Numerical Analysis

Part 3

3^{rd} Year physics

2022-2023

Central difference Interpolation formula

1. Gauss forward interpolation formula

P	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	x_{-2}	y_{-2}				
			Δy_{-2}			
-1	x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
			Δy_{-1}		$\Delta^3 y_{-2}$	
0	x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
			Δy_0		$\Delta^3 y_{-1}$	
1	x_1	y_1		$\Delta^2 y_0$		
			Δy_1			
2	x_2	y_2				

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{u(u-1)(u+1)}{3!}\Delta^3 y_{-1} + \frac{u(u-1)(u+1)(u-2)}{4!}\Delta^4 y_{-2}$$

This equation called Gauss's interpolation formula.

Ex: If $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{12} = 35$ and $u_{16} = 40$. Apply gauss's forward formula to find the value of u_9

Date .... Subject ....

P	x	y (U_x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	x_{-2} 0	y_{-2} 14				
			Δy_{-2} 10			
-1	x_{-1} 4	y_{-1} 24		$\Delta^2 y_{-2}$ -2		
			Δy_{-1} 8		$\Delta^3 y_{-2}$ -3	
0	x_0 8	y_0 32		$\Delta^2 y_{-1}$ -5		$\Delta^4 y_{-2}$ 10
			Δy_0 3		$\Delta^3 y_{-1}$ 7	
1	x_1 12	y_1 35		$\Delta^2 y_0$ 2		
			Δy_1 5			
2	x_2 16	y_2 40				

$$u = \frac{x - x_0}{h} \qquad h = 4 - 0 = 4$$

$$u = \frac{9-8}{4} = \frac{1}{4} = 0.25$$

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{u(u-1)(u+1)}{3!}\Delta^3 y_{-1} + \frac{u(u-1)(u+1)(u-2)}{4!}\Delta^4 y_{-2}$$

$$y = 32 + 0.25 * 3 + \frac{0.25(0.25-1)}{2} * (-5) + \frac{0.25(0.25-1)(0.25+1)}{6} * (7) + \frac{0.25(0.25-1)(0.25+1)(0.25-2)}{24} * (10)$$

$$y = 33.116$$

Date Subject

Gauss Backword Interpolation formula -

$$y = y_0 + u\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-2} + \frac{u(u+1)(u-1)(u+2)}{4!}\Delta^4 y_{-2}$$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	x_{-2}	y_{-2}				
			Δy_{-2}			
-1	x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
			Δy_{-1}		$\Delta^3 y_{-2}$	
0	x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$
			Δy_0		$\Delta^3 y_{-1}$	
1	x_1	y_1		$\Delta^2 y_0$		
			Δy_1			
2	x_2	y_2				

Ex: Using Gauss's backword interpolation formula. Find the population of the town in year 1936 given that.

Year (X)	1901	1911	1921	1931	1942	1951
Population (y)	12	15	20	27	39	52

Date .... Subject ....

J	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-4	1901	12					
			3				
-3	1911	15		2			
			5		0		
-2	1921	20		2		3	
			7		3		-10
-1	1931	27		5		-7	
			12		-4		
0	1941	39		1			
			13				
1	1951	52					

$$u = \frac{x - x_0}{h} \qquad h = 1911 - 1901 = 10$$

$$x = 1936$$

$$u = \frac{1936 - 1941}{10} = -0.5$$

$$y = y_0 + u\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-2}$$

$$y_{1936} = 39 + (-0.5)(12) + \frac{(-0.5)(-0.5+1)}{2}(1) + \frac{(-0.5)(-0.5+1)(-0.5-1)}{6}(-4)$$

$$y_{1936} = 32.625$$

Date Subject

Ex: From the following table find y when $x = 1.35$ using gauss forward central difference formula.

x	1	1.2	1.4	1.6	1.8	2
y	0	-0.112	-0.016	0.336	0.992	2

Solu.

