Chapter Two: PRESSURE MEASUREMENT

Outlines:

2.1. Pressure of a liquid

2.2. Pressure head of a liquid

2.3. Pascal's law

2.4. Absolute and gauge pressures.

2.5. Measurement of pressure–

Manometers–Mechanical gauges

Highlights

2.1. PRESSURE OF A LIQUID

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The *force per unit area is called* **pressure.**

If, P = The force, and

A = Area on which the force acts; then intensity of pressure, $p = \frac{P}{A}$ (2.1)

The pressure of a fluid on a surface will *always act normal to the surface*.

2.2. PRESSURE HEAD OF A LIQUID

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Consider a vessel containing liquid, as shown in Fig. 2.1. The liquid will exert pressure on all sides and bottom of the vessel. Now, let cylinder be made to stand in the liquid, as shown in the figure.

Let, h = Height of liquid in the cylinder,

A = Area of the cylinder base,

w = Specific weight of the liquid,

and, p = Intensity of pressure.

Now, Total pressure on the base of the cylinder = Weight of liquid in the cylinder *i.e.*,

$pA = wAh$

$p = \frac{wAh}{A} = wh$ $\quad$ $i.e., p = wh$ (2.2)

As $p = wh$, *the intensity of pressure in a liquid due to its depth will vary directly with depth*.

As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the *height of the free surface* which would cause the pressure, *i.e.*,

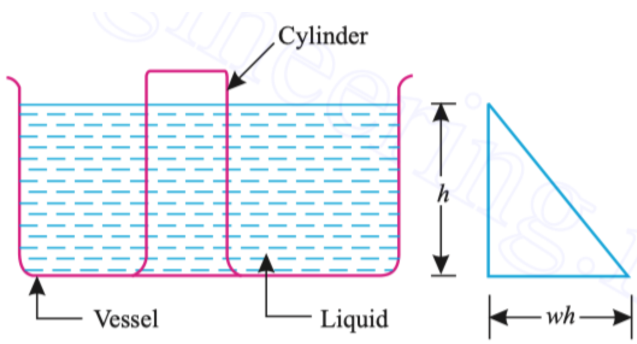


Fig. 2.1. Pressure head.

From equation (2.2) $\quad h = \frac{p}{w}$

The height of the free surface above any point is known as the *static head* at that point. In this case, static head is h.

Hence, the intensity of pressure of a liquid may be expressed in the following two ways:

1. As a force per unit area (*i.e.*, N/mm^2, N/m^2), and
2. As an equivalent static head (*i.e.*, meters, mm or cm of liquid).

Alternatively:

Pressure variation in fluid at rest:

In order to determine the pressure at any point in a fluid at rest "*hydrostatic law*" is used; the law states as follows:

"The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point."

The proof of the law is as follows.

Refer to Fig. 2.2

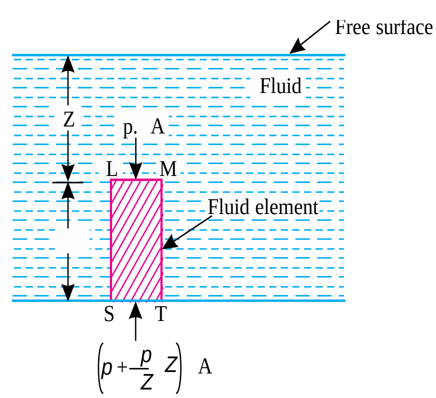


Fig. 2.2. Forces acting on a fluid element.

Let, p = Intensity of pressure on face LM, Fluid element

ΔA = Cross-sectional area of the element,

Z = Distance of the fluid element from free surface, and S T

ΔZ = Height of the fluid element.

The forces acting on the element are: _Z

(*i*) Pressure force on the face.

$LM = p \times \Delta A$ (acting downward)

(*ii*) Pressure force on the face $ST = \left(p + \frac{dp}{dZ} X \Delta Z\right) X \Delta A$ (acting upward)

(*iii*) Weight of the fluid element = Weight density × volume

$= w \times (\Delta A \times \Delta Z)$

(*iv*) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have:

$$pX\Delta A - \left[p + \frac{dp}{dZ} X\Delta Z\right] X\Delta A + wX(\Delta AX\Delta Z) = 0$$

Or, $pX\Delta A - pX\Delta A - \frac{dp}{dZ} X\Delta ZX\Delta A + wX\Delta AX\Delta Z = 0$

Or, $\frac{dp}{dZ}\Delta ZX\Delta A + wX\Delta AX\Delta Z = 0$

Or, $\frac{dp}{dZ} = w(cancellinng\ \Delta ZX\Delta A\ from\ both\ sides)$

Or, $\frac{dp}{dZ} = \rho Xg$ $(\because w = \rho Xg)$ (2.3)

Eqn. (2.3.) states *that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point*. This is "**hydrostatic law**".

On integrating the eqn. (2.3), we get:

$\int dp = \int \rho g.dZ$

Or, $p = \rho g.Z\ (= wZ)$ (2.4)

Where, p is the pressure above atmospheric pressure.

From equ.(2.4), we have: $Z = \frac{p}{\rho.g} (= \frac{p}{w})$

Here Z is known as *pressure head*.

Example 2.1. *Find the pressure at a depth of 15 m below the free surface of water in a reservoir.*

Solution. Depth of water, h = 15 m

Specific weight of water, w = 9.81 kN/m^3 **Pressure *p*:**

$p = wh = 9.81 \times 15 = 147.15$ kN/m^2

We know that, *i.e.,*

p=147.15kN/m^2=**147.15kPa**

Example 2.2. *Find the height of water column corresponding to a pressure of* 54 *kN/m*^{2}.

