Numerical Analysis

Part 2

3rd Year physics

2022-2023

Date .... Subject ....

Interpolation –

Interpolation is a process of determining the unknown value that lie in between the known data point.

EX:

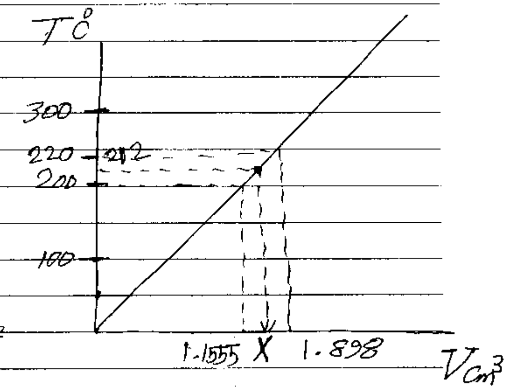
T	V
20	1.0006
40	1.0067
100	1.0280
180	1.1261

y_1 — 200   1.1555 x_1

y — 212 x

y_2 220   1.1898 x_2

230   1.1973



$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{220 - 200}{212 - 200} = \frac{1.898 - 1.1555}{X - 1.1555}$$

$$(212 - 200)(1.898 - 1.1555) = (220 - 200)(x - 1.1555)$$

$$\frac{(212 - 200)(1.898 - 1.1555)}{(220 - 200)} = x - 1.1555$$

$$X = \frac{12 * 0.7425}{20} + 1.1555$$

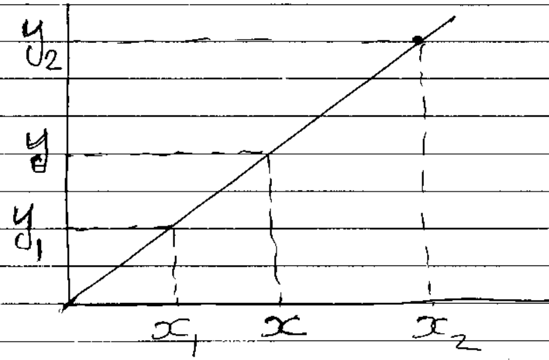
$$X = \frac{8.91}{20} + 1.1555 = 0.4455 + 1.1555$$

$$X = 1.601\ cm^3$$

Date Subject

$$\therefore x = x_1 + \frac{(x_2 - x_1)}{(y_2 - y_1)}(y - y_1)$$

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

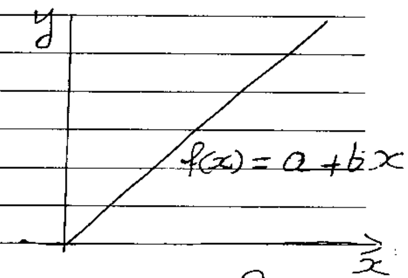


Date ________ Subject ________

Linear Interpolation

Ex:

$t(s)$	$v(m/s)$
0	0
10	222
15	362
20	517
22.5	602
30	901



Find velocity at $t = 16$ sec?

Solution

$$v(t) = a + bt \quad \text{--- (1)}$$

at $t = 15\,s \Rightarrow v = 362$

$t = 20\,s \Rightarrow v = 517$

using equation (1):

$$362 = a + 15b \quad \text{--- (2)}$$

$$\mp 517 = \mp a \mp 20b \quad \text{--- (3)}$$

$$-155 = -5b \Rightarrow b = \frac{155}{5} = 31 \quad \text{--- (4)}$$

From equation 4 and 2.

$$362 = a + 15 * 31$$

$$362 = a + 465$$

$$a = 362 - 465$$

$$\boxed{a = -103}$$

$$\therefore \boxed{v(t) = -103 + 31t}$$

$$v(16) = -103 + 31 * 16 = -103 + 496$$

$$v = 393 \text{ m/s}$$

Date ________ Subject ________

H.W// Solve the following by Interpolation.

x	y
0.26	5.8
0.46	?
0.65	12.8

2- What is the value of temperature at time t = 4 sec.

t(s)	T °C
1	10
3	15
5	20
6	30

Divided difference Interpolation

Let the function $y = f(x)$ take the values $f(x_0), f(x_1) \dots f(x_n)$ at the arguments $x_0, x_1, \dots x_n$ respectively, whe x_K, $K = 0, 1 \dots n$ are not equally spaced.

