زانکۆی سەلاحەدین – هەولێر
University of Salahaddin – Erbil

Calculation of speed of sound for triatomic gases as a function of altitude

Research Project

Submitted to the department of physics in partial fulfillment of the requirements for the degree of BSc. In Physics.

By

Triska Sardar Khalid

Supervised by:

Mohammad M. Othman

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بسم الله الرحمن الرحيم

" وما أوتيتم من العلم إلا قليلا"

صدق الله العظيم

(سورة الاسراء الاية 85)

To My

Respectful Parents

Dear Brothers and Sisters

Lovely Nieces and Nephews

Triska Sardar Khalid

Acknowledgement

My thanks to the Deanery of the college of Education – Physics departments for their support during this work particularly to Dr. Rashad the head of physics department.

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My most gratefulness and respect to my parents.

Triska Sardar Khalid

Abstract

We made an attempt to calculate how the speed of sound for monoatomic gases changes as a function of altitude in the normal region above sea level as the relevant data was taken at altitude $\leq$ 11 Km. The gravitation of earth at each layer taken into consideration. And the variation of speed of sound on earth atmosphere as function of temperature and altitude taken in consideration.

Chapter One

1. Introduction

A sound wave is a pressure disturbance that travels through a medium by means of particle-to-particle interaction. As one particle becomes disturbed, it exerts a force on the next adjacent particle, thus disturbing that particle from rest and transporting the energy through the medium. Like any wave, the speed of a sound wave refers to how fast the disturbance is passed from particle to particle. While frequency refers to the number of vibrations that an individual particle makes per unit of time, speed refers to the distance that the disturbance travels per unit of time. Since the speed of a wave is defined as the distance that a point on a wave (such as a compression or a rarefaction) travels per unit of time, it is often expressed in units of meters/second. In equation form, this is speed = distance/time. The faster a sound wave travels, the more distance it will cover in the same period of time. If a sound wave were observed to travel a distance of 700 meters in 2 seconds, then the speed of the wave would be 350 m/s. A slower wave would cover less distance - perhaps 660 meters - in the same time period of 2 seconds and thus have a speed of 330 m/s. Faster waves cover more distance in the same period of time. speed of the wave would be 350 m/s. A slower wave would cover less distance - perhaps 660 meters - in the same time period of 2 seconds and thus have a speed of 330 m/s. Faster waves cover more distance in the same period of time.

The speed of sound is the distance travelled per unit of time by a sound wave as it propagates through an elastic medium. At 20 °C, the speed of sound in air is about 343 m/s. It depends strongly on temperature as well as the medium through which a sound wave is propagating. At 0 °C, the speed of sound in air is about 331 m/s. The speed has a weak dependence on frequency and pressure in ordinary air, deviating slightly from ideal behavior.

The effect of gravity on the propagation of sound in a gas was first studied by (Rayleigh,1954) about a century ago. He started with the assumption of an adiabatic atmosphere under gravitational stress.

A long range sound propagation topic which has received recent attention is the propagation of sounds through the atmosphere at high altitudes (Besset and Blanc, 1994). Altitudes of concern include the thermosphere up to about 160 km. Secondary sonic booms from proposed supersonic transports, a source of potentially annoying low frequency sounds on the ground, can propagate initially upward before being refracted down to the ground. Sounds generated by atmospheric explosions can also travel along a path that goes up to the thermosphere giving rise to shadow zones on the ground spaced by thousands of miles (Quinton et al., 2018).

It should be noted that standard values in the 1962 US Standard Atmosphere for atmospheric parameters, such as viscosity, at altitudes above 90 km. Thus, as stated earlier, the viscosity-dependent values for classical and rotational relaxation losses at altitudes above 90 km must be considered only as best estimates. Nevertheless, these estimates do account accurately for the dominant effect of temperature on viscosity assuming a constant atmospheric composition. One atmospheric attenuation effect that was not considered due to its negligible contribution is thermal radiation (Bass, 1981; Bass, et al., 1984).

The thermosphere is a complex and externally-forced deterministic system (*Forbes*, 2007). These forces include solar EUV radiation, high-latitude electrodynamics (*Killeen and Roble*, 1988), particle precipitation (*Akasofu*, 1976) and waves propagating from the lower atmosphere (*Hines*, 1967). In the last several decades, many modeling efforts have been made to improve to our understanding of this complex thermosphere system. Some of the well-known global thermosphere models are the Thermosphere Ionosphere Electrodynamic General Circulation Model (TIEGCM) and its predecessors (*Richmond*,1992).

In colloquial speech, speed of sound refers to the speed of sound waves in air. However, the speed of sound varies from substance to substance: typically, sound travels most slowly in gases, faster in liquids, and fastest in solids. For example, while sound travels at 343 m/s in air, it travels at 1,481 m/s in water (almost 4.3 times as fast) and at 5,120 m/s in iron (almost 15 times as fast). In an exceptionally stiff material such as diamond, sound travels at 12,000 meters per (Dean, 1979) about 35 times its speed in air and about the fastest it can travel under normal conditions.

Sound waves in solids are composed of compression waves (just as in gases and liquids), and a different type of sound wave called a shear wave, which occurs only in solids. Shear waves in solids usually travel at different speeds than compression waves, as exhibited in seismology. The speed of compression waves in solids is determined by the medium's compressibility, shear modulus and density. The speed of shear waves is determined only by the solid material's shear modulus and density (Tahani, 2011).

Sir Isaac Newton's 1687 Principia includes a computation of the speed of sound in air (298 m/s). This is too low by about 15%. (Wong and Zhu, 1995). The discrepancy is due primarily to neglecting the (then unknown) effect of rapidly-fluctuating temperature in a sound wave (in modern terms, sound wave compression and expansion of air is an adiabatic process, not an isothermal process). This error was later rectified by Laplace (Bannon and Kaputa, 2014).

