Chapter Two
Analysis and Design of Rectangular Beams

2-1 Introduction:

The analysis and design of a structural member may be regarded as the process of selecting the proper materials and determining the member dimensions such that the design strength is equal or greater than the required strength. The required strength is determined by multiplying the actual applied loads, the dead load, the assumed live load, and other loads, such as wind, seismic, earth pressure, fluid pressure, snow, and rain loads, by load factors. These loads develop external forces such as bending moments, shear, torsion, or axial forces, depending on how these loads are applied to the structure.

Structural members by strength design approach are based on the following **Assumptions:**

1. Strain in concrete is the same as in reinforcing bars at the same level, provided that the bond between the steel and concrete is adequate.
2. Strain in concrete is linearly proportional to the distance from the neutral axis, i.e., Plane cross-sections continue to be plane after bending.
3. The modulus of elasticity of all grades of steel is taken as Es = 200,000MPa
4. Tensile strength of concrete is neglected.
5. At high stresses, non-elastic behavior is assumed, which is in close agreement with the actual behavior of concrete and steel.
6. At failure, the maximum strain at the extreme compression fibers is assumed to equal to 0.003 by the ACI Code provision **or** steel bars reached to yield at.
7. For design strength, the shape of the compressive concrete stress distribution may be assumed rectangular.

2-2 Whitney Block`s:

Although the actual stress distribution given in Fig. 2-1 may seem to be necessary, in practice, any assumed shape (rectangular, parabolic, trapezoidal, etc.) can be used if the resulting equations compare favorably with test results.

Whitney replaced the curved stress block with an equivalent rectangular block of intensity $f`_c$ (where $\beta_1=2\beta$) and depth $a = \beta_1 c$, (where $c = a / \beta_1$) as shown in Fig 2-1. This rectangular block area should equal that of the curved stress block, and the centroids of the two blocks should coincide. Sufficient test results are available for concrete beams to provide the depths of the equivalent rectangular stress blocks.

For concretes with $f_c` > 28$MPa, β_1 can be determined with the following formula (ACI Table 22.2.2.4.3):

$\beta_1 = 0.85$ $\quad$ for $f_c` \leq 28MPa$

$\beta_1 = 0.85 - \frac{0.05(f`c-28)}{7}$ $\quad$ $28 \leq f_c` \leq 56$MPa

$\beta_1 = 0.65$ $\quad$ for $f_c` > 56MPa$

Based on these assumptions regarding the stress block, statics equations can easily be written for the sum of the horizontal forces and the resisting moment produced by the internal couple. These expressions can then be solved separately for *a* and the moment, M_n.

Where M_n is defined as the theoretical or nominal resisting moment of a section. The usable flexural strength of a member, $ØM_n$, must at least be equal to the calculated factored moment, M_u, caused by the factored loads

$ØM_n \geq M_u$

For writing the beam expressions, reference is made to Fig. 2-1. Equating the horizontal forces *C* and *T* and solving for *a*, we obtain :

$0.85f`_c\, ab = A_s f_y$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c}$$

Where $\rho = A_s / b\, d$

Because the reinforcing steel is limited to an amount such that it will yield well before the concrete reaches its ultimate strength, the value of the nominal moment, M_n, can be written as

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right)$$

and the usable flexural strength is

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

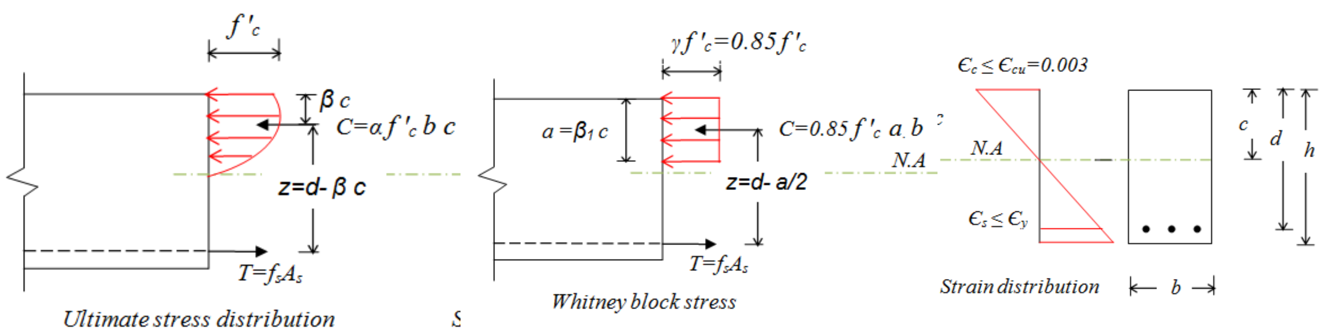


Fig.2-1 Stress-strain curve for concrete in bending

2-3 Structural Safety

The structural safety of the strength design in which uncertainty is considered. The working loads are multiplied by certain load factors that are larger than 1. The resulting factored loads are used for designing the structure. The values of the load factors vary depending on the type and combination of the loads.

