Q1/ Fill in The Blanks the Correct Answer (or Answers)

1.Given $h_1 = 1$, $h_2 = r$, $h_3 = r \sin\theta$ then the velocity is ....

2. Newton's theory of motion starts from a set of terms which are .... within the theory.

(a) well define, (b) undefined, (c) mass and time, (d) axiomatic type.

3. $\vec{A}$ x $\vec{B}$ =0 when ....

$(a)\vec{A} \parallel \overrightarrow{B}$, (b) $\vec{A} \perp \vec{B}$, (c) $\overrightarrow{A}$ $and\ \vec{B}\ in\ same\ plane$,(d) $\vec{A}\ or\ \vec{B}\ is\ null\ vector$

4. The direction of average velocity is same as ....

(a) $velocity\ \vec{v}$, (b) $tangent\ \vec{\tau}$, (c) $accelaration\ \vec{a}$, (d) $\Delta\vec{\boldsymbol{r}}$

5. The acceleration of a particle is given by ....

(a) $\frac{dv}{dt}\vec{\tau}$, (b) $v\frac{d\vec{\tau}}{dt}$, (c) $\frac{dv}{dt}\vec{\tau} + v\frac{d\vec{\tau}}{dt}$, (d) $\frac{dv}{dt}\vec{\tau} + kv^2\vec{n}$.

6. If the velocity of a particle is $-6sin2t\vec{\boldsymbol{\imath}} + 6cos2t\vec{\boldsymbol{\jmath}}$ then the speed is .....

(a) 6, (b) 12, (c) 36, (d) 10

7. The curvature of straight line is ....

(a) Infinite, (b) zero, (c) undefined (d) $3/25$

8. The position vector of a particle is given by $\vec{r} = \vec{\imath}b\ sin\omega t + \vec{\jmath}bcos\ \omega t$, then the speed is ....

(a) $-w^2\vec{r}$, (b) bw ,(c) $\vec{\imath}bw\ cos\ wt - \vec{\jmath}bw\ sin\ wt$ (d) $bw\ cos\ wt - bw\ sin\ wt$

9. For a circular motion, when the angular frequency ω changes with time then a_θ is .... and a_r is ....

10. In polar coordinate system the transverse component of $\vec{a}$ is ...

(a) $\left(\overset{\circ\circ}{r} - r\overset{\circ}{\theta}^2\right)$ (b) $\left(r\overset{\circ\circ}{\theta} + 2\overset{\circ}{r}\overset{\circ}{\theta}\right)$, (c) $\frac{1}{r}\frac{d}{dt}\left(r^2\overset{\circ}{\theta}\right)$,(d) $\left(r\overset{\circ\circ}{\theta} + 2\overset{\circ}{r}\overset{\circ}{\theta}\right)\overrightarrow{e_\theta}$

Q2/ Given spherical coordinate system then

a) Draw the coordinate system.

b) $x =$............ , $y =$............... , $z =$................

c) $q_1 = \cdots\cdots\cdots$, $q_2 = \cdots\cdots\cdots$, $q_3 = \cdots\cdots\cdots$

d) $h_1 = \cdots\cdots$, $h_2 = \cdots\cdots$, $h_3 = \cdots\cdots$

e) $\vec{r} = \cdots\cdots\cdots \vec{e}_r$

f) $\vec{e}_r = \cdots\cdots\cdots \vec{\imath} + \cdots\cdots\cdots \vec{\jmath} + \cdots\cdots\cdots \vec{k}$

g) $\vec{e}_r \cdot \vec{e}_\theta = \cdots\cdots$

h) $\vec{r} = \cdots \vec{\imath} + \cdots \vec{\jmath} + \cdots \vec{k}$

i) $show\ that: \ \vec{v} = \overset{\circ}{r}\vec{e}_r + r\overset{\circ}{\theta}\vec{e}_\theta + r sin\theta\overset{\circ}{\varphi}\vec{e}_\varphi$

Q3/ An electron moves in a force field due to a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ that is at right angles to $\vec{E}$. Let $\vec{E} = \vec{\jmath}E$ and $\vec{B} = \vec{k}B$. Take the initial position of the electron at the origin with initial velocity $\vec{v_0} = \vec{\imath}v_0$ in the x direction. (a) Find the resulting motion of the particle. (B)Show that the path of motion is a cycloid:

$x = a \sin(\omega t) + bt$

$y = a[1 - \cos(wt)]$

$z = 0$

Q4/

(a) Calculate the gravity acceleration ***g*** at the surface, and the ***escape velocity*** from the surface of *Jupiter* where $r_j = 11.209\, r_e$, $M_j = 317.8\, M_e$.