Solution. Intensity of pressure, p = 54 kN/m^2

Specific weight of water, w = 9.81 kN/m^3 **Height of water column, *h*:**

Using the relation $\boldsymbol{p = wh}$; $\boldsymbol{h = \frac{p}{w} = \frac{54}{9.81} = 5.5m}$

2.3. PASCAL'S LAW

The **Pascal's law** states as follows :

"***The intensity of pressure at any point in a liquid at rest, is the same in all directions***".

Proof. Let us consider a very small wedge shaped element *LMN* of a liquid, as shown in Fig. 2.3.

Let, p_x= Intensity of horizontal pressure on the element of liquid,

p_y= Intensity of vertical pressure on the element of liquid,

p_z= Intensity of pressure on the diagonal of the right angled triangular element,

α = Angle of the element of the liquid,

P_x= Total pressure on the vertical side *LN* of the liquid,

P_y= Total pressure on the horizontal side *MN* of the liquid, and

P_z= Total pressure on the diagonal *LM* of the liquid.

Fig. 2.3. Pressure on a fluid element at rest.

Now, $P_x = p_x \times LN$ ...(*i*)

and, $P_y = p_y \times MN$ ...(*ii*)

and, $P_z = p_z \times LM$ ...(*iii*)

As the element of the liquid is at rest, therefore the *sum of horizontal and vertical components of the liquid pressures must be equal to zero*.

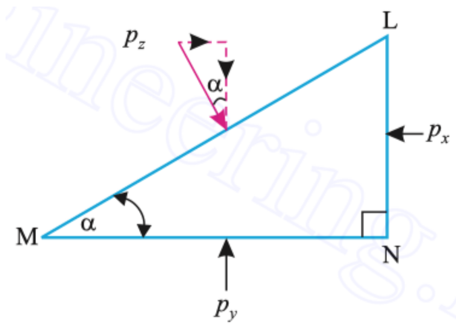


Fig. 2.3. Pressure on a fluid element at rest.

Resolving the forces *horizontally*:

$P_Z \sin\alpha = P_X$

$p_z.LM.sin\alpha = p_x.LN$ $\quad (\because p_z = p_z LM)$

$But \quad LM.sin\alpha = LN$ $\quad from\ equ. 2.3$

$\therefore p_z = p_x \quad s$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots(\ iv)$

Resolving the forces vertically:

$p_z cos\alpha = p_y - W$

(where W=weight of the liquid element)

Since the element is very small, neglecting its weight, we have:

$p_z cos\alpha = p_y \quad or\ p_z.LMcos\alpha = p_y.MN$

$But, \quad LMcos\alpha = MN$ $\quad from\ fig(2.3)$

$\therefore \quad p_z = p_y$ $\quad \ldots\ldots\ldots\ldots\ldots(v)$

$From\ (iv) \quad and\ \ (v),\ we\ get: \quad p_x = p_y = p_z$

Which is independent of α.

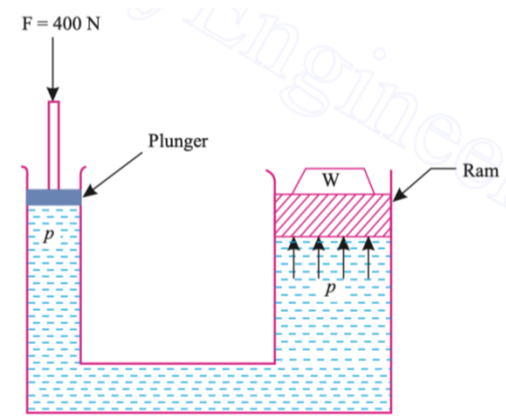
Hence, at *any point in a fluid at rest the intensity of pressure is exerted equally in all directions, which is called* **Pascal's law.**

Example 2.3. *The diameters of ram and plunger(needle) of an hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.*

Solution. Diameter of the ram, *D* = 200 mm = 0.2 m

Diameter of the plunger, *d* = 30 mm = 0.03 m

Force on the plunger, *F* = 400 N



Hydraulic press

Fig.(2.4)

Load lifted, W:

Area of ram, $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}X0.2^2 = 0.0314m^2$

Area of plunger, $a = \frac{\pi d^2}{4} = \frac{\pi}{4}X0.03^2 = 7.068X10^{-4}m^2$

Intensity of pressure due to plunger,

$p = \frac{F}{a} = \frac{400}{7.068X10^{-4}} = 5.66X10^5 N/m^2$

Since the intensity of pressure will be equally transmitted (due to Pascal's law), therefore the intensity of pressure at the ram is also

$= p = 5..66X10^5 N/m^2$

But intensity of pressure at the ram $= \frac{weight}{area\ of\ ram} = \frac{W}{A} = \frac{W}{0.0314} N/m^2$

$\therefore \frac{W}{0.0314} = 5.66X10^5 \;\; or\ W = 0.0314X5.66X10^5 N = 17.77X10^3 N \;\; or\ 17.77kN$

Example 2.4. *For the hydraulic jack shown in Fig. 2.5 find the load lifted by the large piston when a force of 400 N is applied on the small piston. Assume the specific weight of the liquid in the jack is 9810 N/m^3.*

Solution. Diameter of small piston, *d* = 30 mm = 0.03 m and F = 400 N.

