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The Speed of Sound in a Diatomic Gases Bases on the Gravitational Field and Correction of Mach's Number

Research Project

Submitted to the department of physics in partial fulfillment of the requirements for the degree of BSc. In Physics.

By

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Apirl 2024

بسم الله الرحمن الرحيم

" وما أوتيتم من العلم إلا قليلا"

صدق الله العظيم

(سورة الاسراء الاية 85)

Supervisor Certificate

This research project has been written under my supervision and has been submitted for the award of the degree of BSc. in (Physics).

Signature
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Date / /2024

I confirm that all requirements have been completed.

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To My

Respectful Parents

Dear Brothers and Sisters

Lovely Nieces and Nephews

Helin

Acknowledgement

My thanks to the Physics departments for their support during this work particularly to Dr. Assad Hamid the head of physics department.

I am most grateful to my supervisor Dr. Muhammed Mustafa Dzayi for his invaluable guidance, support and patience during this work.

My most gratefulness and respect to my parents.

Helin

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Abstract

The elastic properties of a gas under the action of an external potential field together with the propagation of sound waves in such a medium are discussed in a simple kinetic model. Approximate solution to the wave equation has been obtained explicitly and the velocity of sound is evaluated in the normal region. If the diatomic gas is under the action of gravity the squre of the sound velocity is shown to decrease linearly with altitude, our calculated data was taken to about $\leq$ 11Km. The gravitation in a layer at a given altitude, however, remains approximately constant. The research shows a significant dependence of the speed of sound on the altitude and, consequently, the requirement to adjust the Mach number, which plays an important role in determining the aerodynamic parameters of an aircraft.

Chapter One

1. Introduction

The effect of gravity on the propagation of sound in a gas was first studied by (Lord Rayleigh, 1945) about a century ago. He started with the assumption of an adiabatic atmosphere under gravitational stress.

Sound speed is a characteristic quantity of the medium included in the system of hydrodynamic (gas-dynamic) equations and plays a significant role in study of wave processes within it. Therefore, correct determination of its value is crucial in adequate description of generation and distribution of waves in media. According to the modern theory of sound wave density perturbation is considered as mass variation in constant volume without heat transfer *i.e.* adiabatically and therefore the speed of sound propagation is called adiabatic speed of sound.

Sound, a mechanical disturbance from a state of equilibrium that propagates through an elastic material medium. A purely subjective definition of sound is also possible, as that which is perceived by the ear, but such a definition is not particularly illuminating and is unduly restrictive, for it is useful to speak of sounds that cannot be heard by the human ear, such as those that are produced by dog whistles or by sonar equipment.

The study of sound should begin with the properties of sound waves. There are two basic types of wave, transverse and longitudinal, differentiated by the way in which the wave is propagated. In a transverse wave, such as the wave generated in a stretched rope when one end is wiggled back and forth, the motion that constitutes the wave is perpendicular, or transverse, to the direction (along the rope) in which the wave is moving. An important family of transverse waves is generated by electromagnetic sources such as light or radio, in which the electric and magnetic fields constituting the wave oscillate perpendicular to the direction of propagation.

Sound propagates through air or other mediums as a longitudinal wave, in which the mechanical vibration constituting the wave occurs along the direction of propagation of the wave. A longitudinal wave can be created in a coiled spring by squeezing several of the turns together to form a compression and then releasing them, allowing the compression to travel the length of the spring. Air can be viewed as being composed of layers analogous to such coils, with a sound wave propagating as layers of air "push" and "pull" at one another much like the compression moving down the spring (Halliday and Walker 2013).

In recent years, educators have proposed different designs for experimental setups taking advantage of the new technology of various digital devices. Measurements of the speed of sound, using computer sound cards and audio manipulation software, have been described in the literatures, (Velasco, et al., 2004).

High altitude weather balloons offer platforms to perform very interesting experiments that can illustrate different principles that would not be so fascinating to the students if performed only in the laboratory. Our experience is that students are excited in being involved in such exploration of near space, often demonstrating a sense of ownership of the project, and they are really eager to use different analysis tools to extract any useful information from the data.

A recent study of sound attenuation at high altitude (Sutherland and Bass, 2004) predicts a large increase in sound speed due to rotational relaxation in the atmosphere. In this paper, we explain the physical mechanism which causes this dispersion and investigate its effect on the propagation of infrasound in the thermosphere.

The Laplace speed of sound, C, in a gas can be written C=ϒRT/M, where g is the ratio of the specific heat at constant pressure, C_p, to the specific heat at constant volume, C_v; R is the universal gas constant at 8314.48 J/kmol K; T is the temperature in kelvin; and M is the mean molecular mass at 28.964 kg/kmol (Kinsler and Frey, 1962). This equation is found to be reasonably accurate provided that propagation is adiabatic. In an actual gas, g is frequency-dependent and complex, which gives rise to real and imaginary terms in the sound speed c that are related through a Kramers-Kronig transform (Weaver and Pao, 1981). As a result, if the absorption is known as a function of frequency, dispersion can be computed directly. In this paper, we will adopt a more physical, though no more accurate, description of dispersion.