To accurately estimate the ultimate strength of a structure, it is necessary to consider the uncertainties in material strengths, dimensions, and workmanship. This is done by multiplying the theoretical ultimate strength (called the *nominal strength* herein) of each member by the *strength reduction factor*, *Ø*, which is less than 1. These values generally vary from 0.90 for bending down to 0.65 for some columns.

a. Strength Reduction or Ø Factors

Strength reduction factors are used to take into account the uncertainties of material strengths, inaccuracies in the design equations, approximations in analysis, possible variations in dimension reduction factors for most situations. Among these values are the following:

0.90 for tension-controlled beams and slabs
0.75 for shear and torsion in beams
0.65 or 0.75 for columns
0.65 or 0.75 to 0.9 for columns supporting very small axial loads
0.65 for bearing on concrete

Table 21.2.1—Strength reduction factors ϕ

Action or structural element		ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

b. Ductility Adoption:

Engineers facing two main factors, first the safety of the building, second the economy. Safety can be achieved through the strength and ductility of the materials used in the skeleton of the building. The ductility gain by an ample warning for the user of the building when overloaded. The warning happened with deflection and cracks.

The member may fail in flexural, shear, torsion, axial, bond between steel and concrete or due to a combination of them. The final collapse done when overloaded must be in the beams.

The final failure of the reinforced concrete member depends on the nature of each material and the percent of steel content. Concrete strong and useful in compression, and steel benefits on the tensile side when concrete is cracked. As the steel content is large, the concrete failed firstly (before steel yielded) while, as the steel content is less, the steel failed firstly (before concrete crushed) so that the member failed gradually with advance warning. The final failure of the beam can be controlled by designing the beam to contain an amount of steel equal to or less than maximum steel area ($A_{s,max.}$)or maximum steel ratio.($\rho_{max.}$).

For ductile or tension-controlled beams and slabs where $\epsilon_t \geq (\epsilon_y + 0.003)$, the value of Ø for bending is 0.90. This situation is shown in Fig.3-2.
Members subject to axial loads equal to or less than $0.10 f_c` A_g$ may be used only when $\epsilon_t$ is no lower than ($\epsilon_y$ +0.003) (ACI Section 21.2). Should the members be subject to axial loads ≥ $0.10 f_c` A_g$, then ϵ_t is not limited. When ϵ_t values fall between ϵ_y and(ϵ_y +0.003), they are said to be in the transition range between tension-controlled and compression-controlled sections. In this range, Ø values will fall between 0.65 or 0.70 and 0.90, as shown in Fig.2-2. When $\epsilon_t \leq \epsilon_y$, the member is compression controlled, and the column *Ø* factors apply.

Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

Net tensile stain ε_t	Classification	ϕ — Type of transverse reinforcement — Spirals conforming to 25.7.3		ϕ — Type of transverse reinforcement — Other	
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$	Transition[1]	$0.75 + 0.15\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.003)}$	(c)	$0.65 + 0.25\frac{(\varepsilon_t - \varepsilon_{ty})}{(0.003)}$	(d)
$\varepsilon_t \geq \varepsilon_{ty} + 0.003$	Tension-controlled	0.90	(e)	0.90	(f)

[1]For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

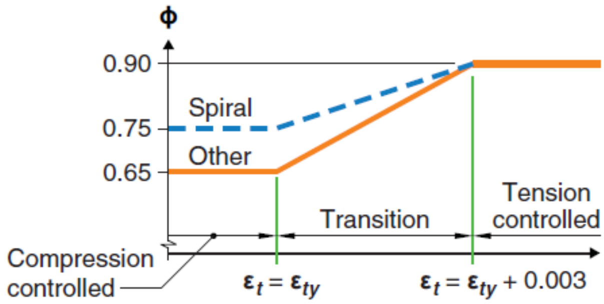


Fig. R21.2.2b—Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.

Fig 2-2. Variation of Ø with tensile strain ϵ_t

In this section, an expression is derived for $A_{s,max}$ *and* ρ_{max}, the percentage of steel to be considered ductile section. At ultimate load for such a beam, the concrete will theoretically fail (at a strain of 0.003), and the steel will be reached to $\epsilon_t = \epsilon_y + 0.003$ simultaneously (see Figure 2.3).
The neutral axis is located by the triangular strain relationships that follow:

$$\frac{\epsilon_{cu}}{c} = \frac{\epsilon_t}{d-c}$$

$$c_{max} = \frac{0.003}{\epsilon_y + 0.006}\, d$$

C = T

Example for **rectangular** section

$$0.85\, f_c`\, \beta_1\, c\, b = A_s\, f_y$$

$$A_{s,\max} = 0.85\, \beta_1 \frac{f`_c}{f_y} \frac{0.003}{(\epsilon_y + 0.006)}\, b\, d$$

Where $\rho = A_s / b\, d$ *then*

$$\rho_{\max} = 0.85\, \beta_1 \frac{f`_c}{f_y} \frac{0.003}{(\epsilon_y + 0.006)}$$

Maximum acceptable percentage of steel to be in tension control where steel stain at $\epsilon_t = \epsilon_y + 0.003$ and reduction factor Ø=0.9. The value of ρ_{max} can be easily determined for different $f`_c$ and f_y.