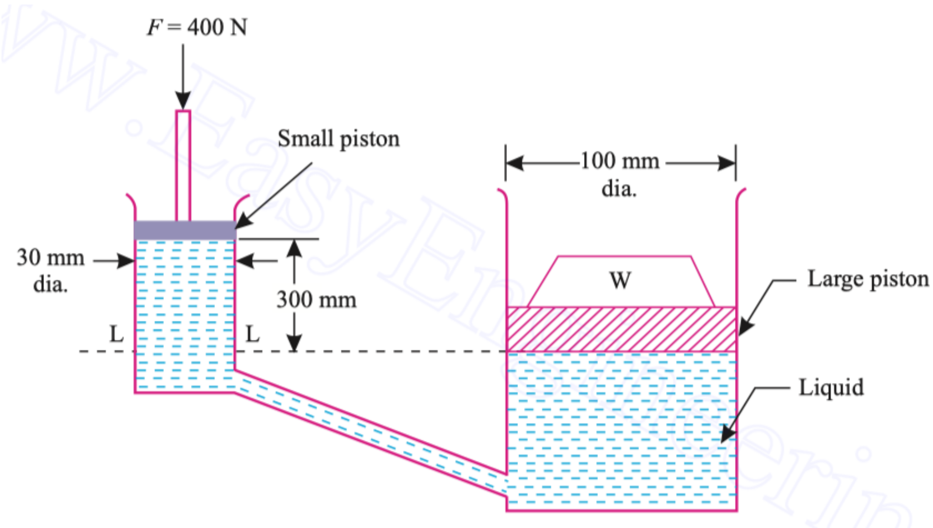


Fig.(2.5)

Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} X 0.03^2 = 7.068 X 10^{-4} m^2$

Dimeter of the large piston, $D = 100mm = 0.1m$

Area of large piston, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} X 0.01^2 = 7.854 X 10^{-3} m^2$

Force on small piston,F=400N

Load lifted, *W*:

Pressure intensity on the small piston, $p = \frac{F}{a} = \frac{400}{7.068 X 10^{-4}} = 5.66 X 10^5 N/m^2$

Pressure intensity on the section LL,

$p_{LL} = \frac{F}{a} + pressure\ intensity\ due\ to\ hieght\ of\ 300mm\ of\ liquid$

$= \frac{F}{a} + wh = 5.66 X 10^5 + 9810 X \frac{300}{1000}$

$= 5.66 X 10^5 + 2943 = 5.689 X 10^5 N/m^2$

Pressure intensity transmitted to the large piston = $5.689 \times 10^5 N/m^2$

Force on the large piston = Pressure intensity × area of large piston

= $5.689 \times 10^5 \times 7.854 \times 10^{-3}$ = 4468 N

Hence, *load lifted by the large piston* = **4468 N**

2.4.ABSOLUTE AND GAUGE PRESSURES

Atmospheric pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as *atmospheric pressure*. The atmospheric pressure is also known as *'Barometric pressure'*.

The atmospheric pressure at sea level (above absolute zero) is called *'Standard atmospheric pressure'*.

The local atmospheric pressure may be a little lower than these values if the place under question is higher than sea level, and higher values if the place is lower than sea level, due to the corresponding decrease or increase of the column of air standing, respectively.

Gauge pressure:

It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum. The *atmospheric pressure* on the scale is marked as *zero*.

Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is *below* the local atmospheric pressure, then the gauge is designated as '*vacuum gauge*' and the recorded value indicates the amount by which the pressure of the liquid is *below* local atmospheric pressure, i.e. *negative pressure*.

(*Vacuum pressure* is defined as the pressure *below the atmospheric pressure*).

Absolute pressure:

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum.

Any pressure measured above the absolute zero of pressure is termed as an '**absolute pressure**'**.**

A schematic diagram showing the gauge pressure, vacuum pressure and the absolute pressure is given in Fig. 2.6.

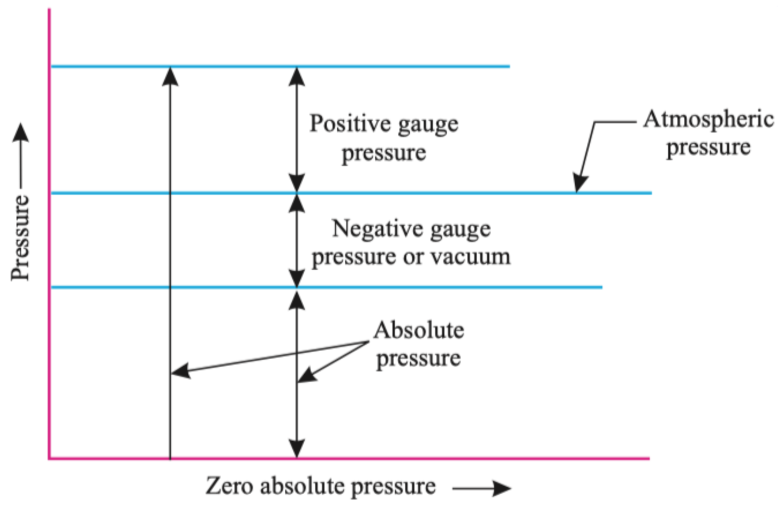


Fig.(2.6): relationship Between pressures.

Mathematically:

1. *Absolute pressure* = Atmospheric pressure + gauge pressure

i.e., $p_{abs} = p_{atm} + p_{gauge}$

2. *Vacuum pressure* = Atmospheric pressure – absolute pressure **Units for pressure:**

The fundamental S.I. unit of pressure is newton per square metre (N/m^2). This is also known as *Pascal*.

Low pressures are often expressed in terms of mm of water or mm of mercury. This is an abbreviated way of saying that the pressure is such that will support a liquid column of stated height.

When the local atmospheric pressure is not given in a problem, it is taken as 100 kN/m^2 or 10 m of water for simplicity of calculations.

Standard atmospheric pressure has the following equivalent values:

101.3 kN/m^2 or 101.3 kPa; 10.3 m of water; 760 mm of mercury; 1013 mb (millibar) ;

1 bar ;100 kPa = 10^5 N/m^2.

Example 2.5. *Given that*:

Barometer reading = 740 mm of mercury; Specific gravity of mercury = 13.6; Intensity of pressure = 40 kPa. Express the intensity of pressure in S.I. units, both gauge and absolute.