At low frequency, the specific heats at constant pressure and volume can be taken to be their static values given by C_{po} and C_{vo}. For gases at pressures of 1 atm and below, C_p and C_v differ by R, so only one is needed to compute ϒ. For a diatomic molecule, C_{vo} = (3/2)R as a consequence of three translational degrees of freedom plus R as a consequence of two rotational degrees of freedom. The complete value of C_{vo} also has a small contribution from a vibrational motion but that value is small enough to be ignored for the purpose of this work (though it is the dominant term when computing absorption at audible frequencies at sea level). Nonlinear polyatomic molecules derive a C_{vo} contribution from rotation of (3/2)R. In practice, dispersion discussed in the following can be determined without considering the contribution to C_{vo} from nonlinear molecules but we have included that very small term, as well as vibrational contributions, for completeness. As the frequency of an acoustic wave increases, at some point, the acoustic variations of pressure and temperature are too fast for the rotational exchange of energy to track translational variations. When the rotational energy can no longer follow translational energy changes, we say that rotation has relaxed and no longer plays a role in propagating sound. At higher frequencies, this relaxation reduces C_v to (3/2)R from (5/2)R and ϒ increases to 1.67 from 1.4. This increases the sound speed by about a third given the same molecular mass and temperature. Rotational relaxation occurs at a frequency of several hundred megahertz at sea level. At higher altitudes, the atmospheric pressure decreases, which in turn decreases the rate at which molecules can collide and exchange energy. The effect is exactly equivalent to increasing the frequency, hence relaxation frequencies depend on the ratio *f/P*. At a pressure of 10^{-6} atm, an acoustic wave of frequency 1 Hz experiences the same dispersion (and absorption) as a 1-MHz wave at a pressure of 1 atm (Bass and Respet., 2007).

At ratios of *f/P* greater than those required to observe the effects of rotational relaxation, the acoustic wavelength approaches the mean free path. At this very rarified extreme, the quantities that enter into viscosity and thermal conduction

begin to become frequency-dependent. The ‘‘translational relaxation’’ regime has been explored by (Bauer,1972) among others, and has been treated in great detail theoretically; the detailed theory is beyond the scope of this paper. Instead, we will adopt an presented by (Sutheland and Bass, 2004), which adequately agrees with Greenspan’s measurements at f/P ratios below 1000 MHz/atm, as well as with the theory based upon gas kinetics. We can look upon this contribution as due to translational dispersion though it is probably better physics not to try too hard to separate the terms.

The speed of any wave depends upon the properties of the medium through which the wave is traveling. Typically there are two essential types of properties that affect wave speed - inertial properties and elastic properties. Elastic properties are those properties related to the tendency of a material to maintain its shape and not deform whenever a force or stress is applied to it. A material such as steel will experience a very small deformation of shape (and dimension) when a stress is applied to it. Steel is a rigid material with a high elasticity. On the other hand, a material such as a rubber band is highly flexible; when a force is applied to stretch the rubber band, it deforms or changes its shape readily. A small stress on the rubber band causes a large deformation. Steel is considered to be a stiff or rigid material, whereas a rubber band is considered a flexible material. At the particle level, a stiff or rigid material is characterized by atoms and/or molecules with strong attractions for each other. When a force is applied in an attempt to stretch or deform the material, its strong particle interactions prevent this deformation and help the material maintain its shape. Rigid materials such as steel are considered to have a high elasticity. (Elastic modulus is the technical term). The phase of matter has a tremendous impact upon the elastic properties of the medium. In general, solids have the strongest interactions between particles, followed by liquids and then gases. For this reason, longitudinal sound waves travel faster in solids than they do in liquids than they do in gases. Even though the inertial factor may favor gases, the elastic factor has a greater influence on the speed (**v**) of a wave, thus yielding this general pattern:

$$\mathbf{v_{solids} > v_{liquids} > v_{gases}}$$

Inertial properties are those properties related to the material's tendency to be sluggish to changes in its state of motion. The density of a medium is an example of an inertial property. The greater the inertia (i.e., mass density) of individual particles of the medium, the less responsive they will be to the interactions between neighboring particles and the slower that the wave will be. As stated above, sound waves travel faster in solids than they do in liquids than they do in gases. However, within a single phase of matter, the inertial property of density tends to be the property that has a greatest impact upon the speed of sound. A sound wave will travel faster in a less dense material than a more dense material. Thus, a sound wave will travel nearly three times faster in Helium than it will in air.

The speed of sound is the distance travelled per unit of time by a sound wave as it propagates through an elastic medium. At 20 °C, the speed of sound in air is about 343 meters per second. It depends strongly on temperature as well as the medium through which a sound wave is propagating. At 0 °C (32 °F), the speed of sound is about 331 meters per second.

The speed of sound in an ideal gas depends only on its temperature and composition. The speed has a weak dependence on frequency and pressure in ordinary air, deviating slightly from ideal behavior.

In colloquial speech, speed of sound refers to the speed of sound waves in air. However, the speed of sound varies from substance to substance: typically, sound travels most slowly in gases, faster in liquids, and fastest in solids. For example, while sound travels at 343 m/s in air, it travels at 1,481 m/s in water (almost 4.3 times as fast) and at 5,120 m/s in iron (almost 15 times as fast). In an exceptionally stiff material such as diamond, sound travels at 12,000 meters per second (39,000 ft/s), (Gomes and Trusler, 1998) about 35 times its speed in air and about the fastest it can travel under normal conditions.

Sound waves in solids are composed of compression waves (just as in gases and liquids), and a different type of sound wave called a shear wave, which occurs only in solids. Shear waves in solids usually travel at different speeds than compression waves, as exhibited in seismology. The speed of compression waves in solids is determined by the medium's compressibility, shear modulus and density. The speed of shear waves is determined only by the solid material's shear modulus and density (Pereira and Precker, 2005).

Sir Isaac Newton's 1687 Principia includes a computation of the speed of sound in air (298 m/s). This is too low by about 15%. The discrepancy is due primarily to neglecting the (then unknown) effect of rapidly-fluctuating temperature in a sound wave (in modern terms, sound wave compression and expansion of air is an adiabatic process, not an isothermal process). This error was later rectified by Laplace (Estrada and Trusler, 1998).

During the 17th century there were several attempts to measure the speed of sound accurately, including attempts by Marin Mersenne in 1630 (420.624 m/s), Pierre Gassendi in 1635 (448.970 m/s) and Robert Boyle (342.9). In 1709, the Reverend William Derham, published a more accurate measure of the speed of sound, at 326.745 m/s. the speed of sound at 20 °C =321.564 m/s.