	J	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
	-2	1	0				
				-0.112			
	-1	1.2	-0.112		0.208		
				0.096		0.048	
x_0	0	1.4	-0.016		0.256		0
				0.352		0.048	
	1	1.6	0.336		0.656		0
				0.656		0.048	
	2	1.8	0.992		0.352		
				1.008			
	3	2	2				

$$u = \frac{x - x_0}{h} \qquad h = 0.2$$
$$x = 1.35$$

$$u = \frac{1.35 - 1.4}{0.2} = -0.25$$

$$y_{1.35} = (-0.016) + (-0.25)(0.352) + \frac{(-0.25)(-0.25-1)}{2}(0.256)$$
$$+ \frac{(-0.25)(-0.25-1)(-0.25+1)}{6}(0.048)$$

$$y = -0.0621$$

Date Subject

H-W

Ex! Find y when $x = 2.36$ by using gauss's backword interpolation formula

x	1.6	1.8	2.0	2.2	2.4 (x_0)	2.6
y	4.95	6.05	7.39	9.03	11.02	13.46
J	-4	-3	-2	-1	0	1

$u = \frac{x - x_0}{h}$ $\qquad h = 1.8 - 1.6 = 0.2$

$X = 2.36$

$u = \frac{2.36 - 2.4}{0.2} = -0.2$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-4	1.6	4.95					
			1.1				
-3	1.8	6.05		0.24			
			1.34		0.06		
-2	2	7.39		0.30		-0.01	
			1.64		0.05		0.06
-1	2.2	9.03		0.35		0.05	
			Δy_{-1} 1.99		$\Delta^3 y_{-2}$ 0.10		
0	x_0 $\boxed{2.4}$	y_0 $\boxed{11.02}$		$\Delta^2 y_{-1}$ 0.45			
			2.44				
1	2.6	13.46					

$$y_{2.36} = 11.02 + (-0.2)(1.99) + \frac{(-0.2)(-0.2+1)}{2}(0.45) + \frac{(-0.2)(-0.2+1)(-0.2+1)}{6}(0.10)$$

$y = 10.5892$

Date ...... Subject ......

Bessel's Interpolation formula –

$$y = \frac{1}{2}\left[y_0 + y_1\right] + \left[u - \frac{1}{2}\right]\Delta y_0 + \frac{1}{2}\,\frac{u(u-1)}{2!}\left[\Delta^2 y_{-1} + \Delta^2 y_0\right]$$
$$+ \frac{u(u-\frac{1}{2})(u-1)}{3!}\,\Delta^3 y_{-1} +$$
$$+ \frac{u(u+1)(u-1)(u-2)}{4!}\left[\Delta^4 y_{-1} + \Delta^4 y_{-1}\right] + \dots$$

Ex: Apply Bessel's formula to obtain $f(32)$ given that $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3388$, $f(40) = 0.3794$

J	x_J	y_J	Δy_J	$\Delta^2 y_J$	$\Delta^3 y_J$
-1	25	0.2707			
			Δy_{-1} 0.032		
0	x_0 $\boxed{30}$	y_0 $\boxed{0.3027}$		$\Delta^2 y_{-1}$ 0.0041	
			Δy_0 0.0361		$\Delta^3 y_{-1}$ 0.004
1	35	y_1 $\boxed{0.3388}$		$\Delta^2 y_0$ 0.0045	
			0.0406		
2	40	0.3794			

$$u = \frac{x - x_0}{h} = \frac{32-30}{5} = 0.4$$

$$y = y_0 + u\,\Delta y_{-1} + \frac{u(u+1)}{2!}\,\Delta^2 y_{-1} + \frac{u(u+1)(u-1)}{3!}\,\Delta^3 y_{-1}$$

$$y = 0.3027 + (0.4)(0.032) + \frac{(0.4)(0.4+1)}{2}(0.0041)$$
$$+ \frac{(0.4)(0.4+1)(0.4-1)}{6}(0.004) = 0.3166$$

Date .... Subject ....

Ex! Apply Bessel's formula to obtain y at $x = 25$.