Solution. Intensity of pressure, p = 40 kPa **Gauge pressure:**

(*i*) p = 40 kPa = 40 kN/m^2= 0.4 × 10^{5}N/m^2= **0.4 bar**. (1 bar = 10^{5}N/m^2)

(*ii*) $h = \frac{p}{w} = \frac{0.4X10^5}{9.81x10^3} = 4.077m\ of\ water$

(*iii*) $h = \frac{p}{w} = \frac{0.4X10^5}{9.81X10^3X13.6} = 0.299m\ of\ mercury$

Where, w = specific weight;

For water : w = 9.81 kN/m^3

For mercury : w = 9.81 × 13.6kN/m^3

Absolute pressure:

Barometer reading (atmospheric pressure)

= 740 mm of mercury = 740 × 13.6 mm of water

$= \frac{740X13.6}{1000} = 10.6m\ of\ water$

$absolute\ pressure(p_{abs}) = atmospheric\ pressure(p_{atm}) + gauge\ pressure(p_{gauge}).$

$p_{abs} = 10.06 + 4.077 = 14.137m\ of\ water$

$= 14.137X(9.81X10^3) = 1.38X10^5 N/m^2$ $(p = w)$

$= 1.38\ bar$ $(1bar = 10^5 N/m)$

$= \frac{14.137}{13.6} = 1.039m\ of\ mercury.$

Example 2.6. *Calculate the pressure at a point 5 m below the free water surface in a liquid that has a variable density given by relation:*

$\rho = (350 + Ay)kg/m^3$

Where $A = 8kg/A^4$ and y is the distance in meters measured from the free surface.

Solution. As per hydrostatic equation

$dp = \rho. g. dy = g(350 + Ay)dy$

Integrating both sides, we get:

$\int dp = \int_0^5 g(350 + Ay)dy = g\int_0^5(350 + gy)dy$

$p = g\left|350y + 8X\frac{y^2}{2}\right| \ {}^5_0$

$= 9.81\left(350X5 + 8X\frac{5^2}{2}\right) = \frac{18148N}{m^2} = 18.15kN/m^2$

Example 2.7. *On the suction side of a pump a gauge shows a negative pressure of 0.35 bar. Express this pressure in terms of:*

(*i*) *Intensity of pressure, kPa,*

(*ii*) *N/m ^{2}absolute,*

(*iii*) *Metres of water gauge,*

(*iv*) *Meters of oil (specific gravity 0.82) absolute, and*

(*v*) *Centimeters of mercury gauge,*

Take atmospheric pressure as 76 cm of Hg and relative density of mercury as 13.6.

Solution. *Given*: Reading of the *vacuum* gauge = 0.35 bar

(*i*) Intensity of pressure, kPa:

Gauge reading = 0.35 bar = 0.35×10^5N/m^2

= 0.35×10^5Pa = **35 kPa**

(*ii*) N/m^2 absolute:

Atmospheric pressure, $p_{\text{atm.}}$= 76 cm of Hg

76 = (13.6 × 9810) × 101396 N/m^2 =100

Absolute pressure = Atmospheric pressure – *Vacuum* pressure

$p_{\text{abs.}} = p_{\text{atm}} - p_{\text{vac.}}$

= 101396 – 35000 = **66396 N/m^2 absolute**

(*iii*) Meters of water gauge:

$p = \rho gh = wh$

$$h_{WATER}(gauge) = \frac{p}{w} = \frac{0.35X10^5}{9810} = 3.567m(gauge)$$

(*iv*) Meters of oil (sp. gr. = 0.82) absolute:

$$h_{oil}(absolute) = \frac{66396}{0.82X9810} = 8.254m\ of\ water(absolute)$$

(*v*) Centimeters of mercury gauge:

$$h_{mercury}(gauge) = \frac{0.35X10^5}{13.6X9810} = 0.2623m\ of\ mercury$$

=26.23cm of mercury.

Example 2.8. *The inlet to pump is 10.5 m above the bottom of sump from which it draws water through a suction pipe. If the pressure at the pump inlet is not to fall below 28 kN/m ^{2}absolute, work out the minimum depth of water in the tank.*
Assume atmospheric pressure as 100 kPa.

Solution. *Given*: $p_{atm.}$ = 100 kPa = 100 kN/m^2; $p_{abs.}$ = 28 kN/m^2.

Minimum depth of water in the tank:

Pressure at the pump inlet.

Let, $p_{vac} = the\ vacuum(suction)pressure\ at\ the\ pump$

Then, $p_{vac} = p_{atm} - p_{abs} = (100 - 28) = 72kN/m^2 \quad or \quad 72000N/m^2$

Further, let h be the distance between the pump inlet and free water surface in the sump. Invoking hydrostatic equation, we have:

$p = wh$

$72000 = 9810 \times h$ or,

$h = \frac{72000}{9810} = 7.339m$

∴ Minimum depth of water in the tank

= 10.5 – 7.339 = **3.161 m**

Example 2.9. **(*a*)** *What is hydrostatic paradox?*

(*b*) *A cylinder of 0.25 m diameter and 1.2 m height is fixed centrally on the top of a large cylinder of 0.9 m diameter and 0.8 m height. Both the cylinders are filled with water. Calculate:*

(*i*) *Total pressure at the bottom of the bigger cylinder, and*

(*ii*) *Weight of total volume of water.*

What is hydrostatic paradox between the two results and how this difference can be reconciled?

Solution. (*a*) Hydrostatic paradox:

Fig. 2.7 shows three vessels 1, 2 and 3 having the same area *A* at the bottom and each filled with a liquid upto the same height *h*.