Chapter Two

2. Theory

2.1. Speed of sound

The speed of sound is the distance travelled per unit of time by a sound wave as it propagates through an elastic medium. At 20 °C, the speed of sound in air is about 343 meters per second. It depends strongly on temperature as well as the medium through which a sound wave is propagating. At 0 °C (32 °F), the speed of sound is about 331 meters per second.

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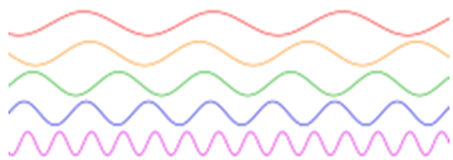
2. 2 **Waves**

Sound is transmitted through gases, plasma, and liquids as longitudinal waves, also called compression waves. It requires a medium to propagate. Through solids, however, it can be transmitted as both longitudinal waves and transverse waves. Longitudinal sound waves are waves of alternating pressure deviations from the equilibrium pressure, causing local regions of compression and rarefaction, while transverse waves (in solids) are waves of alternating shear stress at right angle to the direction of propagation (Resnick and Halliday, 1969).

The energy carried by an oscillating sound wave converts back and forth between the potential energy of the extra compression (in case of longitudinal waves) or lateral displacement strain (in case of transverse waves) of the matter, and the kinetic energy of the displacement velocity of particles of the medium.



Sounds can be represented as a mixture of their component Sinusoidal waves of different frequencies. The bottom waves have higher frequencies than those above. The horizontal axis represents time.

2. 3 Sound absorption

The attenuation is proportional to the square of the sound wave's frequency, as expressed in the formula α/f^2, where α is the attenuation coefficient of the medium and f is the wave frequency. The amplitude of an attenuated wave is then given by $A(x) = A_o e^{-\alpha x,}$ where A_o is the original amplitude of the wave and $A(x)$ is the amplitude after it has propagated a distance x through the medium.

2. 4 Impedance

One of the important physical characteristics relating to the propagation of sound is the acoustic impedance of the medium in which the sound wave travels. Acoustic impedance (Z) is given by the ratio of the wave's acoustic pressure (p) to its volume velocity (U) [Z=P/U]. Like its analogue, electrical impedance (or electrical resistance), acoustic impedance is a measure of the ease with which a sound wave propagates through a particular medium. For the simplest case of a plane wave, specific acoustic impedance is the product of the equilibrium density (ρ) of the medium and the wave speed (S) [Z=ρS].

The unit of specific acoustic impedance is the pascal second per metre, often called the rayl, after Lord Rayleigh. The unit of acoustic impedance is the pascal second per cubic metre, called an acoustic ohm, by analogy to electrical impedance (Greenspan, 1965).

2. 5 Factors affecting the velocity of sound

We proceed in a manner analogous to (Richardson, 1963) in order to set up the differential equation for a longitudinal wave disturbance traveling along the z- direction within the gas . Let f(z,t) be displacement produced at height z at time t. The net elastic restoring force on a strip of cross sectional area s and width dx is,

$$s\frac{\partial}{\partial z}\left(E\frac{\partial f}{\partial z}\right)dz, \qquad (1)$$

Whereas the net external force on this strip of mass (ρsdz) equals,

$$-\frac{s\rho}{m}\frac{dW}{dz}dz, \qquad (2)$$

W being the potential energy per molecule. Application of Newton's second law to the motion of this strip leads immediately to the desired inhomogeneous partial differential equation,

$$\rho\frac{\partial^2 f}{\partial t^2}-\frac{\partial}{\partial z}\left(E\frac{\partial f}{\partial z}\right)=-\frac{\rho}{m}\frac{dW}{dz}. \qquad (3)$$

A general solution of Equ. (3) can be written as a sum of $f(z,t)=f_c(z,t)+f_p(z,t)$, where f_p is a particular integral of Equ. (3) obtained by using the retarded Green's function, f_c is the complimentary function satisfying the homogeneous equation,

$$\rho\frac{\partial^2 f_c}{\partial t^2}-\frac{\partial}{\partial z}\left(E\frac{\partial f_c}{\partial z}\right)=0. \qquad (4)$$

For the purpose of finding the velocity we need to consider only the f_c function because it mainly describes the formation and propagation of progressive waves in the medium, while the f_p function describes the emission of extra casual waves associated with the source term on the right hand side of Equ. (3). Although Equ. (4) is of the same form as the wave equation for a free medium [ ] there is an important difference arising from the z dependence of ρ and E.

Let us seek a periodic solution of Equ. (4) in the form,

$$f_c(z,t)=u(z)/\sqrt{E(z)}e^{-i\omega t}+\text{complex conjugate}, \qquad (5)$$

Where *w* is the angular frequency of the wave, and *u(z)* satisfies the exact differential equation,

$$\frac{d^2u}{dz^2} + (k_n^2 + k_a^2)u = 0 \qquad (6)$$

With

$$k_n^2 = \frac{w^2\rho}{E} \quad \text{and} \quad k_a^2 = -\frac{1}{\sqrt{E}}\frac{d^2\sqrt{E}}{dz^2}. \qquad (7)$$

Equation ( ) leads to,

$$f_c \approx \frac{Ae^{i\phi}}{(E\rho)^{1/4}} + \text{complex conjugate}, \qquad (8)$$

Where

$$\phi \equiv \int^z dz\, k_n - wt \qquad (9)$$

And ***A*** is some constant. The phase velocity of the wave in Equ. (8) is computed from,

$$c(z) \equiv \left| \frac{\partial\phi}{\partial t} \Big/ \frac{\partial\phi}{\partial z} \right| = \sqrt{\frac{E}{\rho}} = \sqrt{\gamma v_3(z)}, \qquad (10)$$

Which is again of the same from as for free gas (Richardson, 1963). However, Equ. (10) is valid only in the normal region where E and ρ are slowly varying.

