Chapter Three: isothermal expansion

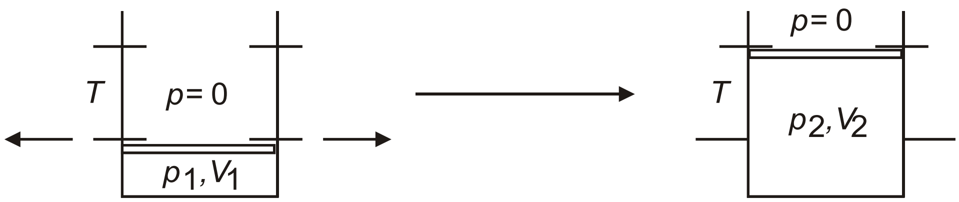
Isothermal Gas Expansion

(Δ T= 0)

gas (p1 , V1 , T) = gas (p2 , V2 , T)

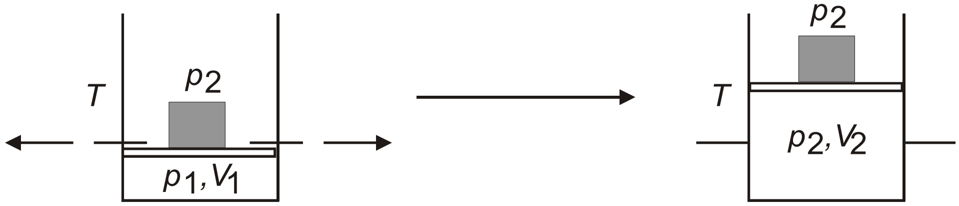
Irreversibly (many ways possible) (1)

1- Set $p_{ext} = 0$

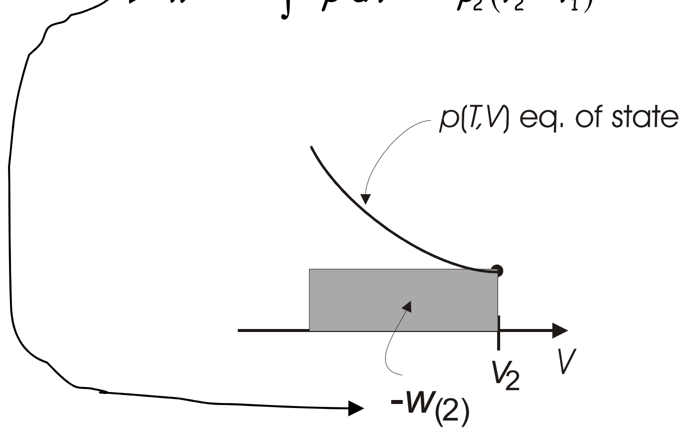


$$w_{(1)} = -\int_{V_1}^{V_2} p_{ext}\, dV = 0$$

2- set Pext=P2



$$w = -\int^{V_2} p\, dV = -p_2 (V_2 - V_1)$$

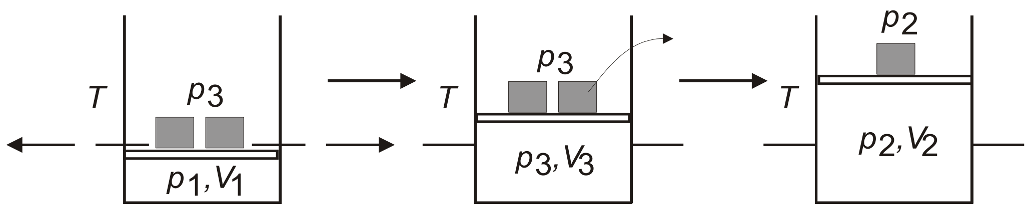


Note, work is negative: system expands against surroundings

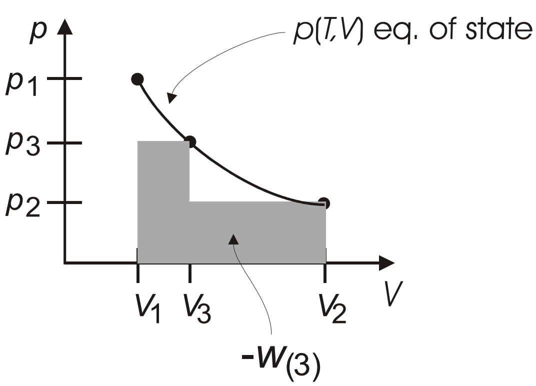
(3) Carry out change in two steps

gas (p_1, V_1, T) = gas (p_3, V_3, T) = gas (p_2, V_2, T)