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	x_{-1} 20	y_{-1} 24			
			Δy_{-1} 8		
0	x_0 24	y_0 32		$\Delta^2 y_{-1}$ -5	
			Δy_0 3		$\Delta^3 y_{-1}$ 7
1	x_1 28	y_1 35		$\Delta^2 y_0$ 2	
			Δy_1 5		
2	x_2 32	y_2 40			

$$u = \frac{x - x_0}{h} = \frac{25 - 24}{4} = 0.25$$

$$y = \frac{1}{2}[32+35] + \left[0.25 - \frac{1}{2}\right](3) + \frac{1}{2} \cdot \frac{0.25(0.25-1)}{2} * [-5+2] + \frac{(0.25)(0.25+1)(0.25-1)(0.25-2)}{6}(7)$$

$$y = 32.9453$$

Stirling Interpolation formula

$$y = y_0 + \frac{u(\Delta y_0 + \Delta y_{-1})}{2} + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u-1)(u+1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2} + \frac{u(u^2-1)(u^2-4)}{5!}\left[\Delta^5 y_{-2} + \Delta^5 y_{-3}\right] + \cdots$$

EX: Apply Stirling formula to obtain y at $x = 34$, from the following data.

Solution:

J	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	x_{-2} 20	y_{-2} 11.4699				
			Δy_{-2} 1.3135			
-1	x_{-1} 25	y_{-1} 12.7834		$\Delta^2 y_{-2}$ -0.3321		
			Δy_{-1} 0.9814		$\Delta^3 y_{-2}$ 0.0841	
0	x_0 30	y_0 $\boxed{13.7648}$		$\Delta^2 y_{-1}$ -0.2480		$\Delta^4 y_{-2}$ -0.0214
			Δy_0 0.7334		$\Delta^3 y_{-1}$ 0.0627	
1	x_1 35	y_1 14.4982		Δy_0 -0.1853		
			Δy_1 0.5481			
2	x_2 40	y_2 15.0467				

$$u = \frac{x - x_0}{h} = \frac{34-30}{5} = 0.8 \qquad h = 25-20 = 5, \quad x = 34$$

using Stirling formula

$$y_{34} = 13.7648 + \frac{0.8(0.7334 + 0.9814)}{2} + \frac{(0.8)^2}{2}(-0.248) + \frac{0.8(0.8-1)(0.8+1)}{6}\left[\frac{0.0627 + 0.0841}{2}\right] + \frac{0.8^2(0.8^2-1)}{24}(-0.0214) = 14.3681$$

Date Subject

Laplace - Everett's interpolation formula -

$$y = vy_0 + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-4)}{5!}\Delta^4 y_{-2} + \cdots$$
$$uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + \frac{u(u^2-1)(u^2-4)}{5!}\Delta^4 y_{-1} + \cdots$$

where $v = 1 - u$

$$u = \frac{x - x_0}{h}$$

Ex: Apply Laplace - Everett's formula to obtain $y(25)$, given $y(20) = 2854$, $y(24) = 3162$, $y(28) = 3544$, $y(32) = 3932$

Solution:

Here $X = 25$ $x_0 = 24$ $h = 4$

$$u = \frac{x - x_0}{h} = \frac{25-24}{4} = \frac{1}{4}$$

$$v = 1 - u = 1 - \frac{1}{4} = \frac{3}{4}$$

J	X	y	Δy_J	$\Delta^2 y_J$	$\Delta^3 y_J$
-1	x_{-1} 20	2854 y_{-1}			
			Δy_{-1} 308		
0	x_0 24	3162 y_0		$\Delta^2 y_{-1}$ 74	
			Δy_0 382		$\Delta^3 y_{-1}$ -8
1	x_1 28	3544 y_1		$\Delta^2 y_0$ 66	
			Δy_1 448		
2	x_2 32	3932 y_2			

Date Subject

$$y_{25} = \frac{3}{4}(3162) + \frac{3}{4}\left(\frac{9-1}{16}\right)\left(\frac{74}{6}\right) +$$

$$+ \frac{1}{4}(3544) + \frac{1}{4}\left(\frac{1-1}{16}\right)\left(\frac{66}{6}\right)$$

$$y_{25} = 2371.5 + (-4.0468) + 886 - 2.5781$$

$$y_{25} = 3257.5 - 6.62499$$

$$y_{25} = 3250.87501 \,//$$

Date Subject Numerical Analysis

System of linear Equation

1_ Direct method,
Gauss elimenation $\rightarrow$ exact Solution.