During the 17th century there were several attempts to measure the speed of sound accurately, including attempts by Marin Mersenne in 1630 (420.624 m/s), Pierre Gassendi in 1635 (448.970 m/s) and Robert Boyle (342.9). In 1709, the Reverend William Derham, published a more accurate measure of the speed of sound, at 326.745 m/s. the speed of sound at 20 °C =321.564 m/s.

In gases, adiabatic compressibility is directly related to pressure through the heat capacity ratio, while pressure and density are inversely related to the temperature and molecular weight, thus making only the completely independent properties of temperature and molecular structure important (heat capacity ratio may be determined by temperature and molecular structure, but simple molecular weight is not sufficient to determine it). Sound propagates faster in low molecular weight gases such as helium than it does in heavier gases such as xenon. For monatomic gases, the speed of sound is about 75% of the mean speed that the atoms move in that gas. For a given ideal gas the molecular composition is fixed, and thus the speed of sound depends only on its temperature. At a constant temperature, the gas pressure has no effect on the speed of sound, since the density will increase, and since pressure and density (also proportional to pressure) have equal but opposite effects on the speed of sound, and the two contributions cancel out exactly. In a similar way, compression waves in solids depend both on compressibility and density just as in liquids but in gases the density contributes to the compressibility in such a way that some part of each attribute factors out, leaving only a dependence on temperature, molecular weight, and heat capacity ratio which can be independently derived from temperature and molecular composition. In non-ideal gas behavior regimen, for which the Van der Waals gas equation would be used, the proportionality is not exact, and there is a slight dependence of sound velocity on the gas pressure. Humidity has a small but measurable effect on the speed of sound (causing it to increase by about 0.1%–0.6%), because oxygen and nitrogen molecules of the air are replaced by lighter molecules of water. This is a simple mixing effect.

1-1 Speed of sound in gases

When sound approaches a liquid or solid, the speed of sound is independent of the density of the medium. Since gases expand to fill a given vacuum, their density is very uniform regardless of the type of gas. This is obviously not the case for solids and liquids. Density and pressure decrease smoothly with altitude,. The speed of sound depends only on the complicated temperature variation at altitude and can be calculated from it since isolated density and pressure effects on the speed of sound cancel each other. The speed of sound increases with height in two regions of the stratosphere and thermosphere, due to heating effects in these regions. In the Earth's atmosphere, the chief factor affecting the speed of sound is the temperature. For a given ideal gas with constant heat capacity and composition, the speed of sound is dependent solely upon temperature; see § Details below. In such an ideal case, the effects of decreased density and decreased pressure of altitude cancel each other out, save for the residual effect of temperature (Jearal 2014).

Since temperature (and thus the speed of sound) decreases with increasing altitude up to 11 km, sound is refracted upward, away from listeners on the ground, creating an acoustic shadow at some distance from the source. The decrease of the speed of sound with height is referred to as a negative sound speed gradient. However, there are variations in this trend above 11 km. In particular, in the stratosphere above about 20 km, the speed of sound increases with height, due to an increase in temperature from heating within the ozone layer. This produces a positive speed of sound gradient in this region. Still another region of positive gradient occurs at very high altitudes, in the aptly-named thermosphere above 90 km.

The limitations of the concept of speed of sound due to extreme attenuation are also of concern. The attenuation which exists at sea level for high frequencies applies to successively lower frequencies as atmospheric pressure decreases, or as the mean free path increases. For this reason, the concept of speed of sound (except for frequencies approaching zero) progressively loses its range of applicability at high altitudes. The standard equations for the speed of sound apply with reasonable accuracy only to situations in which the wavelength of the sound wave is considerably longer than the mean free path of molecules in a gas (Kirtskhalia 2021).

The molecular composition of the gas contributes both as the mass (M) of the molecules, and their heat capacities, and so both have an influence on speed of sound. In general, at the same molecular mass, monatomic gases have slightly higher speed of sound (over 9% higher) because they have a higher γ (5/3 = 1.66...) than diatomics do (7/5 = 1.4). Thus, at the same molecular mass, the speed of sound of a monatomic gas goes up by a factor of:

$$\frac{c_{\mathrm{gas,monatomic}}}{c_{\mathrm{gas,diatomic}}} = \sqrt{\frac{5/3}{7/5}} = \sqrt{\frac{25}{21}} = 1.091\ldots$$

This gives the 9% difference, and would be a typical ratio for speeds of sound at room temperature in helium vs. deuterium, each with a molecular weight of 4. Sound travels faster in helium than deuterium because adiabatic compression heats helium more since the helium molecules can store heat energy from compression only in translation, but not rotation. Thus helium molecules (monatomic molecules) travel faster in a sound wave and transmit sound faster. (Sound travels at about 70% of the mean molecular speed in gases; the figure is 75% in monatomic gases and 68% in diatomic gases). Note that in this example we have assumed that temperature is low enough that heat capacities are not influenced by molecular vibration. However, vibrational modes simply cause gammas which decrease toward 1, since vibration modes in a polyatomic gas give the gas additional ways to store heat which do not affect temperature, and thus do not affect molecular velocity and sound velocity. Thus, the effect of higher temperatures and vibrational heat capacity acts to increase the difference between the speed of sound in monatomic vs. polyatomic molecules, with the speed remaining greater in monatomic.

The most important factor influencing the speed of sound in air is temperature. The speed is proportional to the square root of the absolute temperature, giving an increase of about 0.6 m/s per degree Celsius. For this reason, the pitch of a musical wind instrument increases as its temperature increases. The speed of sound is raised by humidity. The difference between 0% and 100% humidity is about 1.5 m/s at standard pressure and temperature, but the size of the humidity effect increases dramatically with temperature. The dependence on frequency and pressure are normally insignificant in practical applications. In dry air, the speed of sound increases by about 0.1 m/s as the frequency rises from 10 Hz to 100 Hz. For audible frequencies above 100 Hz it is relatively constant. Standard values of the speed of sound are quoted in the limit of low frequencies, where the wavelength is large compared to the mean free path (Estrada-Alexanders, 2008).