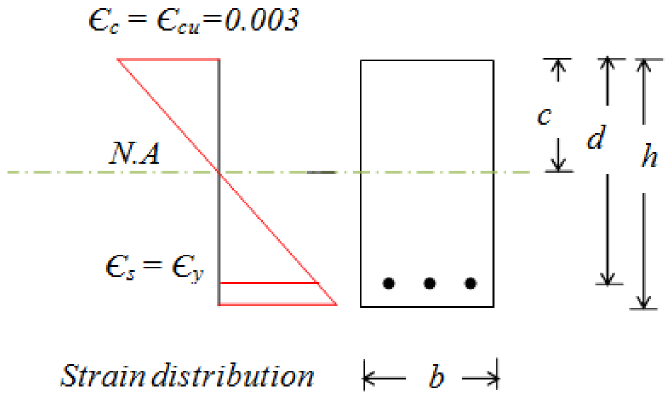


Fig 2-3. Strain distributions

2-4 Minimum Percentage of Steel

Sometimes, because of architectural or functional requirements, beam dimensions are selected that are much larger than are required for bending alone. Such members theoretically require very small amounts of reinforcing.

Actually, another mode of failure can occur in very lightly reinforced beams. If the ultimate resisting moment of the section is less than its cracking moment, the section will fail immediately when a crack occurs. This type of failure may occur without warning. To prevent such a possibility, the ACI (9.6.1.2) specifies a certain minimum amount of reinforcing that must be used at every section of flexural members where tensile reinforcing is required by analysis, whether for positive or negative moments. In the following equations, b_w represents the web width of beams.

$$A_{S,min.} = \frac{1.4}{f_y} b_w\, d \quad \geq \quad \frac{0.25\sqrt{f`_c}}{f_y}\, b_w\, d$$

The $(1.4\ b_w d)/ f_y$ value was obtained by calculating the cracking moment of a plain concrete section and equating it to the strength of a reinforced concrete section of the same size, applying a safety factor of 2.5, and solving for the steel required. It has been found, however, that when *it* is higher than about 35MPa, this value may not be sufficient. Thus, the $b_w d$ value is also required to be met, and it will actually control when f_c` is greater than 31MPa.

$$or\ \ \rho_{min.} = \frac{0.25\sqrt{f`_c}}{f_y} \quad \geq \quad \frac{1.4}{f_y}$$

2-5 Balanced Sections

A beam that has a *balanced steel ratio* is one for which the tensile steel will theoretically just reach its yield point at the same time the extreme compression concrete fibers attain a strain equal to 0.003. Should a flexural member be so designed that it has a balanced steel ratio or be a member whose compression side controls (i.e., if its compression strain reaches 0.003 before the steel yields), the member can suddenly fail without warning. As the load on such a member is increased, its deflections will usually not be particularly noticeable, even though the concrete is highly stressed in compression, and failure will probably occur without warning to users of the structure.

2-6 Beam proportions

Unless architectural or other requirements dictate the proportions of reinforced concrete beams, the most economical beam sections are usually obtained for shorter beams (up to 8m lengths), when the ratio of d to b is in the range of 1.5 to 2. For longer spans, a better economy is usually obtained if deep, narrow sections are used. The depths may be as large as 3 or 4 times the widths.

However, today's reinforced concrete designer is often confronted with the need to keep members rather shallow to reduce floor heights. As a result, wider and shallower beams are used more frequently than in the past. For simplicity, in constructing forms or for the rental of forms which are often selected in multiples of 50mm increments.

2-7 Deflections

The ACI Code in Table 9.3.1.1 provides minimum thicknesses of beams and one-way slabs for which such deflection calculations are not required. The purpose of such limitations is to prevent deflections of such magnitudes as would interfere with the use of or cause injury to the structure. If deflections are computed for members of lesser thicknesses than those listed in the table and are found to be satisfactory, it is not necessary to abide by the thickness rules.

The minimum thicknesses provided apply only to members that are not supporting or attached to partitions or other construction, likely to be damaged by large deflections.

2-8 Estimated beam weight

The weight of the beam to be selected must be included in the calculation of the bending moment to be resisted because the beam must support itself and the external loads. Because concrete weighs approximately 2300 to 2500kg/m^3 a quick-and-dirty calculation of self-weight is simply
$b \times h$ because the concrete weight approximately cancels the 24kN/m^3 conversion factor.