2_ Indirect method,
a_ Jacobi method } approximation
b_ Gauss - Seidel method } Solution

Indirect method,
For example,

$$a_{11}x_1 + a_{12}x_2 + a_{13}X_3 = A$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = B$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 = C$$

Not a_{11}, a_{22} and a_{33} must be Control element.

then,

$$\underset{i=J}{a_{iJ}} \leq \sum_{i\neq J} |a_{iJ}|$$

Jacobi

$$X_1^{n+1} = \frac{1}{a_{11}}\left[-a_{12}X_2^n - a_{13}X_3^n + A\right]$$

$$X_2^{n+1} = \frac{1}{a_{22}}\left[-a_{21}X_1^n - a_{23}X_3^n + B\right]$$

$$X_3^{n+1} = \frac{1}{a_{33}}\left[-a_{31}X_1^n - a_{32}X_2^n + C\right]$$

Date Subject

Gauss - seidel

$$X_1^{n+1} = \frac{1}{a_{11}}\left[-a_{12}X_2^{n} - a_{13}X_3^{n} + A\right]$$

$$X_2^{n+1} = \frac{1}{a_{22}}\left[-a_{21}X_2^{n+1} - a_{23}X_3^{n} + B\right]$$

$$X_3^{n+1} = \frac{1}{a_{33}}\left[-a_{31}X_1^{n+1} - a_{32}X_3^{n+1} + C\right]$$

Example! Solve the linear equation.

$$4x_1 + 2X_2 + X_3 = 14$$
$$X_1 + 5X_2 - X_3 = 10$$
$$X_1 + X_2 + 8X_3 = 20$$

Control element = $\underset{a_{11}}{4}$, $\underset{a_{22}}{5}$ and $\underset{a_{33}}{8}$

Jacobi	Gauss-seidel
$X_1^{n+1} = \frac{1}{4}\left[-2X_2^{n} - X_3^{n} + 14\right]$	$X_1^{n+1} = \frac{1}{4}\left[-2X_2^{n} - X_3^{n} + 14\right]$
$X_2^{n+1} = \frac{1}{5}\left[-x_1^{n} + X_3^{n} + 10\right]$	$X_2^{n+1} = \frac{1}{5}\left[-X_1^{n+1} + X_3^{n} + 10\right]$
$X_3^{n+1} = \frac{1}{8}\left[-X_1^{n} - X_2^{n} + 20\right]$	$X_3^{n+1} = \frac{1}{8}\left[-X_1^{n+1} - X_2^{n+1} + 20\right]$

n	x_1	x_2	x_3
0	0	0	0
1	3.5	2	2.5
2	1.875	1.8	1.813
3			

step 1 $\Rightarrow$ $n=0$

$$x_1^1 = \frac{1}{4}\left[-2x_2^0 - x_3^0 + 14\right]$$

$$x_1^1 = \frac{1}{4}\left[-2(0) - (0) + 14\right] = \frac{14}{4} = 3.5$$

$$x_2^1 = \frac{1}{5}\left[-x_1^0 + x_3^0 + 10\right]$$

$$x_2^1 = \frac{1}{5}\left[-(0) + (0) + 10\right] = \frac{10}{5} = 2$$

$$x_3^1 = \frac{1}{8}\left[-x_1^0 + x_2^0 + 20\right]$$

$$x_3^1 = \frac{1}{8}\left[-(0) + (0) + 20\right] = \frac{20}{8} = 2.5$$

step2 $\Rightarrow$ $n=1$

$$x_1^2 = \frac{1}{4}\left[-2x_2^1 - x_3^1 + 14\right]$$

$$x_1^2 = \frac{1}{4}\left[-2(3.5) - 2.5 + 14\right] = 1.875$$

$$x_2^2 = \frac{1}{5}\left[-x_1^1 + x_3^1 + 10\right]$$

$$x_2^2 = \frac{1}{5}\left[-(3.5) + 2.5 + 10\right] = 1.8$$

$$x_3^2 = \frac{1}{8}\left[-x_1^1 - x_2^1 + 20\right]$$

$$x_3^2 = \frac{1}{8}\left[-3.5 - 2 + 20\right] = 1.813$$

step 3 $n=2$

$$x_1^3 = \frac{1}{4}\left[-2x_2^2 - x_3^2 + 14\right] = \frac{1}{4}\left[-2(1.875) - 1.813 + 14\right] =$$