As shown above, the approximate value 1000/3 = 333.33... m/s is exact a little below 5 °C and is a good approximation for all "usual" outside temperatures (in temperate climates, at least), hence the usual rule of thumb to determine how far lightning has struck: count the seconds from the start of the lightning flash to the start of the corresponding roll of thunder and divide by 3: the result is the distance in kilometers to the nearest point of the lightning bolt. Mach number, a useful quantity in aerodynamics, is the ratio of air speed to the local speed of sound. At altitude, for reasons explained, Mach number is a function of temperature. Aircraft flight instruments, however, operate using pressure differential to compute Mach number, not temperature. The assumption is that a particular pressure represents a particular altitude and, therefore, a standard temperature. Aircraft flight instruments need to operate this way because the stagnation pressure sensed by a Pitot tube is dependent on altitude as well as speed.

1-2 Speed of sound in solids

The speed of sound for pressure waves in stiff materials such as metals is sometimes given for "long rods" of the material in question, in which the speed is easier to measure. In rods where their diameter is shorter than a wavelength, the speed of pure pressure waves may be simplified and is given by (Raymond et al., 2014)

$$c_{\text{solid}} = \sqrt{\frac{E}{\rho}},$$

where *E* is Young's modulus. This is similar to the expression for shear waves, save that Young's modulus replaces the shear modulus. This speed of sound for pressure waves in long rods will always be slightly less than the same speed in homogeneous 3-dimensional solids, and the ratio of the speeds in the two different types of objects depends on Poisson's ratio for the material.

1-3 Speed of sound in liquids

Hence the speed of sound in a fluid is given by

$$c_{\text{fluid}} = \sqrt{\frac{K}{\rho}},$$

where *K* is the bulk modulus of the fluid. In fresh water, sound travels at about 1481 m/s at 20 °C (see the External Links section below for online calculators). Applications of underwater sound can be found in sonar, acoustic communication and acoustical oceanography.

In salt water (Seawater), sound travels at about 1500 m/s. The speed of sound in seawater depends on pressure (depth), temperature An empirical equation for the speed of sound in sea water is provided by Mackenzie (Leroy, 2008; Npl, 2021),

$$c(T,S,z) = a_1 + a_2T + a_3T^2 + a_4T^3 + a_5(S-35) + a_6z + a_7z^2 + a_8T(S-35) + a_9Tz^3,$$

where

- *T* is the temperature in degrees Celsius;
- *S* is the salinity in parts per thousand;
- *z* is the depth in metres.

The constants a_1, a_2, ..., a_9 are,

$$\begin{aligned} a_1 &= 1{,}448.96, & a_2 &= 4.591, & a_3 &= -5.304\times10^{-2},\\ a_4 &= 2.374\times10^{-4}, & a_5 &= 1.340, & a_6 &= 1.630\times10^{-2},\\ a_7 &= 1.675\times10^{-7}, & a_8 &= -1.025\times10^{-2}, & a_9 &= -7.139\times10^{-13}, \end{aligned}$$

with check value 1550.744 m/s for T = 25 °C, S = 35 parts per thousand, z = 1,000 m. This equation has a standard error of 0.070 m/s for salinity between 25 and 40 ppt.

1-4 Speed of sound in plasma

The speed of sound in a plasma for the common case that the electrons are hotter than the ions is given by the formula,

$$c_s = (\gamma ZkT_e/m_i)^{1/2} = 90.85(\gamma ZT_e/\mu)^{1/2}\ \mathrm{m/s},$$

where

m_i is the ion mass;

μ is the ratio of ion mass to proton mass $\mu = m_i/m_p$;

T_e is the electron temperature;

Z is the charge state;

k is Boltzmann constant;

γ is the adiabatic index.

The speed of sound as a function of altitude is generally difficult to represent analytically. We considered temperature, pressure, and humidity as the main factors affecting the speed of sound in our experiment. There are well-known models which relate the speed of sound to temperature and pressure. However, it has been a challenge to find a simple explanation for humidity's influence. The pressure is related to the temperature through the ideal gas law for this model. For the range of temperatures expected, γ changes minimally . So, we anticipated the temperature to have the greatest influence of the factors mentioned above. As far as the effect of humidity, room temperature tests have shown that it has minimal impact on the speed of sound. We have yet to explore how humidity plays a role at different temperature.

Chapter Two

2. Theory

2. 1 Factors affecting the speed of sound

We proceed in a manner analogous to (Richardson, 1963) in order to set up the differential equation for a longitudinal wave disturbance traveling along the z- direction within the gas . Let f(z,t) be displacement produced at height z at time t. The net elastic restoring force on a strip of cross sectional area s and width dx is,

$$s\frac{\partial}{\partial z}\left(E\frac{\partial f}{\partial z}\right)dz, \qquad (1)$$

Whereas the net external force on this strip of mass (ρsdz) equals,

$$-\frac{s\rho}{m}\frac{dW}{dz}dz, \qquad (2)$$

W being the potential energy per molecule. Application of Newton's second law to the motion of this strip leads immediately to the desired inhomogeneous partial differential equation,

$$\rho\frac{\partial^2 f}{\partial t^2}-\frac{\partial}{\partial z}\left(E\frac{\partial f}{\partial z}\right)=-\frac{\rho}{m}\frac{dW}{dz}. \qquad (3)$$

A general solution of Equ. (3) can be written as a sum of $f(z,t)=f_c(z,t)+f_p(z,t)$, where f_p is a particular integral of Equ. (3) obtained by using the retarded Green's function, f_c is the complimentary function satisfying the homogeneous equation,

$$\rho\frac{\partial^2 f_c}{\partial t^2}-\frac{\partial}{\partial z}\left(E\frac{\partial f_c}{\partial z}\right)=0. \qquad (4)$$

For the purpose of finding the velocity we need to consider only the f_c function because it mainly describes the formation and propagation of progressive waves in the medium, while the f_p function describes the emission of extra casual waves associated with the source term on the right hand side of Equ. (3). Although Equ. (4) is of the same form as the wave equation for a free medium [ ] there is an important difference arising from the z dependence of ρ and E.