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

2-9 Selection of bars.

After the required reinforcing area is calculated. For the usual situations, bars of sizes Ø35mm and smaller are practical to be used. It is usually convenient to use bars of one size only in a beam. However, occasionally two sizes will be used. Bars for compression steel and stirrups are usually a different size, however. Otherwise, the workmen may become confused.

2-10 Cover

The reinforcing for concrete members must be protected from the surrounding environment; that is, fire and corrosion protection need to be provided. In addition, the cover improves the bond between the concrete and the steel. In Section 20.6.1.3 of the ACI Code, a specified cover is given for reinforcing bars under different conditions. Values are given for reinforced concrete beams, columns, and slabs; for cast-in-place members; for precast members; for prestressed members; for members exposed to earth and weather; for members not so exposed; and so on.

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, mm
Cast against and permanently in contact with ground	All	All	75
Exposed to weather or in contact with ground	All	No. 19 through No. 57 bars	50
		No. 16 bar, MW200 or MD200 wire, and smaller	40
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 43 and No. 57 bars	40
		No. 36 bar and smaller	20
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	40

2-11 Minimum spacing of bars

The code (25.2.1) states that the clear distance between parallel bars cannot be less than **25mm, d_b or 4/3 d_{agg}.** If the bars are placed in more than one layer, those in the upper layers are required to be placed **directly over** the ones in the lower layers, and the clear distance between the layers must be not less than 25mm.

For example: if b=350mm and h=550 mm, find d (assume all-steel reached yield) see Fig. 2-5:

Cover of 1st layer = 40+10+20/2 = 60 mm

Cover for 2nd layer = 60 + 20/2+25+20/2 =105mm

Center of the bars from bottom edge is yb=(5x314x60 + 2x314x 105)/(7x314)=73mm

Then d=550 –y_b = 477mm

A major purpose of these requirements is to enable the concrete to pass between the bars. The ACI Code further relates the spacing of the bars to the maximum aggregate sizes for the same purpose. In the code Section, maximum permissible aggregate sizes are limited to the smallest of:

(a) one-fifth of the narrowest distance between side forms,

(b) one-third of slab depths, and

(c) three-fourths of the minimum clear spacing between bars.

A reinforcing bar must extend an appreciable length in both directions from its point of highest stress in order to develop its stress by bonding to the concrete. The shortest length in which a bar's stress can be increased from 0 to f_y is called its *development length*.

The designer should always strive for simple spacing, for such dimensions will lead to a better economy.

Each time a beam is designed, it is necessary to select the spacing and arrangement of the bars. The minimum beamwidths required for different numbers of bars can be calculated. For example:

Minimum beam width = 40+10+5x20+4x25+10+40=300mm

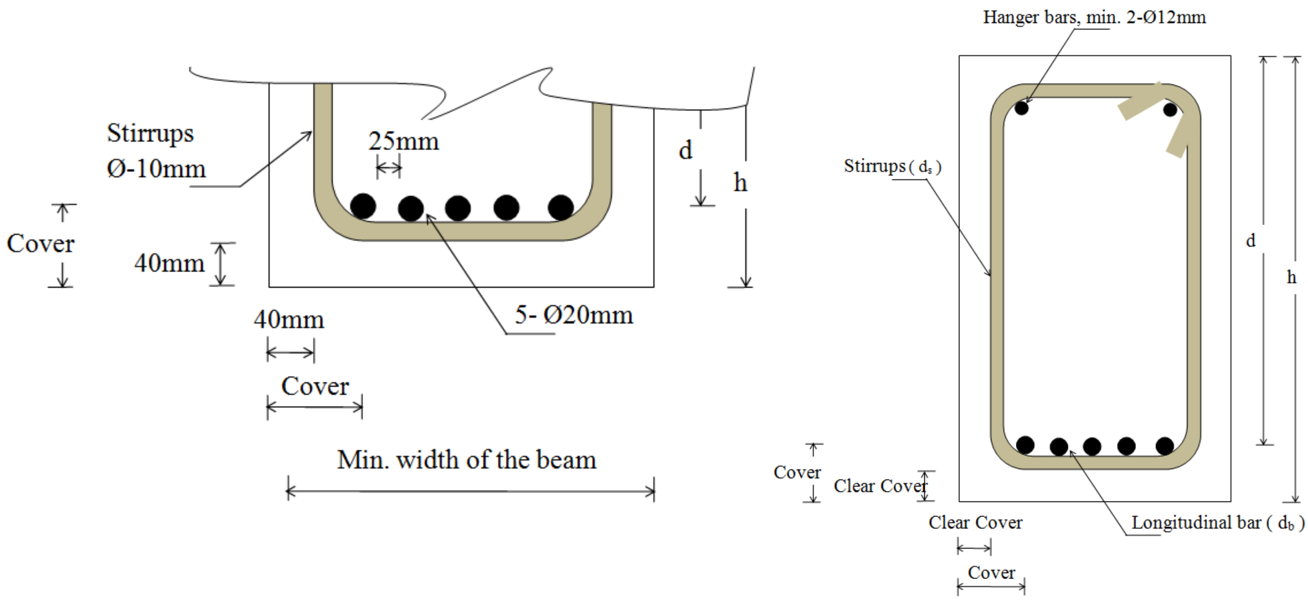


Fig 2-4. Cover of reinforcement.

