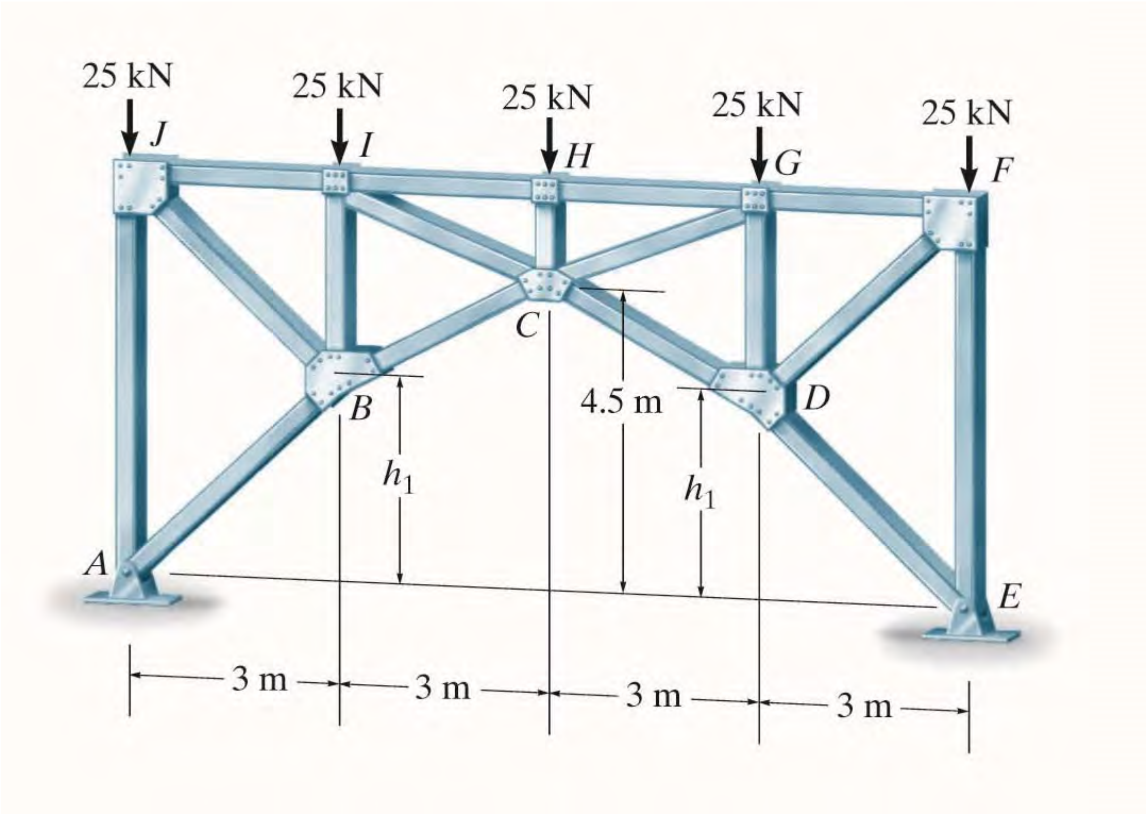
$\Rightarrow M_D = 0$

$M_D + 80\,\text{kN}(5\,\text{m}) - 160\,\text{kN}(2.5\,\text{m}) = 0$

Three-Hinged Arch

Example 5.6

The three-hinged trussed arch supports the symmetric loading. Determine the required height of the joints *B* and *D*, so that the arch takes a funicular shape. Member *HG* is intended to carry no force.



Three-Hinged Arch

Example 5.6 (Solution)

For a symmetric loading, the funicular shape for the arch must be parabolic as indicated by the dashed line. Here we must find the eqn which fits this shape.

With the x, y axes having an origin at C, the eqn is of the form of $y = -cx^2$. To obtain the constant c, we require:

$$-(4.5\,\text{m}) = -c(6\,\text{m})^2$$

$$c = 0.125/\text{m}$$

Therefore,

$$y_D = -(0.125/\text{m})(3\,\text{m})^2 = -1.125\,\text{m}$$

From Fig 5.12(a)

$$h_1 = 4.5\,\text{m} - 1.125\,\text{m} = 3.375\,\text{m}$$