Use: $G = 6.67 \times 10^{-11} Nm^2/kg^2$,

$r_e = 6378.137\ km\ (Earth's\ radius)$

$M_e = 5.97237 \times 10^{24}\ kg\ (Earth's\ mass)$

(b) Discuss: The *moon* has no atmosphere.

Q5/ A particle moves in a spiral path such that the radial distance decrease at a constant rate [$r = b - ct$], while the angular speed increase at a constant rate [$\overset{\circ}{\theta} = kt$].

(a) Find the velocity

(b) Find the speed when $t = \frac{b}{c}$

Q6/

a) If $\vec{\boldsymbol{n}}$ is a unit normal vector to a space curve C, show that $d\vec{\mathbf{n}}/dt$ is normal to $\vec{\boldsymbol{n}}$.

b) Given $\vec{r} = \vec{\imath}b\ sin\omega t + \vec{\jmath}bcos\ \omega t$. Show that the velocity and acceleration are orthogonal.

c) Given spherical coordinate system: $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$: Find h_3

Q7 / If a particle moves on a curve of constant radius. Discuss this motion using polar coordinate system when the angular frequency changes with time.

Q8/

a) Find the force acting on a particle affected by potential $V = 2x^2 + 3y + 4$.

b) Find the angular momentum of a particle affected by force $\vec{F} = f(r)\vec{e}_r$

c) Damped HO has $x = 2e^{-1.0t}\cos 3t$, $m = 15g$. Calculate energy loos per time.

Q9/ Consider a 2-D central force. Set up

a) Lagrangian, generalized momentum, and generalized force for this system. Hamilton equations which describe this system.

Q10/ Calculate the orbital speed of earth around sun.

Q11/

(a)Find the orbit equation of a planet travelling around the sun

using the Lagrange equation.

(b) Show that the path is ellipse.

Q12/

Write Lagrange equation for a single particle in a central force field.Then

find the orbit equation and apply that to a planet under the effect of inverse square force.

Q13/ Given a space curve C with position vector

$\vec{\boldsymbol{r}} = 3\cos 2t\vec{\mathbf{i}} + 3\sin 2t\vec{\mathbf{j}} + (8t - 4)\vec{\mathbf{k}}$

Find the (a) unit tangent vector $\vec{\boldsymbol{\tau}}$ to the curve., (b) curvature, (c) radius of curvature and (d) unit principal normal $\vec{\boldsymbol{n}}$ to any point of the space curve C

Q14/ Given $\vec{r} = \vec{a}cos\omega t + \vec{b}sin\omega t$, where $\vec{a}$ and $\vec{b}$ are any constant non-collinear unit vectors and ω is a constant scalar. Prove that

(a) $\vec{r} \times \frac{d\vec{r}}{dt} = w(\vec{a} \times \vec{b})$, (b) $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r} = 0$

Q15/ show that the velocity vector in (three dimensional)3D-curvilinear coordinate (q_1, q_2, q_3) system is given by:

$$\vec{v} = h_1\mathring{q}_1\overrightarrow{e_1} + h_2\mathring{q}_2\overrightarrow{e_2} + h_3\mathring{q}_3\overrightarrow{e_3}$$

Q16/ Find the velocity x and the position x as functions of the time t for a particle of mass which starts from rest at $x = 0$ and $t = 0$, subject to the following force functions:
(a) $F_x = F_0 + ct$
(b) $F_x = F_0 \sin\ ct$
(c) $F_x = F_0 e^t$
where F_0 and c are positive constants.
Q17/ Find the velocity $\dot{x}$ as a function of the displacement x for a particle of mass m, which start from rest at $x = 0$, subject to the following force functions:
(a) $F_x = F_0 + cx$
(b) $F_x = F_0 e^{-ax}$
(c) $F_x = F_0 \cos\ cx$
where F_0 and c are positive constants.
Q18/ Find the potential energy function $V(x)$ for each of the forces in Problem (2.2)
Q19/ A particle of mass m moves along a frictionless, horizontal plane with a speed given by $v(x) = \alpha/x$, where x is its distance from the origin and α is a positive constant. Find the force $F(x)$ to which the particle is subject.
Q20/ Given that the velocity of a particle in rectilinear motion varies with the displacement x according to the equation. $\dot{x} = bx^{-3}$ where b is a positive constant, find the force acting on the particle as a function of x

Q21/ A block of wood is projected up an inclined plane with initial speed v_0. If the inclination the plane is 30° and the coefficient of sliding friction $\mu_x = 0.1$, find the total time for block to return to the point of projection.