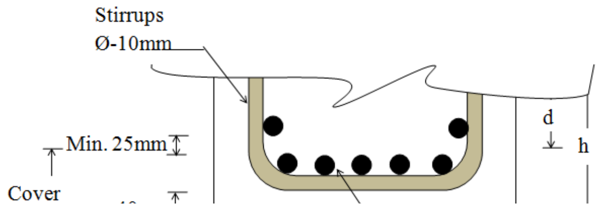
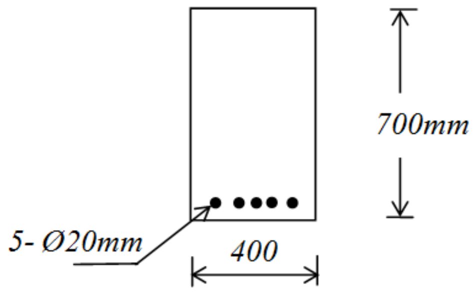


Fig 2-5. Spacing between the vertical longitudinal reinforcements.

Example 2-1

Determine the ACI moment capacity of the beam shown, and then check the beamwidth. *Use fc`=28MPa* *f_y=420MPa, maximum* aggregate size 20mm and Ø10mm for stirrup.

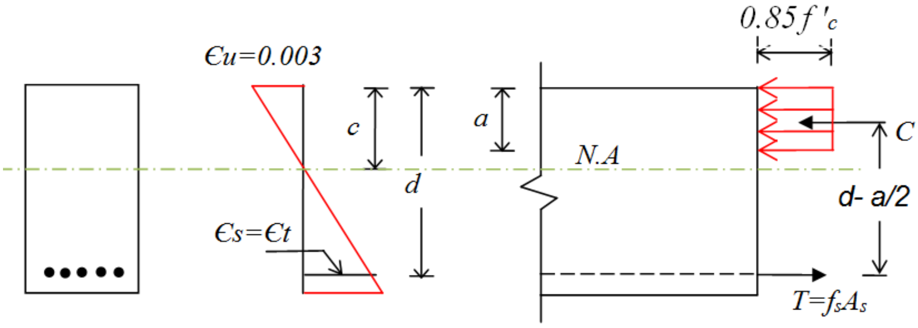


Sol.:

$$\epsilon_y = \frac{f_y}{E_s} = \frac{420}{200000} = 0.0021$$

d= 700 – 40 – 10 – 20/2

= 640mm



A_b= 3.14/4 x 20^2=314mm^2

As= 5 x 314 =1570mm^2

$$\rho_{actual} = \frac{As}{b\ d} = \frac{1570}{400\ x\ 640} = 0.00613 = 0.613\%$$

$$\rho_{min} = \frac{1.4}{f_y} \geq \frac{0.25\sqrt{f_{c`}}}{f_y}$$

$$\rho_{min} = \frac{1.4}{420} = 0.00333$$

$$\rho_{actual} \geq \rho_{min} \quad ok$$

$$a = \frac{A_s f_y}{0.85\, f_{c`} b_w} \qquad a = \frac{1570\ x\ 420}{0.85\ x\ 28\ x400} = 69.3mm$$

$$c = \frac{a}{\beta} = \frac{69.3}{0.85} = 81.5 = 82mm$$

Since: $\epsilon_t = \frac{d-c}{c} x\ 0.003 = \frac{640-82}{82} x 0.003 = 0.0203 > \left(\epsilon_y + 0.003\right) = (0.0021 + 0.003) = 0.0051$

The section is ductile (tension control failure), Therefore: Ø=0.9

$$M_u = \emptyset\ M_n = \emptyset\ A_s f_y (d - \frac{a}{2})$$

$$M_u = 0.9x\ 1570\ x\ 420\left(640 - \frac{69.3}{2}\right) = 0.9x\ 35925100N.mm = 0.9x\ 359kN.m = 323.3\text{kN.m}$$

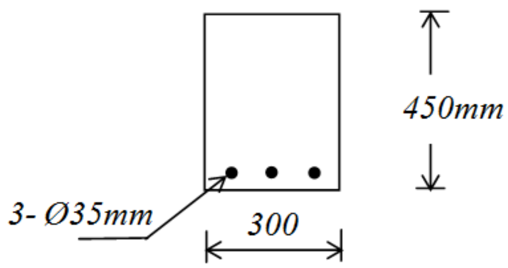
Check width:

(400-40 x 2- 10x 2 -5 x 20)/4 spaces =50mm > { d_b =20mm or 25 mm or 4/3$d_{agg.}$= 4/3 x 20 =26.7~ 27mm }

Ans.