Let us seek a periodic solution of Equ. (4) in the form,

$$f_c(z,t)=u(z)/\sqrt{E}(z)e^{-i\omega t}+\text{complex conjugate}, \qquad (5)$$

Where *w* is the angular frequency of the wave, and *u(z)* satisfies the exact differential equation,

$$\frac{d^2u}{dz^2} + (k_n^2 + k_a^2)u = 0 \qquad (6)$$

With

$$k_n^2 = \frac{w^2\rho}{E} \quad \text{and} \quad k_a^2 = -\frac{1}{\sqrt{E}}\frac{d^2\sqrt{E}}{dz^2}. \qquad (7)$$

Equation ( ) leads to,

$$f_c \approx \frac{Ae^{i\phi}}{(E\rho)^{1/4}} + \text{complex conjugate}, \qquad (8)$$

Where

$$\phi \equiv \int^z dz\, k_n - wt \qquad (9)$$

And ***A*** is some constant. The phase velocity of the wave in Equ. (8) is computed from,

$$c(z) \equiv \left| \frac{\partial\phi}{\partial t} \Big/ \frac{\partial\phi}{\partial z} \right| = \sqrt{\frac{E}{\rho}} = \sqrt{\gamma v_3(z)}, \qquad (10)$$

Which is again of the same from as for free gas (Richardson, 1963). However, Equ. (10) is valid only in the normal region where E and ρ are slowly varying.

The effect of gravity on the propagation of sound in a gas was first studied by (Lord Rayleigh, 1945) about a century ago. He started with the assumption of an adiabatic atmosphere under gravitational stress. The equation of statute for the system is,

$$p = \text{const}\, \rho^{\gamma}; \qquad (11)$$

And the equation for the hydrostatic equilibrium reads

$$dp = -g\rho\, dz, \qquad (12)$$

Where p, ρ, g, and z, respectively, the pressure, density, ratio of the specific heats at constant pressure to constant volume, the acceleration due to gravity, and altitude. The elimination of pressure between these two equations leads to the expression for the density,

$$\rho = \rho_0 \left(1 - \frac{\gamma - 1}{\gamma} \frac{\rho_0 g z}{p_0}\right)^{1/(\gamma - 1)}. \qquad (13)$$

Here ρ_o and p_o are the density and the pressure at the ground where the gravitational potential is regarded to be zero. By differentiating the pressure with respect to the density and using Equ. (13), Rayleigh obtained the following expression for the velocity of sound,

$$C(z)=[C^2(0) - \gamma(\gamma-1)gz]^{1/2} \qquad (14)$$

where C(0) is the velocity at the ground level.

for monoatomic gas $\gamma=(5/3)= 1.666$, and for tridiatomic gas $\gamma=(9/7)=1.285$. The velocity of sound at 20C^o equal to 343 m/s.

In the case of a monoatomic gas $\gamma=(5/3)$, the equation (14) become,

$$C^2 (z)=C^2(0) - (10/9)gZ \qquad (15)$$

In the case of a triatomic gas $\gamma=(9/7)$, the equation (14) become,

$$C^2 (z)=C^2(0) - (18/14)gZ$$

As is well known, standard kinetic theory (Resnick, and Halliday, 2013) of a free ideal gas (i.e., when there are no force acting on the molecules) leads to an expression for the velocity of sound in term of free elasticity E_o and density ρ_o as,

$$c_0 = (E_0/\rho_0)^{1/2} = (\gamma p_0/\rho_0)^{1/2}, \qquad (16)$$

From which the effect of pressure, density, temperature, humidity, etc., follow as a natural consequence. When the gas is acted upon by gravity we anticipate that Equ. (16) will be so modified as if the sound itself experiences an acceleration of the order of g. This is in view of the fact that the molecules execute random mechanical motion under gravity and the molecular speeds are of the same order as the speed of sound (Resnick, and Halliday, 2013).

The speed of sound at the ground level at temperature T is given by,

$$C(0)=\sqrt{\frac{\gamma RT}{M}}. \qquad (17)$$

Note that the spees of sound is faster at higher temperatures and slower for heavier gases. For air, γ = 1.4, M = 0.02897 kg/mol, and R = 8.314 J/mol K. If the temperature is T= 20 °C (T = 293 K), the speed of sound is C(0) = 343 m/s.

The equation for the speed of sound in air $C(0) = (\gamma RT/M)^{1/2}$ can be simplified to give the equation for the speed of sound in air as a function of absolute temperature (Vladimir, 2012)

$$C(0)=\sqrt{\frac{\gamma RT(273K)}{M\,(273K)}} = \sqrt{\frac{\gamma R(273K)}{M}}\ \sqrt{\frac{T}{273K}}$$

$$C(0)=331m/s \sqrt{\frac{T}{273K}} \quad (18)$$

2. 2 Effect of Earth gravity by altitude

The acceleration due to gravity at an altitude above sea level calculated by the relation (Deng, et al., 2008),

$$g_{altitude}=g.[\ R_e/(R_e+Z)]^2 \quad (19)$$

where, Re is mean radius of earth, R_e=6371.009 km

Z is the altitude above sea level in meter.

$g_{altitude}$ is the acceleration due to gravity at specific altitude.

g is acceleration due to gravity at sea level.

$g=9.80665\ m/s^2$

2.3 Effect of temperature by altitude

Atmosphere of the Earth represents multilayered structure and in each layer, dependences of physical parameters on geometrical altitude *z* are different. The scientific literature (Vladimir, 2012)) shows that in interval of altitudes from $z = 0$ to $z = 11$ km temperature changes under linear law

$$T= -0.0064Z + 288.15 \quad (\text{in } K^o) \quad (20)$$

and in the interval from $z = 51$ km to $z = 85$ km approximately

$$T= -0.0026(Z-0.051)+270.5 \quad (\text{in } K^o) \quad (21)$$

under the law.

2.4 Estimation, the method of least square fit

Estimation procedure, that was developed independently by Gauss (1795), Legendre (1805) and Adrain (1808) and published in the first decade of the nineteenth century.