The first divided deference of $f(x)$ for the arguments x_0 and x_1 is denoted by $[x_0, x_1]$ and is defined as

$$\boxed{x_0, x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \Rightarrow \overset{1}{\Delta} y$$

The second divided difference of $f(x)$ for the argument x_0, x_1 and x_2 is difined as,

$$\boxed{x_0, x_1, x_2} = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0} \Rightarrow \overset{2}{\Delta} y$$

The 3rd divided difference

$$\boxed{x_0, x_1, x_2, x_3} = \frac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{x_3 - x_0} \Rightarrow \overset{3}{\Delta} y$$

x	$f(x)$	divided difference $\overset{1}{\Delta} y$	$\overset{2}{\Delta} y$	$\overset{3}{\Delta} y$
x_0	$f(x_0) = y_0$			
		$[x_0, x_1]$		
x_1	$f(x_1) = y_1$		$[x_0, x_1, x_2]$	
		$[x_1, x_2]$		$[x_0, x_1, x_2, x_3]$
x_2	$f(x_2) = y_2$		$[x_1, x_2, x_3]$	
		$[x_2, x_3]$		
X_3	$f(x_3) = y_3$			

Ex! Find the divided difference of $f(x) = x^3 + x + 2$ for the arguments $x = 1, 3, 6, 11$.

x	y	$\Delta^1 y$	$\Delta^2 y$	$\Delta^3 y$
1	4			
		$\frac{32-4}{3-1} = 14$		
			$\frac{64-14}{6-1} = 10$	
3	32			$\frac{20-10}{11-1} = 1$
		$\frac{224-32}{6-3} = 64$		
			$\frac{224-64}{11-3} = 20$	
6	224			
		$\frac{1344-224}{11-6} = 224$		
11	1344			

Newton's interpolating Polynomial

Ex! given points $(0,1)$, $(\frac{3}{2}, \frac{1}{2})$, $(1,0)$
Find 2nd degree polynomial, and find the fit curve.

	x	y	$\Delta^1 y$	$\Delta^2 y$
x_0	0	1		
			$\frac{\frac{1}{2} - 1}{\frac{3}{2} - 0} = -\frac{3}{4}$	
x_1	$\frac{2}{3}$	$\frac{1}{2}$		$\frac{-\frac{3}{2} - (-\frac{3}{4})}{1-0} = -\frac{3}{4}$
			$\frac{0 - \frac{1}{2}}{1 - \frac{2}{3}} = -\frac{3}{2}$	
x_2	1	0		

$$f(x) = y_0 + (x - x_0)\Delta^1 y + (x - x_0)(x - x_1)\Delta^2 y + \dots$$

$$f(x) = 1 + (x-0)(-\tfrac{3}{4}) + (x-0)(x - \tfrac{2}{3})(-\tfrac{3}{4})$$

$$f(x) = 1 - \tfrac{3}{4}x + x(x - \tfrac{2}{3})(-\tfrac{3}{4})$$

$f(x) = 1 - \frac{3}{4}x - \frac{3}{4}x^2 + \frac{1}{2}x$

$f(x) = 1 - \frac{1}{4}x - \frac{3}{4}x^2$

fitting curve;

x	$f(x) = 1 - \frac{1}{4}x - \frac{3}{4}x^2$
0	1
$\frac{2}{3}$	$\frac{1}{2}$
1	0

Ex;
From above example find the value of y when $x = 2$

$f(x) = 1 - \frac{1}{4}x - \frac{3}{4}x^2$

$f(2) = 1 - \frac{1}{4}(2) - \frac{3}{4}(2)^2$

$f(2) = 1 - \frac{1}{2} - 3$

$f(2) = -2.5$

Ex;- using newton's divided difference formula evaluate, $f(8)$, given the points;
H.W.