The effect of gravity on the propagation of sound in a gas was first studied by (Lord Rayleigh, 1945) about a century ago. He started with the assumption of an adiabatic atmosphere under gravitational stress. The equation of statute for the system is,

$$p = \text{const}\, \rho^{\gamma}; \qquad (11)$$

And the equation for the hydrostatic equilibrium reads

$$dp = -g\rho\, dz, \qquad (12)$$

Where p, ρ, g, and z, respectively, the pressure, density, ratio of the specific heats at constant pressure to constant volume, the acceleration due to gravity, and altitude. The elimination of pressure between these two equations leads to the expression for the density,

$$\rho = \rho_0 \left(1 - \frac{\gamma - 1}{\gamma}\frac{\rho_0 gz}{p_0}\right)^{1/(\gamma - 1)}. \qquad (13)$$

Here ρ_o and p_o are the density and the pressure at the ground where the gravitational potential is regarded to be zero. By differentiating the pressure with respect to the density and using Equ. (13), Rayleigh obtained the following expression for the velocity of sound,

$$C(z)=[C^2(0) - \gamma(\gamma-1)gz]^{1/2} \qquad (14)$$

where C(0) is the velocity at the ground level.

for monoatomic gas $\gamma=(5/3)= 1.666$, and for nonvibrating diatomic gas $\gamma=(7/5)=1.4$. The velocity of sound at 20C^o equal to 343 m/s.

In the case of a diatomic gas $\gamma=(5/3)$, the equation (14) become,

$$C^2(z)=C^2(0) - (2/5)\gamma gZ \qquad (15)$$

The right hand side of Equ. (15) is proportional to the square root of the absolute temperature.

As is well known, standard kinetic theory (Resnick, and Halliday, 1969) of a free ideal gas (i.e., when there are no force acting on the molecules) leads to an expression for the velocity of sound in term of free elasticity E_o and density ρ_o as,

$$c_0 = (E_0/\rho_0)^{1/2} = (\gamma p_0/\rho_0)^{1/2}, \qquad (16)$$

From which the effect of pressure, density, temperature, humidity, etc., follow as a natural consequence. When the gas is acted upon by gravity we anticipate that Equ. (16) will be so modified as if the sound itself experiences an acceleration of the order of g. This is in view of the fact that the molecules execute random mechanical motion under gravity and the molecular speeds are of the same order as the speed of sound (Resnick, and Halliday, 1969).

2. 6 Effect of Earth gravity by altitude

The acceleration due to gravity at an altitude above sea level calculated by the relation (Deng, et al., 2008) ,

$$g_{altitude}=g.[R_e/(R_e+Z)]^2 \qquad (17)$$

where, R_e is mean radius of earth, R_e=6371.009 km

Z is the altitude above sea level in meter.

$g_{altitude}$ is the acceleration due to gravity at specific altitude.

g is acceleration due to gravity at sea level.

$g=9.80665\ m/s^2$

2.7 Mach Number

The Mach number calculator returns the **Mach number** for the given speed. If you wondered **how fast an aircraft is flying** making those loud noises, this calculator tells you much faster it is than the speed of sound. The calculator will also tell you the meaning of the terms – supersonic and subsonic! Scroll down and read on further to understand what is Mach number and how to calculate Mach number.

Sound travels at different speeds in different medium as we explained in our speed of sound in at different temperatures. That speed is compared with the speed of an object, say an aircraft or a boat. The ratio of the speed of an object and the speed of sound is known as the Mach number. The Mach number is a dimensionless quantity. Mathematically, the Mach number equation can be written as:

$$M = \frac{v}{c} \tag{18}$$

Such that ***v*** is the speed of the object and ***c*** is the speed of sound.

You can also use the advanced mode of the calculator if you know the temperature of air. The speed of sound in air at different temperature, *T* (in °*C*) is given by the speed of sound formula:

$$c = 331.3 \cdot \sqrt{1 + \frac{T}{273.15}} \tag{19}$$

Estimate the Mach number for an aircraft flying at a speed 800 m/s. Given the speed of sound as 343 m/s.

- Step 1: Enter the speed of aircraft as 800 m/s.
- Step 2: Enter the speed of sound in air as 343 m/s.
- Step 3: The Mach number or Mach speed for the aircraft is:

  M=800/343=1.805

2.8 Estimation, the method of least square fit

Estimation procedure, that was developed independently by Gauss (1795), Legendre (1805) and Adrain (1808) and published in the first decade of the nineteenth century.

The least-squares method is a form of mathematical regression analysis used to determine the line of best fit for a set of data, providing a visual demonstration of the relationship between the data points. Each point of data represents the relationship between a known independent variable and an unknown dependent variable. This method of regression analysis begins with a set of data points to be plotted on an x- and y-axis graph. An analyst using the least-squares method will generate a line of best fit that explains the potential relationship between independent and dependent variables.

The equation of least square line is given by Y = a + bX (Sastry, 2012)

Normal equation for ‘a’:

$$\sum Y = na + b\sum X \quad (19)$$

Normal equation for ‘b’:

$$\sum XY = a\sum X + b\sum X^2 \quad (20)$$

Solving these two normal equations we can get the required trend line equation.

Thus, we can get the line of best fit with formula y = ax + b

Or

Formula for linear regression equation is given by:

$$y = a + bx \quad (21)$$

a and *b* are given by the following formulas:

$$a\,(intercept) = \frac{\sum y \sum x^2 - \sum x \sum xy}{(\sum x^2) - (\sum x)^2} \quad (22)$$

$$b\,(slope) = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \quad (23)$$

Where,

x and *y* are two variables on the regression line.

b = Slope of the line.

a = *y*-intercept of the line.