The least-squares method is a form of mathematical regression analysis used to determine the line of best fit for a set of data, providing a visual demonstration of the relationship between the data points. Each point of data represents the relationship between a known independent variable and an unknown dependent variable. This method of regression analysis begins with a set of data points to be plotted on an x- and y-axis graph. An analyst using the least-squares method will generate a line of best fit that explains the potential relationship between independent and dependent variables.

The equation of least square line is given by y = a + bx (Sastry, 2012)

Normal equation for 'a':

$$\sum Y = na + b\sum X \quad (22)$$

Normal equation for 'b':

$$\sum XY = a\sum X + b\sum X^2 \quad (23)$$

Formula for linear regression equation is given by:

$$y = a + bx \quad (24)$$

a and *b* are given by the following formulas:

$$a\,(intercept) = \frac{\sum y \sum x^2 - \sum x \sum xy}{(\sum x^2) - (\sum x)^2} \quad (25)$$

$$b\,(slope) = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \quad (26)$$

Where,

x and *y* are two variables on the regression line.

b = Slope of the line.

a = *y*-intercept of the line.

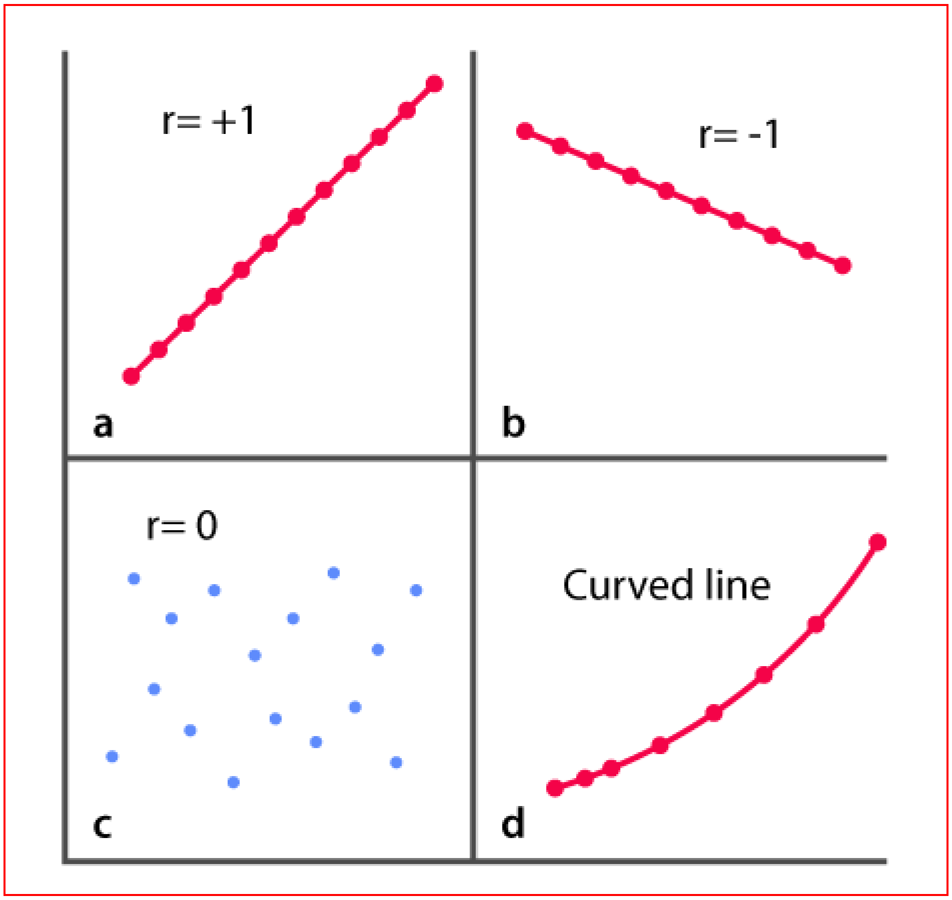
x = Values of the first data set.

y = Values of the second data set.

The sample correlation coefficient formula is:

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\,\sqrt{n\sum y_i^2 - (\sum y_i)^2}} \quad (27)$$

The degree of association is measured by “r_{xy}” after its originator and a measure of linear association. Other complicated measures are used if a curved line is needed to represent the relationship.



The above graph represents the correlation.

The coefficient of correlation is measured on a scale that varies from +1 to -1 through 0. The complete correlation among two variables is represented by either +1 or -1. The correlation is positive when one variable increases and so does the other; while it is negative when one decreases as the other increases. The absence of correlation is described by 0.

Chapter Three

3. Result and Discussion

The universal force of attraction among all the entities or matter in this universe is also known as gravity. It can be considered as the driving force which pulls together all the matter. Gravity is measured in terms of the acceleration or movement that it gives to freely falling objects. At Earth's surface, the value of the acceleration of gravity is about **$9.8\ m/s^2$**. Thus, for every second an object is in free fall, its speed increases by about $9.8\ m/s^2$.

Factors affecting Acceleration due to Gravity

g is majorly affected by the following four factors:

1. The shape of the Earth.
2. Rotational motion of the Earth.
3. Altitude above the Earth's surface.
4. Depth below the Earth's surface.

The variation in apparent gravitational acceleration (*g*) at different locations on Earth is caused by two. First, the Earth is not a perfect sphere, it's slightly flattened at the poles and bulges out near the equator, so points near the equator are farther from the center of mass. The distance between the centers of mass of two objects affects the gravitational force between them, so the force of gravity on an object is smaller at the equator compared to the poles. This effect alone causes the gravitational acceleration to be about 0.18% less at the equator than at the poles.

Second, the rotation of the Earth causes an apparent centrifugal force which points away from the axis of rotation, and this force can reduce the apparent gravitational force. The centrifugal force points directly opposite the gravitational force at the equator, and is zero at the poles. Together, the centrifugal effect and the center of mass distance reduce *g* by about 0.53% at the equator compared to the poles.