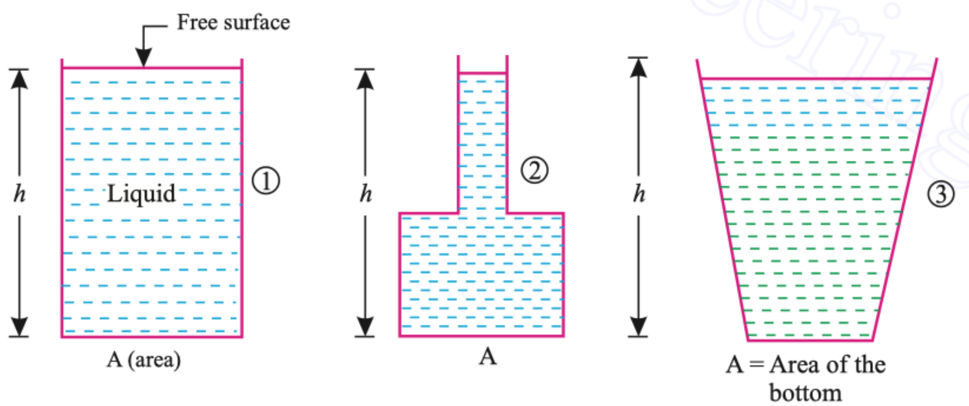


Fig. 2.7. Hydrostatic paradox.

According to the hydrostatic equation, $p = wh$; the intensity of pressure (p) *depends only on the height of the column and not at all upon the size of the column.* Thus, in all these vessels of different shapes and sizes, the same intensity of pressure would be exerted on the bottom of each of these vessels. Since each of the vessels has the same area A at the bottom, the pressure force $P = p \times A$ on the base of each vessel would be same. This is independent of the fact that the weight of liquid in each vessel is different. *This situation is referred to as* **hydrostatic paradox.**

(*b*) Area at the bottom:

$$\boldsymbol{A = \frac{\pi}{4} X(0.9)^2 = 0.6362 m^2}$$

Intensity of pressure at the bottom

$$\boldsymbol{p = wh = 9810 X(1.2 + 0.8) = 19620 N/m^2}$$

Total pressure force at the bottom

$$\boldsymbol{p = pXA = 19620 X 0.6362 = 12482 N}$$

Weight of total volume of water contained in the cylinder,

$$\boldsymbol{W = wXvolume\ of\ water}$$

$$\boldsymbol{= 9810\left[\frac{\pi}{4} X 0.9^2 X 0.8 + \frac{\pi}{4} X 0.25^2 X 1.2\right]}$$

=5571N

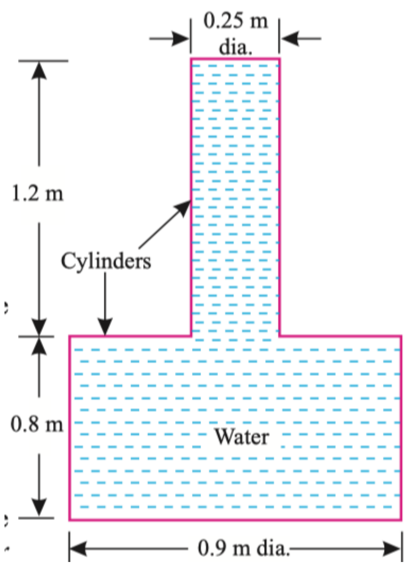


Fig.(2.8)

From the above calculations it may be observed that the *total pressure force at the bottom of the cylinder is greater* 0.9 m dia. *than the weight of total volume of water (W) contained in the* Fig. 2.9 *cylinders. This is* **hydrostatic paradox.**

The following is the *explanation of the hydrostatic paradox*: Refer to Fig. 2.9.

Total pressure force on the bottom of bigger tank = 12482 N (downward). A reaction at the roof of the lower tank is caused by the upward force which equals,

$wAh = 9810X\frac{\pi}{4}(0.9^2 - 0.25^2)X1.2 = 6911N(upward)$

The distance *h* corresponding to depth of water in the cylinder fixed centrally on the top of larger cylinder.

Net downward force exerted by water = 12482 – 6911 = 5571 N and it equals the weight of water in the two cylinder.

2.5. MEASUREMENT OF PRESSURE

The pressure of a fluid may be measured by the following devices:

1. Manometers:

Manometers *are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid.* These are classified as follows:

(*a*) Simple manometers:

(*i*) Piezometer, (*ii*) U-tube manometer, and (*iii*) Single column manometer.

(*b*) Differential manometers.

2. Mechanical gauges:

These are the devices in which the pressure is measured by balancing the fluid column by spring (elastic *element*) *or dead weight.* Generally, these gauges are used for measuring *high pressure* and where *high precision is not required.* Some commonly used mechanical gauges are:

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,

(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.

2.5.1 Manometers

2.5.1.1. Simple manometers

A "simple manometer" is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere.

Common types of simple manometers are discussed below:

1. Piezometer:

A piezometer is the simplest form of manometer which can be used for measuring *moderate pressures* of liquids. It consists of a glass tube (Fig 2.9) inserted in the wall of a vessel or of a pipe, containing liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it. Thus if w is the specific weight of the liquid, then the pressure at point A(p) is given by: $p = wh$

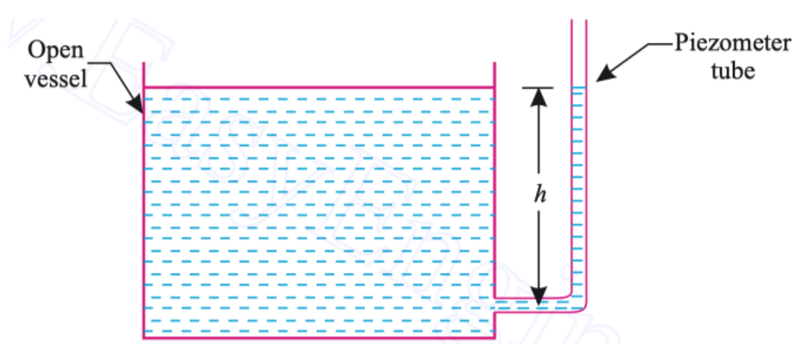


Fig. 2.9. (*a*) Piezometer tube fitted to open vessel.

