Questions Bank

Differential Equations

By Mudhafar Hamed Hamadamen
mudhafar.hamadamen@su.edu.krd

1. What is the difference between particular and singular solutions of an ordinary differential equation.

2. Solve the ordinary differential equation $(xD^3 + 4D^2)y = 8e^x$,where $D = \frac{d}{dx}$.

3. Solve

$$\frac{dy}{dx} + (x+1)y = e^{x^2} y^3 .$$

4. Let $(sinycosy + xcos^2y)dx + xdy = 0$. Test for exactness. If it is not exact then find an integrating factor.

5. Find the orthogonal trajectories of family of curves $4x^2 + y^2 = c$, where c being a parameter.

6. Find a general solution of differential equation $(D^4 + a^4)y = 0$,where $D = \frac{d}{dx}$.

7. Show that e^{3x} and xe^{3x} form a basis of the following differential equation $y'' - 6y' + 9y = 0$. Find also the solution that satisfies the conditions $y(0) = -1.4$, $y'(0) = 4.6$.

8. Use the method of undetermined coefficients to find the particular solution of the differential equation $y'' - 4y' + 4y = 2e^{2x}$.

9. Let $y'' + p(x)y' + q(x)y = 0$. Suppose $y_1(x)$ and $y_2(x)$ are two solutions of given differential equation . Show that linear combination of two solutions $(y_1(x), y_2(x))$ on an open interval I, is again a solution of given differential equation on I.

10. Find a homogeneous linear ordinary differential equation for which to functions x^3 and x^{-2} are solutions. Show also linear independence by considering their Wronskian.

11. Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = -5y_1 + 2y_2 \ , y_1(0) = 1$$

$$\frac{dy_2}{dt} = 2y_1 - 2y_2 \ , y_2(0) = -2 .$$

12. Use the method of variation of parameters to find the general solution of the differential equation $(D^2 - I)y = \frac{1}{coshx}$.

13. Find a general solution of differential equation $x^2y'' - xy' + y = xln|x|$.

14. Find the radius of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n(x-1)^{2n}}{4^n}.$$

15. Find a power series solution of the following differential equation (in power of x) $y'' + xy' - 2y = 0$.

16. Find a partial differential equation by eliminating a and b

i) $z = ax + by + a^2 + b^2$

ii) $z = a(x + y) + b$.

17. Find the general solution of partial differential equation $yzu_x - xzu_y + xy(x^2 + y^2)u_z = 0$.

18. Solve the initial value problem $u_x + 2u_y = 0, u(0, y) = 4e^{-2y}$ using the method of separation of variables.

19. Obtain the canonical form of the equation $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$ and hence find the general solution .

20. Reduce the following partial differential equation with constant coefficients, $u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0$ into canonical form.

21. Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution .

22. Find the solution of quasi-linear partial differential equation $u(x + y)u_x + u(x - y)u_y = x^2 + y^2$, with the Cauchy data $u = 0$ on $y = 2x$.

23. Show that

$$P_n(x) = \frac{1}{2^n n!} \frac{dn}{dx^n} (x^2 - 1)^n.$$

24. Find a solution $(a^2 - x^2)y'' - 2xy' + n(n + 1)y = 0 , a \neq 0$ by reduction to the Legendre equation.

25. Find a general solution of $y'' + 2y' - 24y = 0$ by Conversion of an nth order ODE to a System.