Example 2-2:

Determine the ultimate moment capacity of the beam shown, if *fc`=28MPa fy=420MPa.*



Sol.:

$$\epsilon_y = \frac{f_y}{E_s} = \frac{420}{200000} = 0.0021$$

d= 450 – 40 – 10 – 35/2 = 382mm

A_b= 3.14/4 x 35^2=960mm^2

As= 3 x 960 =2880mm^2

$$\rho_{actual} = \frac{As}{b\ d} = \frac{2880}{300\ x\ 382} = 0.0251$$

$$\rho_{min} = \frac{1.4}{f_y} \geq \frac{0.25\sqrt{f_{c`}}}{f_y}$$

$$\rho_{min} = \frac{1.4}{420} = 0.00333$$

$$\rho_{actual} \geq \rho_{min}\ \ ok$$

$$a = \frac{A_s f_y}{0.85\ f_{c`} b_w} \quad a = \frac{2880\ x\ 420}{0.85\ x\ 28\ x300} = 169mm$$

$$c = \frac{a}{\beta} = \frac{169}{0.85} = 199mm$$

Since: $\in_t = \frac{d-c}{c} x0.003 = \frac{382-199}{199} x0.003 = 0.00276 < (\in_y + 0.003) = (0.0021 + 0.003) = 0.0051$

The section is considered brittle; therefore, the section is rejected.

Either increase the depth if allowed or design the beam as a double reinforced beam. **Ans.**

Example 2-3 Beam Design Example

Design a rectangular beam for 7m simply span if the dead load of 15 kN/m (not include self-weight) and the live load of 30kN/m are to be supported. Use *f_c`=28MPa and f_y=420MPa.* maximum aggregate size 20mm and Ø10mm for stirrup.

Sol.:

Estimate beam dimensions and weight

Assume h= 0.1 span of beam = 0.1 x 7 =0.7m =700 mm

Use Ø25mm and stirrups diameters is 10mm

Therefore d= 700 – 40 – 10 – 25/2 =637.5 ~637mm

Assume width of beam b=0.5 h =0.5 x 700 =350mm

Beam weight = w_t= 0.35 x 0.7 x 24 =5.88kN/m

Applied load:

w_u= *1.2D+1.6L* = 1.2 x (5.88 +15) +1.6 x 30=73.1 kN/m

Since the beam is simply supported moment equal to $wL^2/8$;

Applied moment is:

$$M_u = \frac{w_u L^2}{8} = \frac{73.1\ x\ 7^2}{8} = 447.7kN.m$$

Assume Ø=25mm

Compute A_s from moment equation:

$$M_n = \emptyset A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c^` b}\right)$$

Solve the above equation for A_s:

$$\frac{f_y^2}{1.7\ f_c^`\ b} A_s^2 - f_y\ d\ A_s + \frac{M_u}{\emptyset} = 0$$

$$\frac{420^2}{1.7\ x\ 28\ x\ 350} A_s^2 - 420\ x\ 637\ x A_s + \frac{447.7}{0.9} = 0$$

Solve for A_s:

A_s=2017mm^2

Required 4.1 bars Ø25mm

Use 5- Ø25mm (A_s provided is 5 x 490 =2450mm^2)

Check width of the beam:

(350-40 x 2 – 10 x 2 – 5 x 25) / 4 spacing = 31.3 mm > 25 mm , d_b=25mm and 4/3 x 20 =27mm, therefore min. spacing must be 27mm

Provided width 31.3mm > required Width is 27mm

Width is ok

$$\rho_{max} = \frac{A_s}{b_w d} = \frac{2450}{350\ x\ 637} = 0.011$$

C=T

0.85 x 28 x a x 350 =2450 x 420

a=123.5 =124mm

c= a / β1 = 123.5 / 0.85 =145.3 ~145mm

$$\epsilon_t = \frac{d-c}{c}\ 0.003 = \frac{637-145}{145}\ x\ 0.003 = 0.0102 > \ \epsilon_y + 0.003 = 0.0051$$

Therefore, tension control

resisted moment is:

$$M_u = \emptyset A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c^{`} b}\right) = 0.9\ x\ 2450\ x\ 420\ \left(637 - \frac{2450\ x\ 420}{1.7\ x\ 28\ x350}\right) =$$

$535.7 kN.m$

Applied moment is Mu=447.7

Since resisting moment > applied moment, the beam is safe and ok.

Ans.

Example 2-4 Beam Design Example

A beam is to be selected with half of the maximum steel ratio, applied moment to the beam is 810kN.m. Use f_c`=28MPa and f_y=420MPa.