Piezometers measure *gauge pressure only* (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is *not suitable* for measuring *negative pressure*; as in such a case the air will enter in pipe through the tube.

2. U-tube manometer:

Piezometers cannot be employed when large pressures in the *lighter liquids* are to be measured, since this would require *very long tubes*, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a *gas forms no free atmospheric surface*. These limitations can be overcome by the use of U-tube manometers.

A U-tube manometer consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.10.

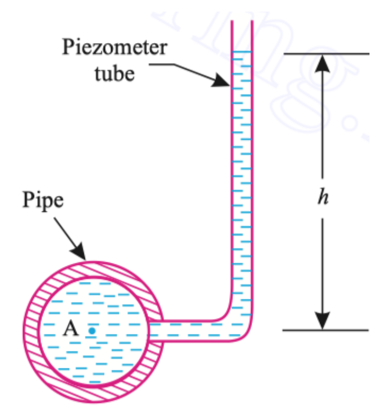


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

***(i)* For positive pressure:** Refer to Fig. 2.11 (*a*).

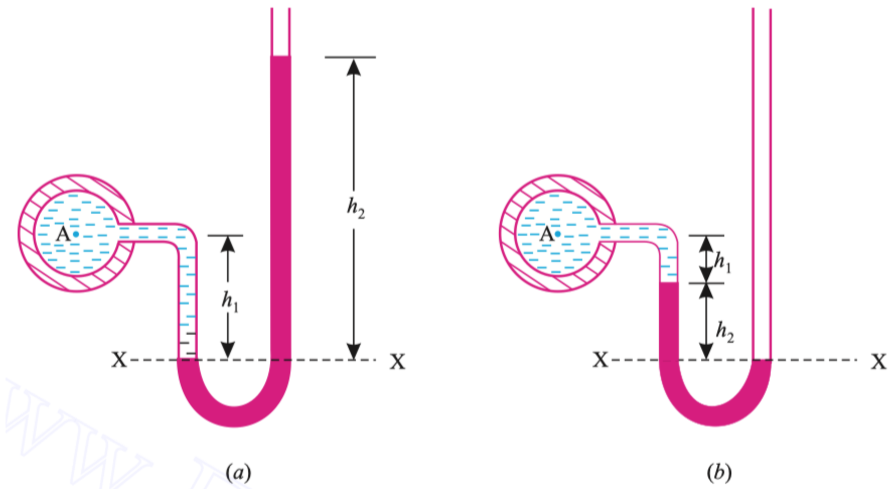


Fig. 2.11. U-tube manometer.

Let, A be the point at which pressure is to be measured. $X–X$ is the datum line as shown in Fig. 2.11 (*a*).

Let, h_1 = Height of the light liquid in the left limb above the datum line,

h_2 = Height of the heavy liquid in the right limb above the datum line,

h = Pressure in pipe, expressed in terms of head,

S_1 = Specific gravity of the light liquid, and

S_2 = Specific gravity of the heavy liquid.

The pressures in the left limb and right limb above the datum line $X–X$ are equal (*as the pressures at two points at the same level in a continuous homogeneous liquid are equal*).

Pressure head above $X–X$ in the *left limb* = $h + h_1S_1$ Pressure head above $X–X$ in the *right limb* = h_2S_2 Equating these two pressures, we get:

$$h + h_1S_1 = h_2S_2 \text{ or } h = h_2S_2 - h_1S_1 \qquad (2.5)$$

(*ii*) For negative pressure:

Refer to Fig. 2.11 (*b*).

Pressure head above $X–X$ in the *left limb* = $h + h_1S_1 + h_2S_2$ Pressure head above $X–X$ in the *right limb* = 0.

Equating these two pressures, we get:

$h + h_1S_1 + h_2S_2 = 0$

or $h = -(h_1 S_1 + h_2 S_2)$.(2.6)

Example 2.10. *In a pipeline water is flowing. A manometer is used to measure the pressure drop for flow through the pipe. The difference in level was found to be 20 cm. If the manometric fluid is CCl_4, find the pressure drop in S.I units (density of CCl_4 = 1.596 g/cm^3). If the manometric fluid is changed to mercury (ρ = 13.6gm/cm^3) what will be the difference in level?*

Solution. *Given:*

$h_{ccl_4} = 20cm = 0.2m;\ \ \rho_{ccl_4} = 1.596g/cm^3$

$= 1.596X10^3 kg/m^3$

$\rho_{Hg} = 13.6X10^3 kg/m^3$

Pressure drop, $\Delta p = \rho_{ccl_4} g h_{ccl_4} = 1.596X10^3 X9.81X0.2N/m^2$

$= \frac{3131.3N}{m^2}\ \ or\ \ pa = 3.131kPa$

Difference in level with mercury,

$h_{Hg} = h_{ccl_4} X \frac{\rho_{ccl_4}}{\rho_{Hg}} = 0.20X\frac{1.596X10^3}{13.6X10^3} = 0.02347m\ \ or\ \ 2.347cm$

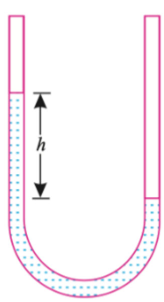


Fig.(2.12)

2.5.1.2. Differential Manometers

A **differential manometer** *is used to measure the difference in pressures between two points in a pipe, or in two different pipes.* In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out. Following are the most commonly used types of differential manometers:

1. U-tube differential manometer.

2. Inverted U-tube differential manometer.

1. U-tube differential manometer:

A U-tube differential manometer is shown in Fig. 2.13.