$(2,15)$, $(3,39)$, $(6,243)$, $(7,375)$ and $(9,771)$

EX: Using a table of divided differences, find Newton's interpolating for the data.

x	y	$\Delta^1 f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0 -5	y_0 -2	$\frac{6-(-2)}{-1-(-5)}=2$	$\frac{-5-2}{0-(-5)}=\frac{-7}{5}$	$\frac{2-(-\frac{7}{5})}{2-(-5)}=\frac{17}{35}$
x_1 -1	y_1 6	$\frac{1-6}{0-(-1)}=-5$	$\frac{1-(-5)}{2-(-1)}=2$	
x_2 0	y_2 1	$\frac{3-1}{2-0}=1$		
x_3 2	y_3 3			

$$f(x)=y_0+(x-x_0)\Delta^1 f(x)+(x-x_0)(x-x_1)\Delta^2 f(x)+(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)$$

$$f(x)=-2+(x-(-5))(2)+(x-(-5))(x-(-1))\left(-\frac{7}{5}\right)+(x-(-5))(x-(-1))(x-0)\left(\frac{17}{35}\right)$$

$$f(x)=-2+(x+5)(2)+(x+5)(x+1)\left(-\frac{7}{5}\right)+(x+5)(x+1)(x)\left(\frac{17}{35}\right)$$

$f(x)=$

Newton's forward Interpolation - equal interval.

Ex: From a difference table and interpolate the value of $f(x)$ when $x=4$ given;

	x_0	x_1	x_2	x_3
x	3	5	7	9
$y=f(x)$	180	150	120	90
	y_0	y_1	y_2	y_3

Solution:

$h = 2 \Rightarrow u = \dfrac{x-x_0}{h}$

$x=4 \quad x_0=3 \quad u = \dfrac{4-3}{2} = 0.5$

$\boxed{u=0.5}$

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	180			
		$150-180=-30$		
5	150		$-30-(-30)=0$	
		$120-150=-30$		0
7	120		$-30-(-30)=0$	
		$90-120=-30$		
9	90			

$$y = y_0 + \frac{u}{1!}\Delta y + \frac{u(u-1)}{2!}\Delta^2 y + \frac{u(u-1)(u-2)}{3!}\Delta^3 y$$

$$y = 180 + \frac{0.5}{1!}(-30) + \frac{(0.5)(0.5-1)}{2!}(0) + \frac{0.5(0.5-1)(0.5-2)}{3!} * (0)$$

$y = 180 - 15$

$y = 165$

Interpolation for equal interval

1- Newton's forward difference,
2- newton's backward difference,

Ex: Estimate the population in 1795 and 1825 from the following data,

x	y	$\Delta^1 f$	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1791	48				
		17			
1801	65		-11		
		6		17	
1811	71		6		-22
		12		-5	
1821	83		1		
		13			
1831	96				

$\ast$ newton's forward difference

$$h = x_1 - x_0 = 1801 - 1791 = 10$$

$$u = \frac{x - x_0}{h}$$

$$u = \frac{1975 - 1791}{10} = 0.4$$

$$y = f(x) = y_0 + \frac{u}{1!}\Delta^1 y + \frac{u(u-1)}{2!}\Delta^2 y + \frac{u(u-1)(u-2)}{3!}\Delta^3 y + \dots + \frac{u(u-1)\cdots(u-n+1)}{n!}\Delta^n y$$

$$y = f(1795) = 48 + \frac{0.4}{1!}(17) + \frac{0.4(0.4-1)}{2!}(-11) + \frac{0.4(0.4-1)(0.4-2)}{3!}(17 + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4!}(-22)$$

$$f(1795) = 48 + \frac{0.4}{1} * 17 + \frac{0.4(0.4-1)}{2 \times 1} * (-11)$$
$$+ \frac{0.4(0.4-1)(0.4-2)}{3 \times 2 \times 1}(17) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4 \times 3 \times 2 \times 1} * (-22)$$

$$y = f(1795) = 58.12$$

Newton's Backword difference –

$$f(x) = f(x_n) + \frac{u}{1!} \Delta^1 f(x) + \frac{u(u+1)}{2!} \Delta^2 f(x)$$
$$+ \frac{u(u+1)(u+2)}{3!} \Delta^3 f(x) + \frac{u(u+1)(u+2)(u+3)}{4!} \Delta^4 f(x)$$