Sol.:

$$\rho_{max} = 0.3148\ \beta_1 \frac{f_c^{`}}{f_y} = 0.3148\ x\ 0.85\ x \frac{28}{420} = 0.0178$$

Take $\rho = \frac{1}{2}\rho_{max} = \frac{0.0178}{2} = 0.00892$

Assume Ø=0.9

Substitute $A_s = \rho\ b\ d$ get:

$$M_u = \emptyset A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c^{`} b}\right) == \emptyset \rho\ b\ d\ \ f_y \left(d - \frac{\rho\ b\ d\ \ f_y}{1.7 f_c^{`}\ b}\right) = \phi \rho\ b\ d^2 f_y \left(1 - \frac{\rho\ f_y}{1.7 f_c^{`}}\right)$$

$$810\ x 10^2 = 0.9\ x\ 0.00892\ b\ d^2\ x\ 420\ (1 - \frac{0.00892\ x\ 420}{1.7\ x\ 28})$$

bd^2 = 257328209

to solve the above equation, use trail and errors methods

b (mm)	d (mm)
250	1015
300	926
350	857
400	802

The reasonable section is 400 x 900 mm

Min. h =802 + 40 + 10 +30 /2 = 867 ~ 900mm

Required As = ρ b d = 0.00892 x 400 x 802 =2903mm^2

4.1 bars –Ø30mm

Use 5–Ø30mm

Check width of the beam:
(400-40 x 2 – 10 x 2 – 5 x 30) / 4 spacing = 37.5 mm > 25 mm, d_b=30mm and 4/3 x 20 =27mm, therefore min. spacing must be 30mm
Provided width 37.5mm > required Width is 30mm
Width is ok
C=T
0.85 x 28 x a x 400 =5 x 706 x 420
a=155.7 =156mm
c= a / β1 = 155.7 / 0.85 =183.2 ~183mm

$$\epsilon_t = \frac{d-c}{c} 0.003 = \frac{835-183}{183} x\ 0.003 = 0.0107 > \epsilon_y + 0.003 = 0.0051$$

Therefore, tension control
Resisted moment is:

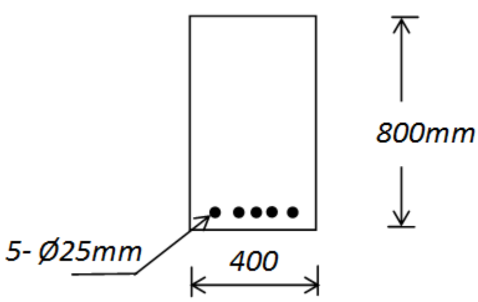
$$M_u = \emptyset A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c^` b}\right) = 0.9\ x\ 3530x\ 420\ \left(835 - \frac{156}{2}\right) = 1010.1kN.m$$

Applied moment is Mu=810
Since resisting moment > applied moment, the beam is safe and ok.

Ans.

Problem 2-1

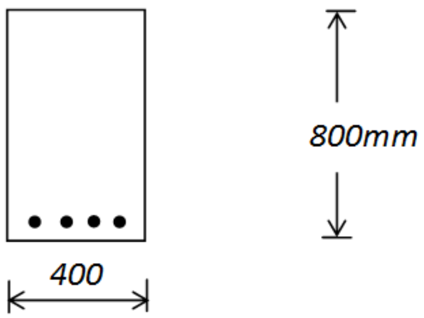
Determine the ACI moment capacity of beam shown, and then check the beamwidth. *Use f_c`=28MPa f_y=420MPa, Ø10mm for stirrup.*
(Ans. Mu=630 kN.m, the width is ok)



Problem 2-2

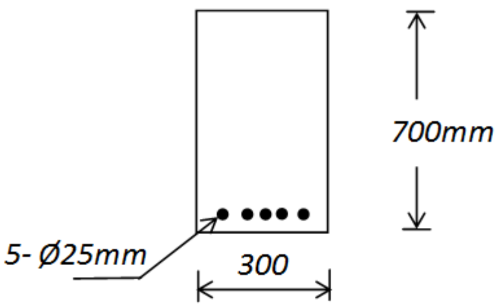
P3-1. Design the beam shown for a moment Mu= 800 kN.m. *Use Ø25mm* , f_c`=28MPa f_y=420MPa , E_s=200GPa, *Ø10mm for stirrup.*

(Ans. A3180mm2 – Use 7- Ø25mm)



Problem 2-3

Determine the ACI moment capacity of the beam shown for different concrete strength, f_c`=28MPa,$f_c$`=100MPa, then compare between the results, use f_y=420MPa, and *Ø10mm for stirrup.*



(Ans. for fc`=28 Mu=523kN.m and for $fc$`=100 MPA Mu=571.2kN.m)