Third, to calculate *earth gravitation at* a certain latitude, gravity decreases with altitude as one rises above the Earth's surface because greater altitude means greater distance from the Earth's center. All other things being equal, an increase in altitude from sea level to 9,000 meters causes a weight decrease of about 0.29%. (An additional factor affecting apparent weight is the decrease in air density at altitude, which lessens an object's buoyancy. This would increase a person's apparent weight at an altitude of 9,000 meters by about 0.08%).

The formula shown here approximates the Earth's gravity variation with altitude.

$g_{altitude}=g.[\ R_e/(R_e+Z)]^2$

where, R_e is mean radius of earth, R_e=6371.009 km

Z is the altitude above sea level in meter.

$g_{altitude}$ is the acceleration due to gravity at specific altitude.

g is acceleration due to gravity at sea level.

$g=9.80665\ m/s^2$

Figure 1 show the variation of gravity as function of altitude, which is explained in table 1. We see that as the altitude is rising the influence of the Earth's gravitational field on the speed of sound increases. The gravitational acceleration decreases with altitude, as shown by the solid line in figure. Earth gravity is equal to 9.7744 m/s^2 at 11 km altitude, which is less than gravity at sea level (g=9.8066 m/s^2). The gravitational force above the Earth's surface is proportional to$1/R^2$, where R is your distance from the center of the Earth, gravity decreases with an increase in height and it becomes zero at an infinite distance from earth.

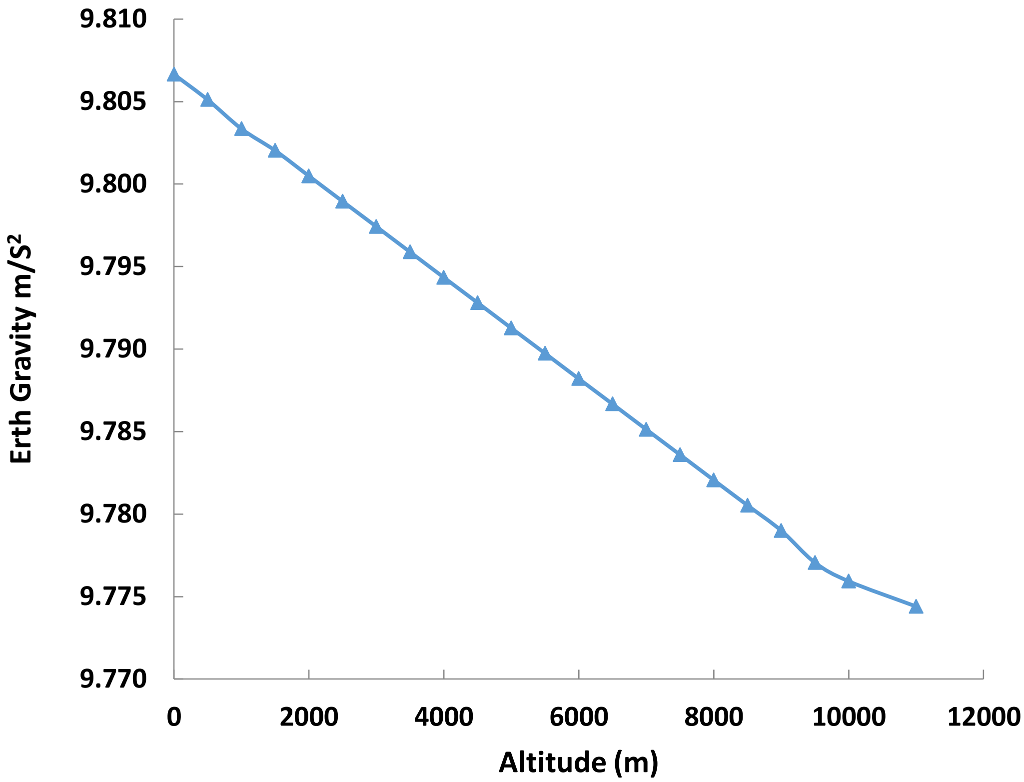


Figure 1: Variation of erth gravity against altitude.

Table 1: Variation of earth gravitation with altitude in the normal region

Altitude (meter)	Earth Gravity m/s^2
0	9.80665
500	9.80511
1000	9.80335
1500	9.80203
2000	9.80049
2500	9.79895
3000	9.79742
3500	9.79588
4000	9.79434
4500	9.79281
5000	9.79127
5500	9.78974
6000	9.7882
6500	9.78667
7000	9.78513
7500	9.7836
8000	9.78206
8500	9.78053
9000	9.779
9500	9.77706
10000	9.77593
11000	9.7744

The starting point of our kinetic model has been a gas in thermal equilibrium in the sense that the averaged properties of the gas do not change with time and the energy equi-partition is built in. This generally means that the molecular mean free path is small compared to other characteristic distances. The fact that the total energy of a given molecule behaves in the same way as if no collisions are present is understandable because the molecules are regarded as perfectly elastic point like bodies.

It is noticed that the coefficient of gz in our formula [Eq. (14)] is $\gamma(\gamma-1)$ unlike $(\gamma-1)$ appearing in Rayleigh's formula, In the case of a triatomic gas ($\gamma = 9/7$) our formula reduces to

$$C(z)=[C^2(0) - 0.3673gz]^{1/2}.$$

The fact that the velocity of sound under gravity decreases with increasing altitude z is to be understood in terms of the variation of temperature from layer to layer as mentioned already in the Introduction

In figure 2 and 3 graphs represent the distribution of the true velocities of sounds in CO_2 and O_3 triatomic gases respectively along the altitude of the troposphere. We see that as the altitude is rising the influence of the Earth's gravitational field on the speed of sound increases.

Table 2 and 3 represents the altitude distribution of the values of the adiabatic and true velocities of sounds in CO_2 and O_3 triatomic gases respectively in the stratosphere.

Relative errors between the values of the true and adiabatic speeds of sound and the corresponding least square fit are also presented in the table. As seen, the relative error in determining the speed of sound at an altitude of 11 km is greater than at an altitude of 1 km.

Figure 4 and table 4 show variation of temperature with altitude in interval 0-11 Km by using equation (20), the temperature is decrease as function of altitude obeys linear law.