$h = 10 \Rightarrow$ Interval

$$u = \frac{x - x_n}{h}$$

$$u = \frac{x - x_4}{h} = \frac{1825 - 1831}{10} = -0.6$$

$$\therefore f(1825) = 96 + \frac{(-0.6)}{1} * (13) + \frac{(-0.6)(-0.6+1)}{2} * 1$$
$$+ \frac{-0.6(-0.6+1)(-0.6+2)}{6}(-5)$$
$$+ \frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{12} * (-22)$$

$$y = f(1825) = ??$$

Ex: From the table find sin57 using newton's interpolation.

	x	y	$\Delta^1 y$	$\Delta^2 y$	$\Delta^3 y$
x_0	45	0.7071			
			0.589		
x_1	50	0.7660		-0.0057	
			0.0532		-0.0007
x_2	55	0.8192		-0.0064	
			0.0468		
x_3	60	0.8660			

$h = 50 - 45 = 5 \Rightarrow$ Interval

$x = 57$

$$u = \frac{x - x_n}{h} \qquad x_n = x_3 \qquad \boxed{n=3}$$

$$\therefore u = \frac{57 - 60}{5} = -0.6$$

$$f(\ ) = y_n + \frac{u}{1!}\Delta^1 y + \frac{u(u+1)}{2!}\Delta^2 y + \frac{u(u+1)(u+2)}{3!}\Delta^3 y$$

$$f(57) = 0.8660 + \frac{(-0.6)}{1}(0.0468) + \frac{(-0.6)(-0.6+1)}{2}(-0.0064) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{6} \times (-0.0007)$$

$$f(57) = 0.835$$

Error in the interpolation formula -

Ex: use Newton's forward interpolation formula, find the value of sin52 from the following data,

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	45°	0.7071			
			0.0589		
				−0.0057	
x_1	50°	0.7660			−0.0007
			0.0532		
				−0.0064	
x_2	55°	0.8192			
			0.0468		
x_3	60°	0.8660			

$x = 52$ $\boxed{h = 50 - 45 = 5}$

$$u = \frac{x - x_0}{h}$$

$$u = \frac{52 - 45}{5} = 1.4$$

From Newton's formula

$$f(x) = y_0 + \frac{u}{1!}\Delta y + \frac{u(u-1)}{2!}\Delta^2 y + \frac{u(u-1)(u-2)}{3!}\Delta^3 y$$

$$f(52) = 0.7071 + \frac{1.4}{1}(0.0589) + \frac{1.4(1.4-1)}{2\times 1}(-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{3\times 2\times 1} * (-0.0007)$$

$$f(52) = 0.7880032$$

$$Error = \frac{u(u-1)(u-2)\cdots(u-n+1)}{(n+1)!}\Delta^{n+1}y$$

$$Error = \frac{u(u-1)(u-2)}{3!}\Delta^{3}y$$

$$Error = \frac{1.4(1.4-1)(1.4-2)}{3\times 2\times 1} * (-0.0007)$$

$$Error = 0.0000392$$

Interpolation with unequal interval

The divided difference table:

Argument x	Entry $f(x)$	$\Delta^1 f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	$f(x_0)=y_0$			
		$f(x_0,x_1)$		
x_1	$f(x_1)=y_1$		$f(x_0,x_1,x_2)$	
		$f(x_1,x_2)$		$f(x_0,x_1,x_2,x_3)$
x_2	$f(x_2)=y_2$		$f(x_1,x_2,x_3)$	
		$f(x_2,x_3)$		
x_3	$f(x_3)=y_3$			

$$f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$f(x_0,x_1,x_2) = \frac{f(x_2)-f(x_0)}{x_2-x_0}$$

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_3)-f(x_0)}{x_3-x_0}$$

$\Delta^1 f(x)$ = divided difference of order one.
$\Delta^2 f(x)$ = " " " " two.
$\Delta^3 f(x)$ = " " " " three.