$$x_2^3 = \frac{1}{5}\left[-x_1^2 + x_3^2 + 10\right] = \frac{1}{5}\left[-1.875 + 1.813 + 10\right] =$$

$$x_3^3 = \frac{1}{8}\left[-x_1^2 - x_2^2 + 20\right] = \frac{1}{8}\left[-1.875 - 1.8 + 20\right] =$$

Date Subject

Gauss - Seidel

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 = A$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 = B$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 = C$$

a_{11} and a_{22} and a_{33} are Control element

$$X_1^{n+1} = \frac{1}{a_{11}}\left[-a_{12}X_2 - a_{13}X_3 + A\right]$$
$$X_2^{n+1} = \frac{1}{a_{22}}\left[-a_{21}X_1 - a_{23}X_3 + B\right]$$
$$X_3^{n+1} = \frac{1}{a_{33}}\left[-a_{31}X_1 + a_{32}X_2 + C\right]$$

Exp: Solve the linear equation using Gaus - seidel method.

$$10x_1 + x_2 + x_3 = 12$$
$$2X_1 + 10X_2 + X_3 = 13$$
$$2X_1 + 2X_2 + 10X_3 = 14$$

Solution:

$$x_1^{n+1} = \frac{1}{10}\left[-x_2^n - x_3^n + 12\right]$$
$$x_2^{n+1} = \frac{1}{10}\left[-2X_1^{n+1} - x_3^n + 13\right]$$
$$x_3^{n+1} = \frac{1}{10}\left[-2x_1^{n+1} - 2x_2^{n+1} + 14\right]$$

$\Rightarrow$

Date Subject

n	x_1	x_2	x_3
0	0	0	0
1	1.2	1.06	0.9948
2	0.9992	1.00536	0.999098
3	0.995542	1.100536	0.980784

1st iteration $x=0$ $n=0$

$$x_1^1 = \frac{1}{10}\left[-x_2^0 - x_3^0 + 12\right]$$

$$X_1^1 = \frac{1}{10}\left[-0-0+12\right] = 1.2$$

$$X_2^1 = \frac{1}{10}\left[-x_1^1 - x_3^0 + 13\right]$$

$$X_2^1 = \frac{1}{10}\left[-1.2 - 0 + 13\right] = 1.06$$

$$X_3^1 = \frac{1}{10}\left[-2x_1^1 - x_2^1 + 14\right]$$

$$X_3^1 = \frac{1}{10}\left[-2(1.2) - 2(1.06) + 14\right]$$

$$= 0.948$$

second iteration, $n=1$

$$X_1^2 = \frac{1}{10}\left[-x_2^1 - x_3^1 + 12\right] = \frac{1}{10}\left[-1.06 - 0.948 + 12\right] = 0.9992$$

$$X_2^2 = \frac{1}{10}\left[-2X_1^2 - x_3^1 + 13\right] = \frac{1}{10}\left[0.9992 - 0.948 + 13\right]$$

$$X_2^2 = 1.00536$$

$$x_3^2 = \frac{1}{10}\left[-2X_1^2 - 2X_2^2 + 14\right] = \frac{1}{10}\left[-2(0.9992) - \right.$$