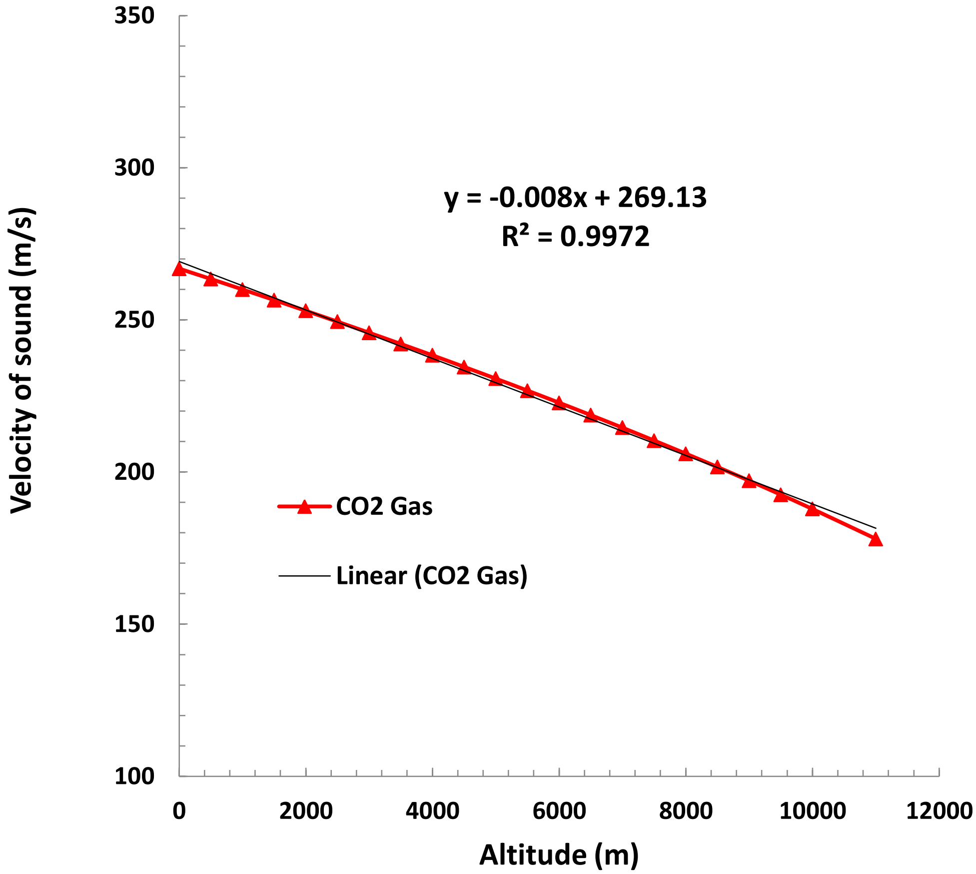


Figure 2. . Velocity of sound against the altitude for CO_2 triatomic gas .

Table 2: Variation of the velocity of sound with altitude in the normal region for CO_2 triatomic gas

Altitude (m)	Earth gravitation m/s^2	Velocity of sound C(z) (m/s)	Velocity of sound (m/s) due to simulation fitting	Error%
0	9.81	266.82	269.13	0.87
500	9.81	263.42	265.13	0.65
1000	9.80	259.98	261.13	0.44
1500	9.80	256.50	257.13	0.25
2000	9.80	252.97	253.13	0.06
2500	9.80	249.39	249.13	0.10
3000	9.80	245.76	245.13	0.25
3500	9.80	242.07	241.13	0.39
4000	9.79	238.33	237.13	0.50
4500	9.79	234.53	233.13	0.60
5000	9.79	230.67	229.13	0.67
5500	9.79	226.75	225.13	0.71
6000	9.79	222.75	221.13	0.73
6500	9.79	218.69	217.13	0.71
7000	9.79	214.55	213.13	0.66
7500	9.78	210.33	209.13	0.57
8000	9.78	206.02	205.13	0.43
8500	9.78	201.63	201.13	0.25
9000	9.78	197.14	197.13	0.00
9500	9.78	192.54	193.13	0.30
10000	9.78	187.83	189.13	0.69
11000	9.77	178.03	181.13	1.74