Case I. Fig. 2.13 (*a*) shows a differential manometer whose two ends are connected with two different points A and B at the same level and containing same liquid.

Let, h = Difference of mercury levels (heavy liquid) in the U-tube,

h_1 = Distance of the center of A from the mercury level in the right limb,

$S_1 (= S_2)$ = Specific gravity of liquid at the two points A and B

S = Specific gravity of heavy liquid or mercury in the U-tube,

h_A = Pressure head at A, and

h_B = Pressure head at B.

We know that the pressures in the left limb and right limb, above the datum line, are equal. Pressure head in the *left limb*

$= h_A + (h_1 + h) S_1$

Pressure head in the *right limb*

$= h_B + h_1 \times S_1 + h \times S$

$h_A + (h_1 + h) S_1 = h_B + h_1 S_1 + hS$

or, $h_A - h_B = h_1 S_1 + hS - (h_1 + h) S_1$

$= h_1 S_1 + hS - h_1 S_1 + hS_1 = h (S - S_1)$ *i.e.,* Difference of pressure head,

$h_A - h_B = h (S - S_1)$.(2.7)

Case II. Fig. 2.13 (*b*) shows a differential manometer whose two ends are connected to two different points A and B at different levels and containing different liquids.

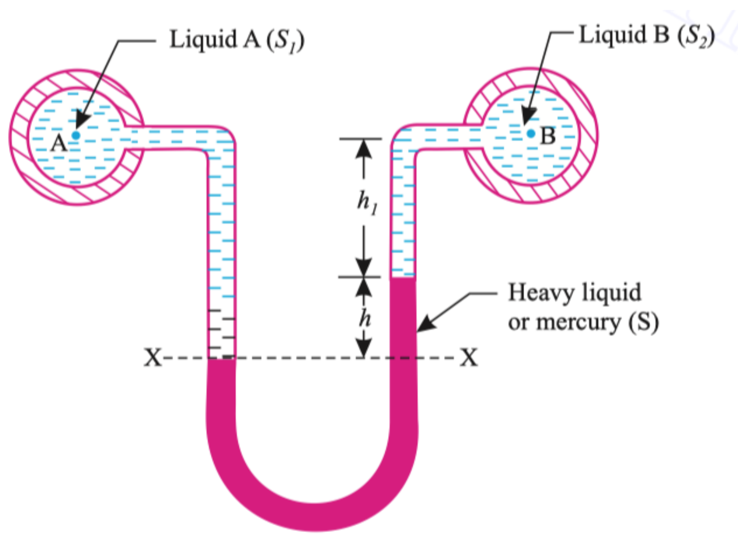


Fig.(2.13. b) U-tube differential manometers

Let,

h = Difference of mercury levels (heavy liquid) in the U-tube,

h_1= Distance of the center of A, from the mercury level in the left limb, h_2= Distance of the center of B, from the mercury level in the right limb, S_1= Specific gravity of liquid in pipe A,

S_2= Specific gravity of liquid in pipe B,

S = Specific gravity of heavy liquid or mercury,

h_A= Pressure head at A, and

h_B= Pressure head at B.

Considering the pressure heads above the datum line $X–X$, we get:

Pressure head in the *left limb* = $h_A + (h_1 + h)\, S_1$

Pressure head in the *right limb* = $h_B + h_2 \times S_2 + h \times S$ Equating the above pressure heads, we get:

$h_A + (h_1 + h)\, S_1 = h_B + h_2 \times S_2 + h \times S$

$(h_A - h_B) = h_2 \times S_2 + h \times S - (h_1 + h)\, S_1$

$= h_2 \times S_2 + h \times S - h_1 S_1 - hS_1 = h\,(S - S_1) + h_2 S_2 - h_1 S_1$ *i.e.,* Difference of pressure heads at A and B,

$h_A - h_B = h\,(S - S_1) + h_2 S_2 - h_1 S_1$ (2.8)

Example 2.11. *A differential manometer connected at the two points A and B in a pipe containing an oil of specific gravity of 0.9 shows a difference in mercury levels as 150 mm. Find the difference in pressures at the two points.*

Solution.

Specific gravity of oil, $S_1 = 0.9$

Specific gravity of mercury, $S = 13.6$

Difference of mercury levels, $h = 150$ mm

Let, $h_A - h_B$ = Difference of pressures between A and B, in terms of head of water, and

$p_A - p_B$ = Difference of pressures between A and B.

Using the relation: $h_A - h_B = h(S - S_1)$

$= 150\,(13.6 - 0.9) = 1905$ mm = **1.905 m of water** **(Ans.)**

Now, using the relation,

$p_A - p_B = wh$, we have, $p_A - p_B = 9.81 \times 1.905 = 18.68$ kN/m^2 = **18.68 kPa** **(Ans.)**

2.5.1.3. Advantages and Limitations of Manometers

Advantages:

1. Easy to fabricate and relatively inexpensive.
2. Good accuracy.
3. High sensitivity.
4. Require little maintenance.
5. Not affected by vibration.
6. Specially suitable for low pressure and low differential pressures.
7. It is easy to change the sensitivity by affecting a change in the quantity of manometric liquid in the manometer.

Limitations:

1. Usually bulky and large in size.
2. Being fragile, get broken easily.
3. Readings of the manometers are affected by changes in temperature, altitude and gravity.
4. A capillary effect is created due to surface tension of manometric fluid.
5. For better accuracy meniscus has to be measured by accurate means.