Newton's general divided difference formula is,

$$f(x) = f(x_0) + (x-x_0)\Delta^1 f(x) + (x-x_0)(x-x_1)\Delta^2 f + \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1})\Delta^{n-1} f(x)$$

Ex! Use the divided difference formula to find $f(5)$ for the following data

x	1	3	4	6	10
y	0	18	58	190	920

Solution

x	$f(x)$	$\Delta^1 f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	0				
		$\frac{18-0}{3-1}=9$			
3	18		$\frac{40-9}{4-1}=10.33$		
		$\frac{58-18}{4-3}=40$		$\frac{7-10.33}{6-1}=0.66$	
4	58		$\frac{61-40}{6-3}=7$		$\frac{0.46-0.66}{10-1}$
		$\frac{190-58}{6-4}=61$		$\frac{10.25-7}{10-3}=0.46$	$\Downarrow$
6	190		$\frac{182.5-61}{10-4}=10.25$		0.12
		$\frac{920-190}{10-6}=182.5$			
10	920				

$$f(x)=f(x_0)+(x-x_0)\Delta^1 f(x)+(x-x_0)(x-x_1)\Delta^2 f(x)$$
$$+(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)+(x-x_0)(x-x_1)(x-x_2)(x-x_3)*\Delta^4 f(x)$$

$$f(5)=0+(5-1)(9)+(5-1)(5-3)(10.33)+(5-1)(5-3)(5-4)(0.66)$$
$$+(5-1)(5-3)(5-4)(5-6)(0.12)$$

$$f(5)=4*9+4*2*10.33+4*2*1*0.66+4*2*1*(-1)(0.12)$$

$$=36+82.64+5.28-0.96$$

$$f(5)=122.96 \;//$$

H.W

EX: Find the form of the function $f(x)$ from the following data.

x	0	1	2	5
y	2	3	12	147

EX: If $f(0)=8$, $f(1)=11$, $f(4)=68$, $f(15)=123$

Find the form of the equation which satisfies the above values.

EX: Find the value $f(8)$ from the following data.

x	4	5	7	10	11	12
y	48	100	294	900	1210	2028

EX: If $f(x)=\frac{1}{x}$ then show that

$$f(a,b,c,d)=\frac{-1}{abcd}$$

EX:

Given the following data find $f(x)$ as a polynomial in power of $(x-5)$

x	0	2	3	4	7	9
y	4	26	58	112	466	922

Lagrange's Interpolation formula.

Let $f(x)$ be a function whic asume the values $f(x_0), f(x_1), f(x_2) \ldots f(x_n)$ corresponding to the values $x = x_0, x_1, x_2, \ldots x_n$, where the value of x are not equispaced.

x	x_0	x_1	x_2	X_3	$\ldots X_n$
$y=f(x)$	$y_0=f(x_0)$	y_1	y_2	y_3	y_n

Le the polynomial be

$$f(x) = \sum_{j=0}^{n} L_j(x) f(x_j)$$

$$= L_0 f(x_0) + L_1(x) f(x_1) + \cdots\cdots L_n(x) f(x_n)$$

$$= L_0 y_0 + L_1 y_1 + \cdots\cdots + L_n y_n$$

$$L_j(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{j-1})\cdots(x-x_n)}{(x_j-x_0)(x_j-x_1)\cdots(x_j-x_{j-1})\cdots(x_j-x_n)}$$

$$L_j(x) = \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{(x-x_i)}{(x_j-x_i)} \qquad j=0,1,2\ldots n$$

The Lagrange form:

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x-x_3)\cdots(x-x_n)} f(x_2)$$

$$\vdots$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})}f(x_n)$$

$\Downarrow$ Lagrange's interpolation formula.