$$X_3^2 = \frac{1}{10}\left[-2(0.9992) - 2(1.00536) + 13\right] = 0.999098$$

Date Subject

3rd iteration — $n=2$

$$x_1^3 = \frac{1}{10}\left[-x_2^2 - x_3^2 + 12\right]$$

$$X_1^3 = \frac{1}{10}\left[-1.00536 - 0.999098 + 12\right]$$

$$X_1^3 = 0.995542$$

$$X_2^3 = \frac{1}{10}\left[-2X_1^3 - x_3^2 + 13\right]$$

$$X_2^3 = \frac{1}{10}\left[-2(0.995542) - 0.999098 + 13\right]$$

$$X_2^3 = 1.100536$$

$$X_3^3 = \frac{1}{10}\left[-2X_1^3 - 2X_2^3 + 14\right]$$

$$X_3^3 = \frac{1}{10}\left[-2(0.995542) - 2(1.100536) + 14\right]$$

$$X_3^3 = 0.980784$$

H.W!
solve the linear equation using Gauss-Jacobi's method

$$5x_1 + 2x_2 + x_3 = 12$$
$$x_1 + 4X_2 + 2X_3 = 15$$
$$x_1 + 2X_2 + 5X_3 = 20$$

H.W! solve by Gauss-seidel method,

$$5X_1 + 2X_2 + X_3 = 12$$
$$X_1 + 4X_2 + 2X_3 = 15$$
$$X_1 + 2X_2 + 5X_3 = 20$$

Numerical differentiation

1- Derivative using Newton's forward interpolation formula.

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \cdots\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y - \Delta^3 y + \frac{11}{12}\Delta^4 y + \cdots\right]$$

$$\left.\frac{d^3y}{dx^3}\right|_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y - \frac{3}{2}\Delta^4 y + \cdots\right]$$

EX! Compute $y'(0.2)$ and $y''(0)$ from the following data

x	0.0	0.2	0.4	0.6	0.8	1.0
y	1.00	1.16	3.56	13.96	41.96	101.00

Solution!

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0.0	1.00					
		$\boxed{0.16}$				
$\boxed{0.2}$	1.16		$\boxed{2.24}$			
		2.4		$\boxed{5.76}$		
0.4	3.56		8.0		$\boxed{3.84}$	
		10.4		9.6		$\boxed{0.0}$
0.6	13.96		17.6		3.84	
		28.0		13.44		
0.8	41.96		31.04			
		59.04				
1.0	101.00					

$$f'(x) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \dots\right]$$

$$= \frac{1}{0.2}\left[2.4 - \frac{1}{2}(8.) + \frac{1}{3}(9.6) - \frac{1}{4}(3.84) + \dots\right]$$

$$f'(x) = 32$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y + \frac{11}{12}\Delta^4 y - \frac{5}{6}\Delta^5 y\right]$$

$$f''(x) = \frac{1}{(0.2)^2}\left[2.24 - 5.76 + \frac{11}{12}(3.84) + 0\right]$$

$$f''(x) = 0$$

2- Gregory-Newton Backward formula for derivative-

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\Delta^4 y_0 + \dots\right]$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \dots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{3}{2}\Delta^4 y + \dots\right]$$

Ex: Find $y'(0.4)$ from the following data-

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
0.1	1.10517			
		0.11623		
0.2	1.22140		0.01223	
		0.12846		0.00127
0.3	1.34986		0.01350	
		0.14196		
0.4	1.49182			

$$f'(x) = \frac{1}{h}\left[\Delta y + \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y + \frac{1}{4}\Delta^4 y\right]$$

$$f'(x) = \frac{1}{0.1}\left[0.14196 + \frac{1}{2}(0.135) + \frac{1}{4}(0.00127)\right]$$

$$f'(x) = 1.4913$$

Ex! From the table of values below Compute $f'(x)$ and $f''(x)$ for $x=1$

x	1	2	3	4	5	6
y	1	8	27	64	125	216

Solution!

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		
		91			
6	216				

$x_0 = 1 \quad h = 1 \quad x = 1$

$$f'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y\right]$$

$$= \frac{1}{1}\left[7 - \frac{1}{2}(12) + \frac{1}{3}(6) - \frac{1}{4}(0)\right]$$

$$= 7 - 6 + 2 = 3$$

$$\frac{d^2y}{dx^2} = f''(x) = \frac{1}{h^2}\left[\Delta^2 y - \Delta^3 y + \frac{11}{12}\Delta^4 y\right]$$

$$= \frac{1}{1^2}\left[12 - 6\right] = 6$$