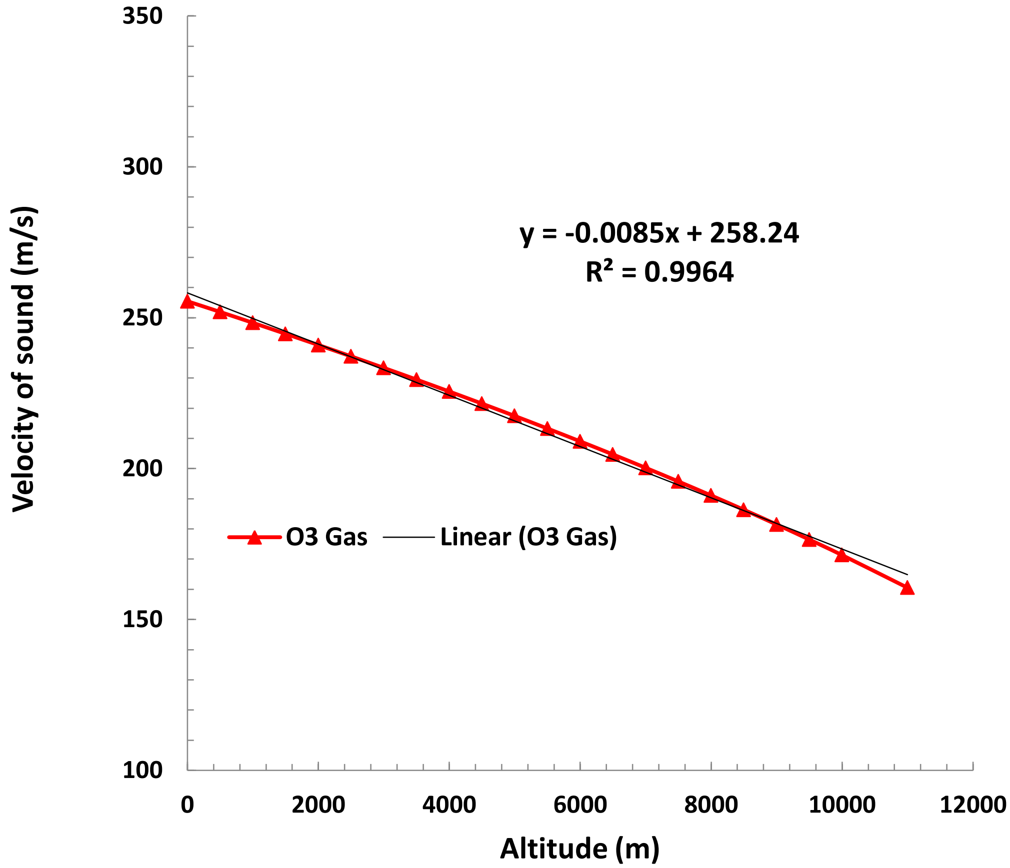


Figure 3. . Velocity of sound against the altitude for O3 triatomic gas .

Table 3: Variation of the velocity of sound with altitude in the normal region for O_3 triatomic gas

Altitude (m)	Earth gravitation m/s^2	Velocity of sound C(z) (m/s)	Velocity of sound (m/s) due to simulation fitting	Error%
0	9.81	255.47	258.24	1.08
500	9.81	251.92	253.99	0.82
1000	9.80	248.32	249.74	0.57
1500	9.80	244.67	245.49	0.33
2000	9.80	240.97	241.24	0.11
2500	9.80	237.20	236.99	0.09
3000	9.80	233.38	232.74	0.28
3500	9.80	229.50	228.49	0.44
4000	9.79	225.55	224.24	0.58
4500	9.79	221.53	219.99	0.70
5000	9.79	217.44	215.74	0.78
5500	9.79	213.27	211.49	0.84
6000	9.79	209.02	207.24	0.85
6500	9.79	204.69	202.99	0.83
7000	9.79	200.26	198.74	0.76
7500	9.78	195.73	194.49	0.63
8000	9.78	191.10	190.24	0.45
8500	9.78	186.35	185.99	0.19
9000	9.78	181.48	181.74	0.14
9500	9.78	176.48	177.49	0.57
10000	9.78	171.33	173.24	1.12
11000	9.77	160.53	164.74	2.63

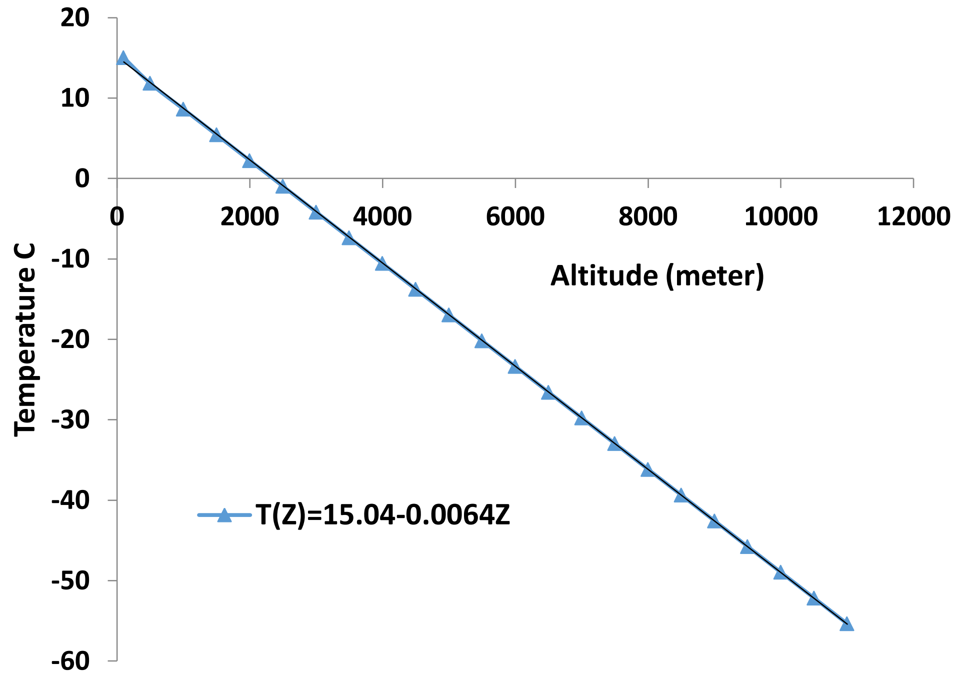


Figure 4 : Variation of Temperature as function of Altitude.

Table 4: Temperature as function of altitude

Z (meter)	Temperature C^o
100	15.04
500	11.84
1000	8.64
1500	5.44
2000	2.24
2500	-0.96
3000	-4.16
3500	-7.36
4000	-10.56
4500	-13.76
5000	-16.96
5500	-20.16
6000	-23.36
6500	-26.56
7000	-29.76
7500	-32.96
8000	-36.16
8500	-39.36
9000	-42.56
9500	-45.76
10000	-48.96
10500	-52.16
11000	-55.36

Chapter Four

4. Conclusion

It is shown above that the existing theory on adiabatic sound in the Earth atmosphere is characterized by deficiencies related to incorrect definition of the sound speed which is understood as speed of distribution of adiabatic density perturbation of the medium. As a result, the sound speed in the Earth atmosphere depends only on temperature. It is demonstrated that contemporary conception on adiabaticity of sound in the Earth atmosphere is fair in sufficient approximation only for altitudes $z \leq 1$ Km. At higher altitudes ($z \leq 11$ Km) the heterogeneity of the atmosphere in gravitation field of the Earth, we show that the speed of sound changes as a function of altitude in Sodium vapor and Neon monoatomic gases. Specifically, the earth gravitation and the speed of sound have a general positive correlation. And the temperature decrease linearly as function of altitude.

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