Ex! We have $x_0=0$, $x_1=1$, $x_2=3$, $X_3=4$

$y_0=f(x_0)=-12$, $y_1=f(x_1)=0$, $y_2=f(x_2)=6$

$y_3=f(x_3)=12$

using Langrange interpolation formula we can write,

$$f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}f(x_0)$$

$$+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}f(x_1)$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}f(x_2)$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}f(x_3)$$

$$f(x)=\frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}*(-12)$$

$$+\frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}*(0)$$

$$+\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}*(6)$$

$$+\frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}*(12)$$

$$f(x) = \frac{(x^3 - 8x^2 - 19x - 12)}{12} * (-12)$$

$$+ \frac{(x^3) - 5x^2 + 4x}{-6} * (6) + \frac{(x^3 - 4x^2 + 3x)}{12} (12)$$

$$f(x) = x^3 - 7x^2 + 18x - 12 \quad //$$

EX! H.W //

using Lagrange's interpolation, find the value of y corresponding to $x = 10$ from the following table.

x	5	6	9	11
Y	12	13	14	16

Ans! " $\frac{42}{3}$ "

Inverse Interpolation –

Use of Lagrange's interpolation formula for inverse interpolation.

In Lagrange's interpolation formula y is expressed as a function of x as given below

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}\, y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}\, y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}\, y_n$$

by interchanging x and y we can express x as a function of y as follows,

$$x = \frac{(y-y_1)(y-y_2)\cdots(y-y_n)}{(y_0-y_1)(y_0-y_2)\cdots(y_0-y_n)}\, x_0 + \frac{(y-y_0)(y-y_1)\cdots(y-y_n)}{(y_1-y_0)(y_1-y_2)\cdots(y_1-y_n)}\, x_1 + \cdots + \frac{(y-y_0)(y-y_1)(y-y_2)\cdots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_2)\cdots(y_n-y_{n-1})}\, x_n$$

the above formula may be used for inverse interpolation.

Date Subject

EX: The following table gives the value of the elliptical integral

$$y = F(\phi) = \int_0^{\phi} \frac{d\phi}{1-\frac{1}{2}\sin^2\phi}$$

for certain value of ϕ. Find the value of ϕ if $y = F(\phi) = 0.3887$

ϕ	21°	23°	25°
$y=F(\phi)$	0.3706	0.4068	0.4433

using the inverse interpolation formula

$$\phi = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)}\phi_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)}\phi_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)}\phi_2$$

$$\phi = \frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)} * 21 + \frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)} * 23 + \frac{(0.3887-0.3706)(0.3887-0.4068)}{(0.4433-0.3706)(0.4433-0.4068)} * 25$$

$$\phi = 7.884 + 17.20 - 3.087 = 21.997$$

$$\phi \simeq 22^\circ$$

Date Subject

EX H.W Find the value of x when $y = 0.3$ by applying inverse formula.

x	0.4	0.6	0.8
y	0.3683	0.3332	0.2897

Ans: $X = 0.7573$

EX: The following table gives the values of the probablity integral $y = \frac{2}{\sqrt{\pi}} \int_0^{n} e^{-x^2} dx$ corresponding to certain value of x. For what value of X the integral equal to $\frac{1}{2}$.

EX: using langrange's interpolation formula by using following data Find $\log_{10} 656$

X	654	658	659	661
$Y = \log_{10} X$	2.8156	2.8182	2.8189	2.8202

$656 = x$

Ans:

$Y = \log_{10} 656 = 2.8170$

Date ______ Subject ______

Ex: Evaluate $\sqrt{155}$ by using Lagrange's interpolation formula from the following data

	x_0	x_1	x_2	x_3
x	150	152	154	156
y	12.247	12.239	12.410	12.490

Sol.

$X = 155$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(X_1-X_0)(X_1-X_2)(X_1-X_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(X_2-x_0)(X_2-X_1)(X_2-X_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(X_3-X_2)} y_3$$

$$f(x) = \frac{(155-152)(155-154)(155-156)}{(150-152)(150-154)(150-156)}(12.247)$$

$$+ \frac{(155-150)(155-154)(155-156)}{(152-150)(152-154)(152-156)}(12.239)$$

$$+ \frac{(155-150)(155-152)(155-156}{(154-150)(154-152)(154-156}(12.410)$$

$$+ \frac{(155-150)(155-152)(155-154)}{(156-150)(156-152)(156-154)}(12.490)$$

$$f(x) = \sqrt{155} = 0.7654 + (-3.8246) + 11.6343 + 3.9032$$

$$\sqrt{155} = 12.478$$