$p_1 > p_3 > p_2$



$$w_{(3)} = -\int_{V_1}^{V_3} p_3 dV - \int_{V_3}^{V_2} p_2 dV = -p_3\left(V_3 - V_1\right) - p_2\left(V_2 - V_3\right)$$



More work delivered to surroundings in this case.

For ideal gas

$$w_{rev} = -\int_{V_1}^{V_2} \frac{nRT}{V} dV = -nRT\ln\frac{V_2}{V_1} = nRT\ln\frac{p_2}{p_1}$$

<u>The Internal Energy U</u>

$dU = đq + đw$ (First Law)

$dU = C_{path}dT - p_{ext}dV$

And U (T ,V) $\quad \Rightarrow \quad dU = (\frac{\partial U}{\partial T})_V + (\frac{\partial U}{\partial V})_T dV$

Some frequent constraints:

Reversible $\quad \rightarrow dU = ðq_{rev} + ðw_{rev} = ðq_{rev} - PdV \,, \qquad p = p_{ext}$

Isolated $\quad \rightarrow ðq = ðw = 0$

$Adiabatic \quad \rightarrow ðq = 0 \quad \rightarrow dU = ðw \quad = -pdV \quad (reversible)$

$constant\ \ V \quad \rightarrow \quad w = 0 \quad \rightarrow dU = ðq_V$

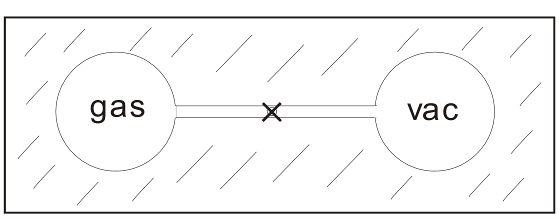
$dU = (\frac{\partial U}{\partial T})_V dT + (\frac{\partial U}{\partial V})_T dV \qquad constant\ V$

But also $\quad ðq_V = (\frac{\partial U}{\partial T})_V dT$

$ðq_V = C_V dT \quad \rightarrow (\frac{\partial U}{\partial T})_V = C_V$ $\qquad$ very important result

$so \quad dU = c_V dT + (\frac{\partial U}{\partial V})_T dV$

Joule free expansion of a gas $\quad (to\ get \quad (\frac{\partial U}{\partial V})_T)$



Adiabatic $\qquad q = 0$

Expansion into Vac. $\quad w = 0$
(p_{ext}=0)

gas (p_1, T_1, V_1) = gas (p_2, T_2, V_2)

Since q = w = 0 ⇒ dU or ΔU = 0

$recall \qquad dU = C_V dT + (\frac{\partial U}{\partial V})_T dV = 0$

$(\frac{\partial U}{\partial V})_T dV_U = C_V dT_U$

$(\frac{\partial U}{\partial V})_T = -C_V (\frac{\partial T}{\partial V})_U \qquad measure\ in\ joule\ exp't\ (\frac{\Delta T}{\Delta V})_U$

$Joule\ did\ this \qquad \lim_{\Delta V \to \to 0} (\frac{\Delta T}{\Delta V})_U = (\frac{\Delta T}{\Delta V})_U = n_J$

$\therefore \quad dU = C_V dT - C_V n_J dV \qquad n_J \quad is\ a\ Joule\ coefficient$

For Ideal gas ⇒ $\eta_J = 0$ exactly

dU = $C_V dT$ Always for ideal gas

U(T) only depends on T

The internal energy of an ideal gas depends only on temperature

Consequences

⇒ Δ U =0 For all isothermal expansions or compressions of ideal gases

⇒ ΔU = $\int C_V\, dT$ For any ideal gas change in state