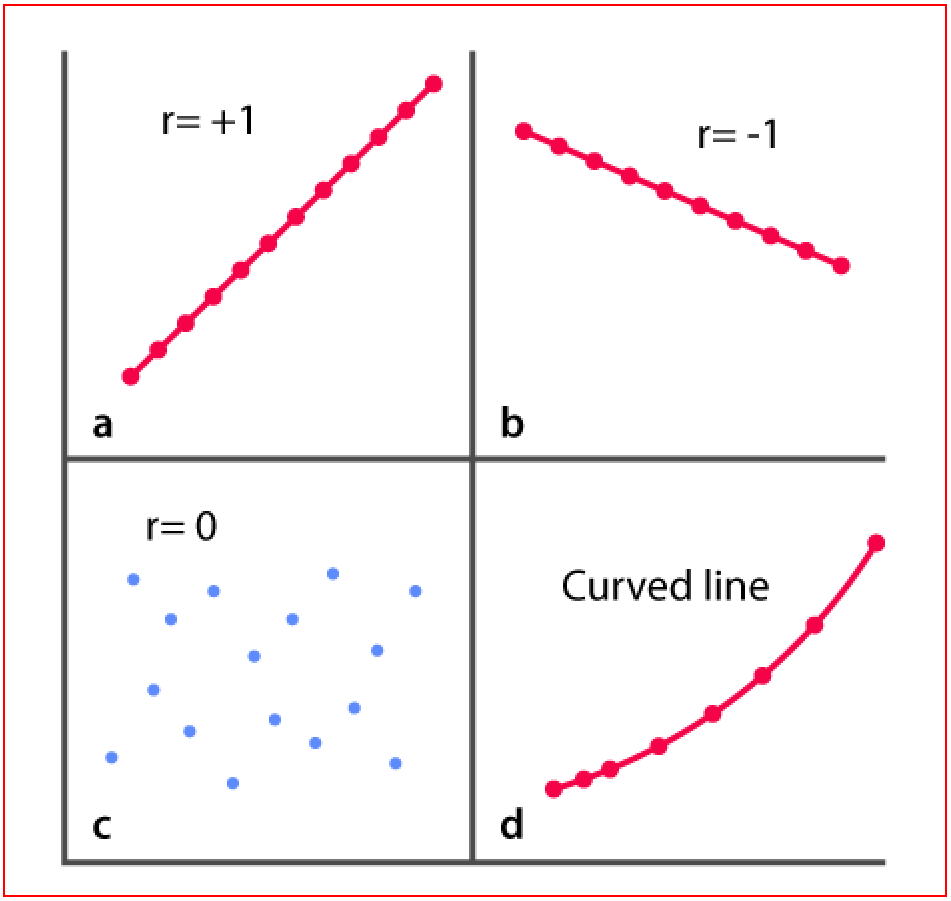
x = Values of the first data set.

y = Values of the second data set.

The sample correlation coefficient formula is:

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\ \sqrt{n\sum y_i^2 - (\sum y_i)^2}} \quad (24)$$

The degree of association is measured by "r" after its originator and a measure of linear association. Other complicated measures are used if a curved line is needed to represent the relationship.



The above graph represents the correlation.

The coefficient of correlation is measured on a scale that varies from +1 to -1 through 0. The complete correlation among two variables is represented by either +1 or -1. The correlation is positive when one variable increases and so does the other; while it is negative when one decreases as the other increases. The absence of correlation is described by 0.

Chapter Three

3. Result and Discussion

The universal force of attraction among all the entities or matter in this universe is also known as gravity. It can be considered as the driving force which pulls together all the matter. Gravity is measured in terms of the acceleration or movement that it gives to freely falling objects. At Earth's surface, the value of the acceleration of gravity is about **9.8 m/s^2**. Thus, for every second an object is in free fall, its speed increases by about 9.8 m/s^2.

factors affecting Acceleration due to Gravity g is majorly affected by the following four factors:

1. The shape of the Earth.
2. Rotational motion of the Earth.
3. Altitude above the Earth's surface.
4. Depth below the Earth's surface.

The variation in apparent gravitational acceleration (*g*) at different locations on Earth is caused by two. First, the Earth is not a perfect sphere, it's slightly flattened at the poles and bulges out near the equator, so points near the equator are farther from the center of mass. The distance between the centers of mass of two objects affects the gravitational force between them, so the force of gravity on an object is smaller at the equator compared to the poles. This effect alone causes the gravitational acceleration to be about 0.18% less at the equator than at the poles.

Second, the rotation of the Earth causes an apparent centrifugal force which points away from the axis of rotation, and this force can reduce the apparent gravitational force. The centrifugal force points directly opposite the gravitational force at the equator, and is zero at the poles. Together, the centrifugal effect and the center of mass distance reduce *g* by about 0.53% at the equator compared to the poles.

Third, to calculate *earth gravitation at* a certain latitude, gravity decreases with altitude as one rises above the Earth's surface because greater altitude means greater distance from the Earth's center. All other things being equal, an increase in altitude from sea level to 9,000 meters causes a weight decrease of about 0.29%. (An additional factor affecting apparent weight is the decrease in air density at altitude, which lessens an object's buoyancy. This would increase a person's apparent weight at an altitude of 9,000 meters by about 0.08%).

The formula shown here approximates the Earth's gravity variation with altitude.

$$g_{altitude}=g.[\,R_e/(R_e+Z)]^2$$

where, R_e is mean radius of earth, R_e=6371.009 km

Z is the altitude above sea level in meter.

$g_{altitude}$ is the acceleration due to gravity at specific altitude.

g is acceleration due to gravity at sea level.

$g=9.80665\ m/s^2$

Figure 1 show the variation of gravity as function of altitude, which is explained in table 1. We see that as the altitude is rising the influence of the Earth's gravitational field on the speed of sound increases. The gravitational acceleration decreases with altitude, as shown by the solid line in figure. Earth gravity is equal to 9.7744 m/s^2 at 11 km altitude, which is less than gravity at sea level (g=9.8066 m/s^2). The gravitational force above the Earth's surface is proportional to1/R^2, where R is your distance from the center of the Earth, gravity decreases with an increase in height and it becomes zero at an infinite distance from earth.