Q22/ A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous resistance that varies as the $\frac{3}{2}$ power of the speed: $F(v) = -cv^{3/2}$. If the initial speed of the block is v_0 at $x = 0$, show that the block cannot travel farther than $2mv_0^{1/2}/c$

Q23/ A particle of mass m is released from rest a distance b from a fixed origin of force that attracts the particle according to the inverse square law:

$F(x) = -kx^{-2}$. Show that the time required for the particle to reach the origin is $\pi\left(\frac{mb^3}{8k}\right)^{1/2}$

Q24 / The force acting on a particle of mass m is given by $F = kvx$, in which k is a positive constant. The particle passes through the origin with speed v_0 at time $t = 0$. Find x as a function of t

Q25/ A particle P of mass 2 moves along the x axis attracted toward origin O by a force whose magnitude is numerically equal to $8x$. If it is initially at $x = 10$ and moves toward origin with speed equal to 2. find (a) the differential equation and initial conditions describing the motion,(b) the position of the particle at any time, (c) the speed and the velocity of the particle at any time and (d) the amplitude, period and frequency of the vibration.

Q26 / Test Yourself by Completing These Sentences

1. The central force $f(r) < 0$ indicate that the force is ………………………
2. The magnitude of a central force depends only on …………………….
3. When the force is central then the orbit of the particle must be in ……………
4. In order to show that the central force is conservative, we calculate …………
5. In order to show that the angular momentum of a particle moves in a central force is constant we calculate ……………….
6. The angular momentum is zero when …………………..
7. According to Kepler's second law, the line joining a planet to the sun sweeps out ……………… areas in ……………………
8. Kepler's second law is nothing other than the ………………….. of a planet about the sun is conserved quantity.
9. The ratio of angular momentum per mass is ………………..
10. The quantity $\frac{h}{2}$ is ……………..

11. For circular orbits the transverse component of acceleration is ……….
12. The attractive force for which all circular orbits have identical areal velocities is the …………………
13. To obtain the *equation of the orbit* in central field we use a parametric equation given by …………………..
14. When $r = \frac{1}{u}$ then $\overset{0}{r} =$ ……………………
15. The quantity $\frac{f(u^{-1})}{mh^2u^2}$ is equal to …………………
16. To find out how the angle θ varies with time, we start with the equation ………………………………………..
17. When a particle subjected to the force of gravity, then the force is given by …………………………………..
18. The following equation $\left(\frac{d^2u}{d\theta^2} + u\right) = \frac{k}{mh^2}$ has the same form of ………… with adaptive constant …………..
19. The following equation $r = \frac{\frac{mh^2}{k}}{1+\frac{Amh^2}{k}\,cos(\theta)}$ describes the equation ……….. in ………. system.
20. Any *conic section*, hyperbola, parabola and ellipse has the general form ……………………..
21. Comets generally have ……………………... orbital eccentricities.
22. In the case of circular motion, the eccentricities is equal to ……………….
23. The ISS speed is equal to …………. While the altitude is ……………………..
24. Kepler's third law: the squares of the orbital periods of the planets are directly proportional to the …………………………..
25. The energy equation of an orbit in a central force field is given by ………….

Q27/ Find the velocity and the potential energy as a function of the isplacement x for aparticle of mass m, which start from rest at $x = 0$, subject to the following force $F_x = F_0\cos(cx)$. where F_0 and c are positive constants.
Q28 /Write the differential equation of harmonic oscillator in the case of motion in a single plane xz

Q29/ Given $m\ddot{x} = -kx$ and $m\ddot{y} = -ky$ than (a) is these equations are separatable or not? (b) is the harmonic oscillator being isotropic or non-isotropic?