2.5.2. Mechanical Gauges

The manometers (discussed earlier) are suitable for *comparatively low pressures.* For high pressures they become unnecessarily larger even when they are filled with heavy liquids. Therefore, for measuring medium and high pressures we make use of *elastic pressure gauges*. They employ different forms of elastic systems such as tubes, diaphragms or bellows etc. to measure the pressure. The elastic deformation of these elements is used to show the effect of pressure. Since these elements are deformed within the elastic limit only, therefore these gauges are sometimes called *elastic gauges.* Sometimes they are also called *secondary instruments*, *which implies that they must be calibrated by comparison with primary instruments such as manometers etc.*

Some of the important types of these gauges are enumerated and discussed below:

1. Bourdon tube pressure gauge,
2. Diaphragm gauge, and
3. Vacuum gauge.

1. Bourdon tube pressure gauge:

Bourdon tube pressure gauge is used for measuring high as well as low pressures. A simple form of this gauge is shown in Fig. 2.14. In this case, the pressure element consists of a metal tube of approximately elliptical cross-section. This tube is bent in the form of a segment of a circle and responds to pressure changes. When one end of the tube which is attached to the gauge case, is connected to the source of pressure, the internal pressure causes the tube to expand, whereby circumferential stress *i.e.*, hoop tension is set up. The free end of the tube moves and is in turn connected by suitable levers to a rack, which engages with a small pinion mounted on the same spindle as the pointer. Thus the pressure applied to the tube causes the rack and pinion to move. The pressure is indicated by the pointer over a dial which can be graduated in a suitable scale.

The Bourdon tubes are generally made of *bronze or nickel steel.* The *former* is generally used for *low pressures* and the *latter for high pressures.*

Depending upon the purpose for which they are required Bourdon tube gauges are made in different forms, some of them are:

(*i*) *Compound Bourdon tube*–used for measuring pressures both *above and below atmospheric pressure.*

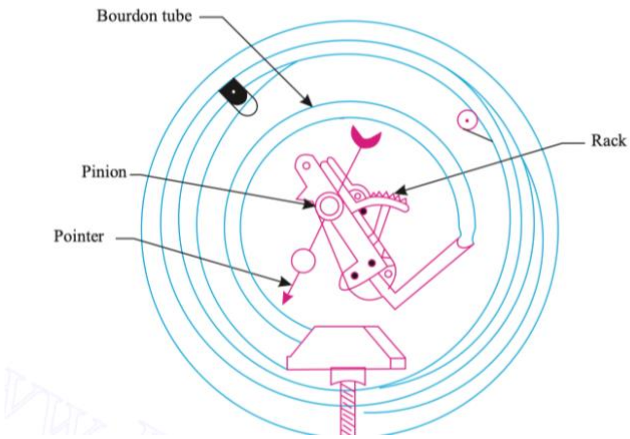


Fig. 2.14. Bourdon tube pressure gauge.

(*ii*) *Double Bourdon tube*–used where *vibrations are encountered.*

2. Diaphragm gauge:

This type of gauge employs a metallic disc or diaphragm instead of a bent tube. This disc or diaphragm is used for *actuating* the indicating device.

Refer to Fig. 2.15. When pressure is applied on the lower side of the diaphragm it is deflected upward. This movement of the diaphragm is transmitted to a rack and pinion. The latter is attached to the spindle of needle moving on a graduated dial. The dial can again be graduated in a suitable scale.

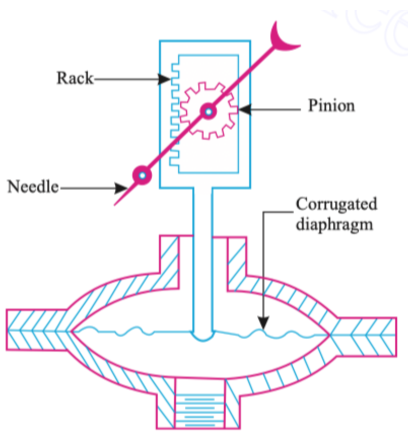


Fig. 2.15. Diaphragm gauge.

3. Vacuum gauge:

Bourdon gauges discussed earlier can be used to measure vacuum instead of pressure. Slight changes in the design are required in this purpose. Thus, in this case, the tube be *bent inward instead of outward* as in pressure gauges. *Vacuum gauges* are graduated in millimeters of mercury below atmospheric pressure. In such cases, therefore, absolute pressure in millimeters of mercury is the difference between barometer reading and vacuum gauge reading.

Vacuum gauges are used to measure the vacuum in the condensers, etc. If there is leakage, the vacuum will drop.

The pressure gauge installation requires the following considerations:

1. Flexible copper tubing and compression fittings are recommended for most installations.
2. The installation of a gauge cock and tee in the line close to the gauge is recommended because it permits the gauge to be removed for testing or replacement without having to shut down the system.
3. Pulsating pressures in the gauge line are not required.
4. The gauge and its connecting line is filled with an inert liquid and as such liquid seals are provided. Trapped air at any point of gauge lines may cause serious errors in pressure reading.

HIGHLIGHTS

1. The force (P) per unit area (A) is called pressure (p); mathematically, $p = \frac{P}{A}$

2. Pressure head of a liquid, $h = \frac{p}{w}$ $(p = wh)$

where, *w* is the specific weight of the liquid.

3. *Pascal's law* states as follows:

"The intensity of pressure at any point in a liquid at rest, is the same in all directions".

4. The atmospheric pressure at sea level (above absolute zero) is called standard atmospheric pressure.

(*i*) Absolute pressure = atmospheric pressure + gauge pressure

$p_{abs.} = p_{atm.} + p_{gauge}$

(*ii*) Vacuum pressure = Atmospheric pressure – absolute pressure (Vacuum pressure is defined as the pressure below the atmospheric pressure)

5. *Manometers* are defined as the devices used for measuring the pressure at a point in fluid by balancing the column of fluid by the same or another column of liquid.

6. *Mechanical gauges* are the devices in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Some commonly used mechanical gauges are:

(*i*) Bourdon tube pressure gauge, (*ii*) Diaphragm pressure gauge,

(*iii*) Bellow pressure gauge, and (*iv*) Dead-weight pressure gauge.