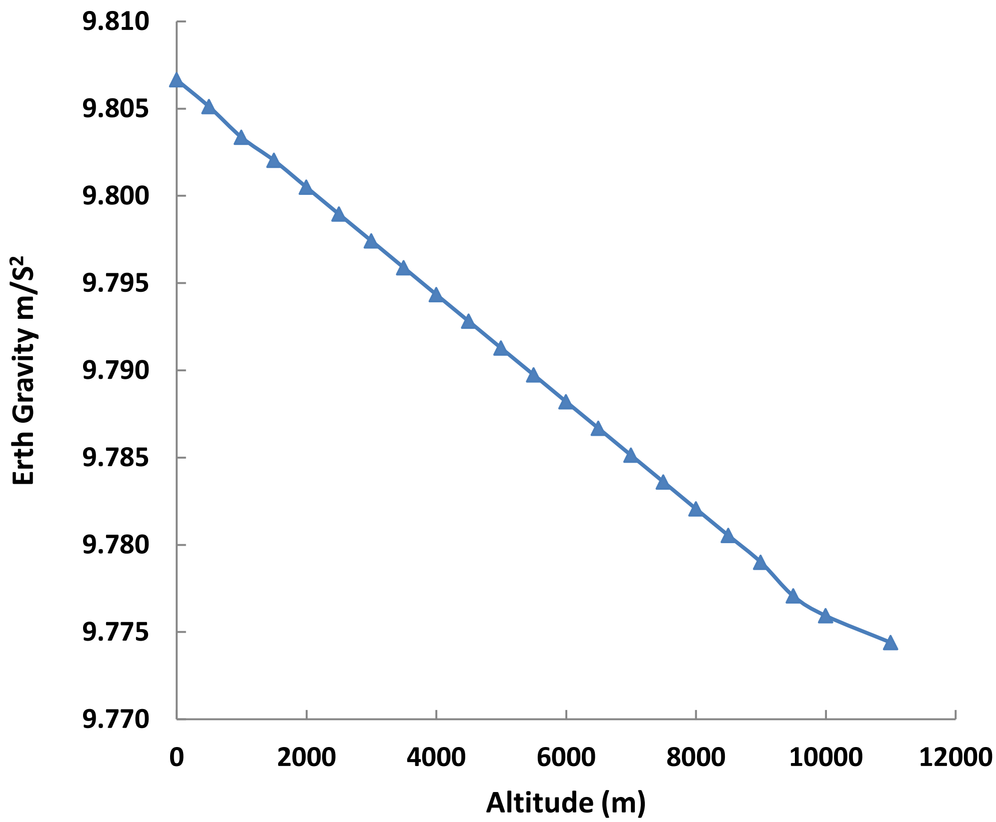


Figure 1: Variation of erth gravity against altitude.

Table 1: Variation of earth gravitation with altitude

Altitude (meter)	Earth Gravity m/s^2
0	9.80665
500	9.80511
1000	9.80335
1500	9.80203
2000	9.80049
2500	9.79895
3000	9.79742
3500	9.79588
4000	9.79434
4500	9.79281
5000	9.79127
5500	9.78974
6000	9.7882
6500	9.78667
7000	9.78513
7500	9.7836
8000	9.78206
8500	9.78053
9000	9.779
9500	9.77706
10000	9.77593
11000	9.7744

The starting point of our kinetic model has been a gas in thermal equilibrium in the sense that the averaged properties of the gas do not change with time and the energy equi-partition is built in. This generally means that the molecular mean free path is small compared to other characteristic distances. The fact that the total energy of a given molecule behaves in the same way as if no collisions are present is understandable because the molecules are regarded as perfectly elastic point like bodies.

It is noticed that the coefficient of gz in our formula [Eq. (14)] is $\gamma(\gamma-1)$ unlike (γ-1) appearing in Rayleigh's formula, In the case of a diatomic gas (γ = 7/5) our formula reduces to

$C(z)=[C^2(0) - 0.56gz]^{1/2}$.

The fact that the velocity of sound under gravity decreases with increasing altitude z is to be understood in terms of the variation of temperature from layer to layer as mentioned already in the Introduction

In figure 2 and 3 graphs represent the distribution of the true velocities of sounds in nitrogen and Chlorine diatomic gases respectively along the altitude of the troposphere. We see that as the altitude is rising the influence of the Earth's gravitational field on the speed of sound increases.

Table 2 and 3 represents the altitude distribution of the values of the adiabatic and true velocities of sounds in nitrogen (N_2) and chlorine (Cl_2) diatomic gases respectively in the stratosphere.

Relative errors between the values of the true and adiabatic speeds of sound and the corresponding least square fit are also presented in the table. As seen, the relative error in determining the speed of sound at an altitude of 11 km is greater than at an altitude of 1 km.

The Mach number plays a key role in determining the aerodynamic parameters for supersonic and hypersonic airplanes. As is known, the Mach number is equal to the ratio of speed of the aircraft relative to the medium, to the speed of sound in this medium. The importance of the Mach number is explained by the fact, that it determines whether the velocity of the flow of a gas medium (or the movement of a body in the gas) exceeds the speed of sound in it or not. Supersonic and subsonic flight regimes have fundamental differences in aviation. This difference is expressed in the fact that in supersonic regimes, narrow layers of rapid and significant changes in flow parameters (shock waves) emerge which lead to an increase in the resistance of bodies while in motion, the concentration of heat flows at their surface, and the possibility of burning through the hull of the bodies, and so on. Based on this, it is clear how important it is to correctly determine the speed of Sound at different altitudes of the Earth's atmosphere.

Table 4 represents the altitude distribution of the values of the speed of sounds in the stratosphere, Mach numbers are also presented in the table. As seen, the Mach number at an altitude of 11 km greater than at an altitude of 1 km. For example, for an airplane flying at the speed of 800 m/s, at an altitude of 11 km, the

Mach number calculated by formula (18), figure 4 show the Mach number as a function of altitude, the Mach number increase with increasing altitude.