Q30/ Given $x = Acos(\omega t + \alpha)$ and $y = Bcos\ (\omega t + \beta)$ is …………… where $A, B, \alpha,$ and β are …………… determined from ………….. to find the equation of the path, we eliminate ………………. from these two equations.

Q31/ In this equation $y = Bcos\ (\omega t + \alpha + \Delta)$ Δ is ……

Q32 / Show that $\alpha = cos^{-1}(\frac{x}{A})$

Q33/ Given $\frac{x^2}{A^2} - xy\frac{2cos\ \Delta}{AB} + \frac{y^2}{B^2} = sin^2\ \Delta$ (a) show this equation is equation of ellipse. (b) suppose $\Delta = \pi/2$ and $A = B$ what will be the path? (c) in which the path equation is $y = -\frac{B}{A}x$ (d) show that in general ,the axis of the elliptical path is inclined to the x-axis by the angle ψ then $tan\ 2\psi = \frac{2ABcos\ \Delta}{A^2-B^2}$

Q34/ Show that $V(x, y, z) = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2y^2 + \frac{1}{2}k_3z^2$

Q35/ Given $V(\mathbf{r}) = \frac{1}{2}k(x^2 + 4y^2)$, calculate ω_x , ω_y , F_x , F_y

Q36/ In problem 35 above, given the initial conditions $t = 0: x = a, y =$ b, $\dot{x} = 0, \dot{y} = v_0$ then find x and y as a function of time.

Q37/ Given $V(\mathbf{r}) = \frac{1}{2}k(x^2 + 2.3\ y^2)$ is the path is repeated harmonically with time? why?

Q38/ A particle in a central field moves in the spiral orbit ($r = c\theta$)
Find the force function by using the energy equation.
How r changes with time?

Q39/ Assuming the variation of gravity with height show that

$$\frac{1}{2}mv^2 - mgr_e^2\left(\frac{1}{r_e+x}\right) = \frac{1}{2}mv_0^2 - mgr_e^2(\frac{1}{r_e+x_0}).$$

Q40/ A particle undergoing simple harmonic motion has a velocity $\dot{x}_1$ when the displacement is x_1 and a velocity $\dot{x}_2$ when the displacement is x_2. Find the angular frequency and the amplitude of the motion in terms of the given quantities.

Q41/ Two springs having stiffness k_1 and k_2, respectively, are used in a vertical position to support a single object of mass m. Show that the angular frequency of oscillation is $[(k_1 + k_2)/m]^{1/2}$ if the springs are tied in parallel, and $[k_1 k_2/(k_1 + k_2)m]^{1/2}$ if the springs are tied in series.

Q42/ A spring of stiffness k supports a box of mass M in which is placed a block of mass m. If the system is pulled downward a distance d from the equilibrium position and then released, find the force of reaction between the block and the bottom of the box as a function of time. For what value of d does the block just begin to leave the bottom of the box at the top of the vertical oscillations?

Q43/ Show that the ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant. (Note: The maxima do not occur at the points of contact of the displacement curve with the curve $Ae^{-\gamma t}$.)

Q44/ A particle is placed on top of a smooth sphere of radius a. If the particle is slightly disturbed, at what point will it leave the sphere?

Q45/ A particle of mass m moving in three dimensions under the potential energy function $V(x, y, z) = \alpha x + \beta y^2 + \gamma z^3$ has speed v_0 when it passes through the origin.
(a) What will its speed be if and when it passes through the point (1,1,1)?
(b) If the point (1,1,1) is a turning point in the motion ($v = 0$), what is v_0?
(c) What are the component differential equations of motion of the particle?
(Note: It is not necessary to solve the differential equations of motion in this problem.)

Q46/ The initial conditions for a two-dimensional isotropic oscillator are as follows: $t = 0, x = A, y = 4A,\ \dot{x} = 0, \dot{y} = 3\omega A$ where ω is the angular frequency. Find x and y as functions of t. Show that the motion takes place entirely within a rectangle of dimensions $2A$ and $10A$. Find the inclination ψ of the elliptical path relative to the x-axis. Make a sketch of the path.

Q47/ A double pendulum, see example 6.2, vibrate in vertical plane then:

a) Find the kinetic energy.
b) Find the potential energy (taking the reference plane a level at $\ell_1 + \ell_2$ below the point of suspicion.

c) Setup the Lagrangian of the system.