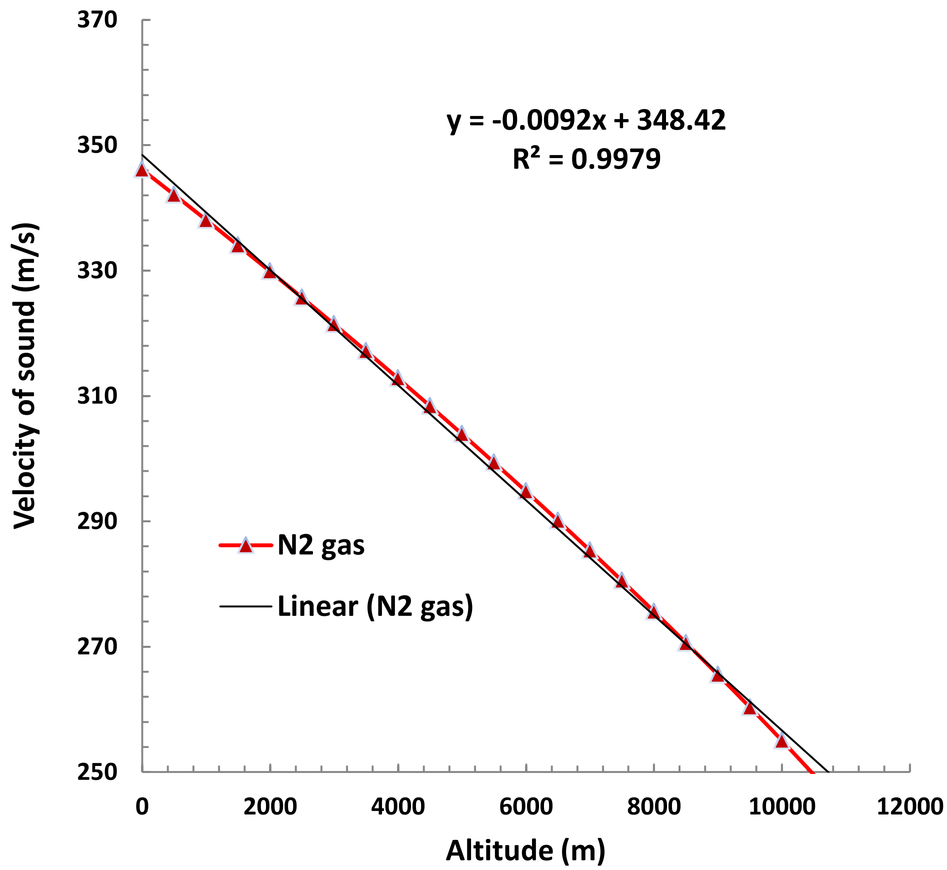


Figure 2. Velocity of sound against the altitude for N2 diatomic gas.

Table 2: Variation of the velocity of sound with altitude for Nitrogen diatomic gas

Altitude (m)	Earth gravitation m/s^2	Velocity of sound C(z) (m/s)	Velocity of sound (m/s) due to simulation fitting	Error%
0	9.807	346.09	348.42	0.67
500	9.805	342.10	343.82	0.50
1000	9.803	338.07	339.22	0.34
1500	9.802	333.98	334.62	0.19
2000	9.800	329.85	330.02	0.05
2500	9.799	325.67	325.42	0.08
3000	9.797	321.43	320.82	0.19
3500	9.796	317.14	316.22	0.29
4000	9.794	312.79	311.62	0.37
4500	9.793	308.38	307.02	0.44
5000	9.791	303.91	302.42	0.49
5500	9.790	299.38	297.82	0.52
6000	9.788	294.77	293.22	0.53
6500	9.787	290.09	288.62	0.51
7000	9.785	285.34	284.02	0.46
7500	9.784	280.51	279.42	0.39
8000	9.782	275.60	274.82	0.28
8500	9.781	270.60	270.22	0.14
9000	9.779	265.50	265.62	0.04
9500	9.777	260.32	261.02	0.27
10000	9.776	255.02	256.42	0.55
11000	9.774	244.07	247.22	1.29

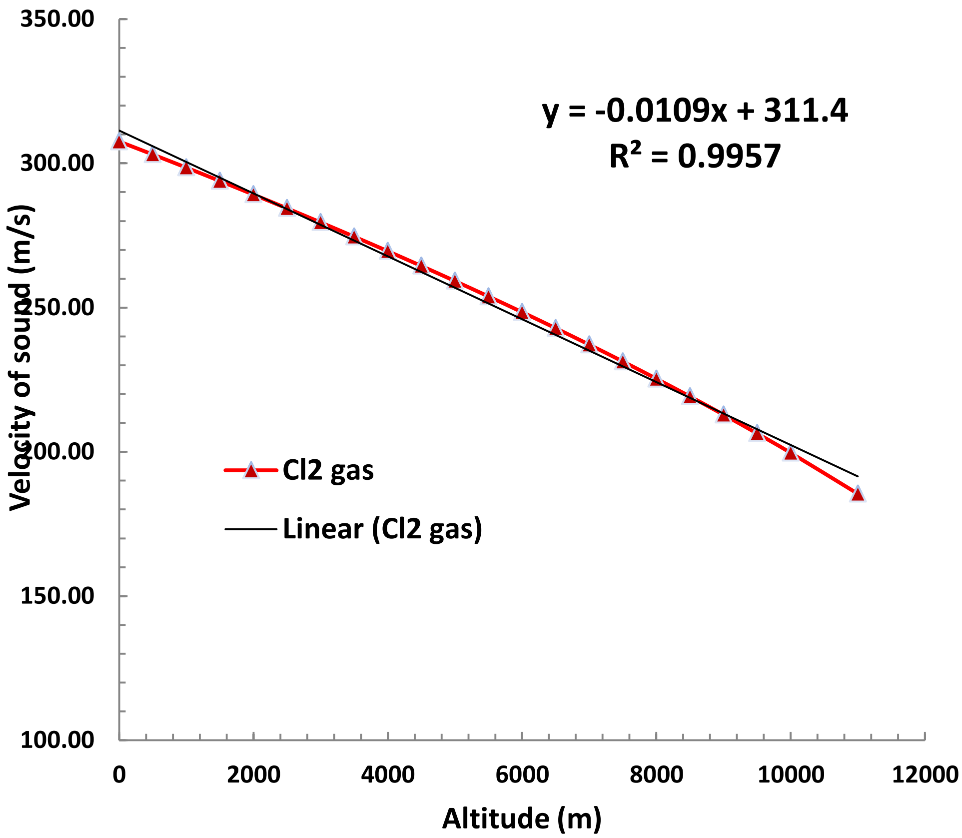


Figure 1. Velocity of sound against the altitude for Cl_2 diatomic gas.

Table 3: Variation of the velocity of sound with altitude for Cl_2 diatomic gas

Altitude (m)	Earth gravitation m/s^2	Velocity of sound C(z) (m/s)	Velocity of sound (m/s) due to simulation fitting	Error%
0	9.807	307.57	311.4	1.25
500	9.805	303.07	305.95	0.95
1000	9.803	298.51	300.5	0.67
1500	9.802	293.88	295.05	0.40
2000	9.800	289.18	289.6	0.15
2500	9.799	284.40	284.15	0.09
3000	9.797	279.53	278.7	0.30
3500	9.796	274.59	273.25	0.49
4000	9.794	269.56	267.8	0.65
4500	9.793	264.43	262.35	0.79
5000	9.791	259.20	256.9	0.89
5500	9.790	253.86	251.45	0.95
6000	9.788	248.42	246	0.97
6500	9.787	242.85	240.55	0.95
7000	9.785	237.15	235.1	0.87
7500	9.784	231.32	229.65	0.72
8000	9.782	225.33	224.2	0.50
8500	9.781	219.19	218.75	0.20
9000	9.779	212.87	213.3	0.20
9500	9.777	206.36	207.85	0.72
10000	9.776	199.63	202.4	1.39
11000	9.774	185.44	191.5	3.27

Table 4: Variation of Mach number with altitude in air.

Z	C(Z)	Mach No.
0	443	1.805869
500	439.8904	1.818635
1000	436.7598	1.83167
1500	433.6073	1.844987
2000	430.4329	1.858594
2500	427.2358	1.872502
3000	424.0157	1.886722
3500	420.772	1.901267
4000	417.5041	1.916149
4500	414.2114	1.931381
5000	410.8935	1.946977
5500	407.5495	1.962952
6000	404.179	1.979321
6500	400.7811	1.996102
7000	397.3554	2.013311
7500	393.9009	2.030968
8000	390.4169	2.049092
8500	386.9027	2.067703
9000	383.3573	2.086826
9500	379.7829	2.106467
10000	376.1699	2.126699
11000	368.8342	2.168996

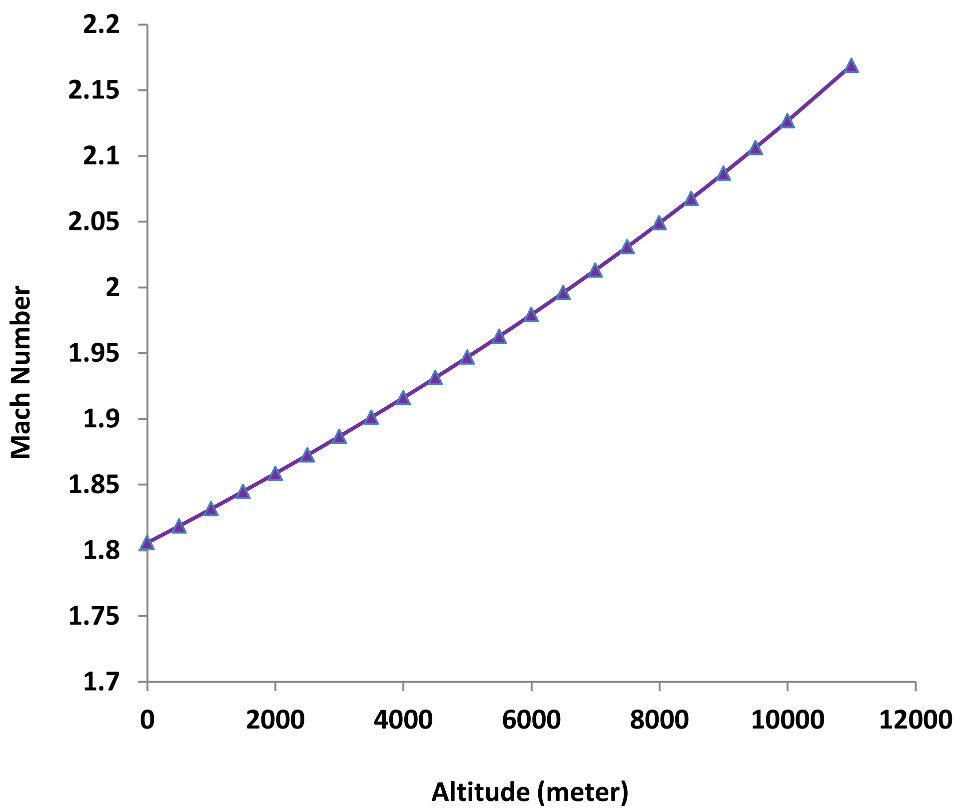


Figure 4. Variation of Mach number against altitude in air.

Chapter Four

4. Conclusion

We were able to successfully show that the speed of sound changes as a function of altitude in nitrogen and oxygen diatomic gases. Specifically, the earth gravitation and the speed of sound have general positive correlation. Our data was only taken to about $\leq$ 11Km. Also the Mach number depend on altitude which increase with increasing altitude, the Mach number at an altitude of 11 km greater than at an altitude of 1 km.

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