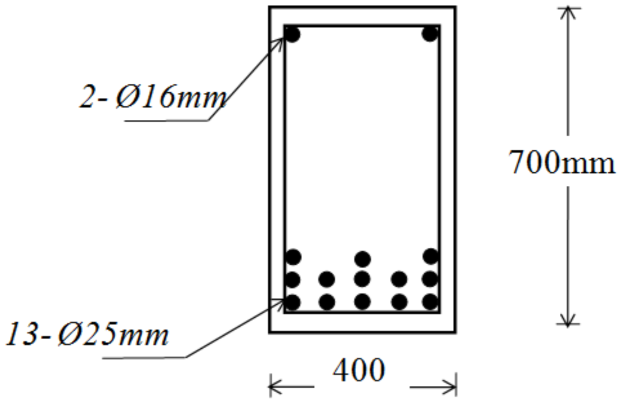
Problem 2-4

Determine the ACI moment capacity of the beam shown, f_y=350MPa and f_y=690MPa, then compare between the results, use f_c`=28MPa, and *Ø10mm for stirrup.*

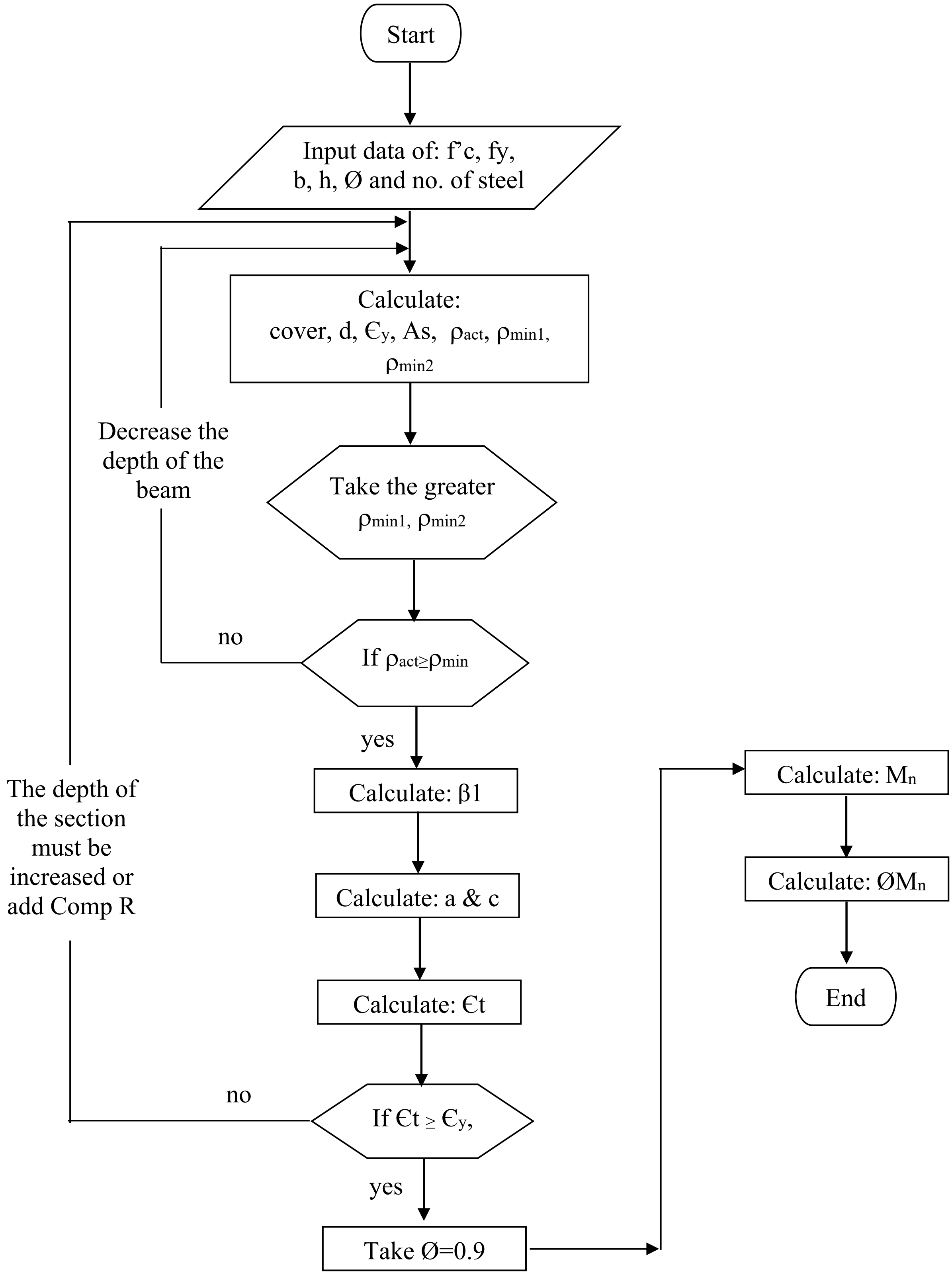
(Ans. for f_y = 350 Mu=445kN.m and for f_y = 690 MPA M_u= kN.m)

Problem 2-5

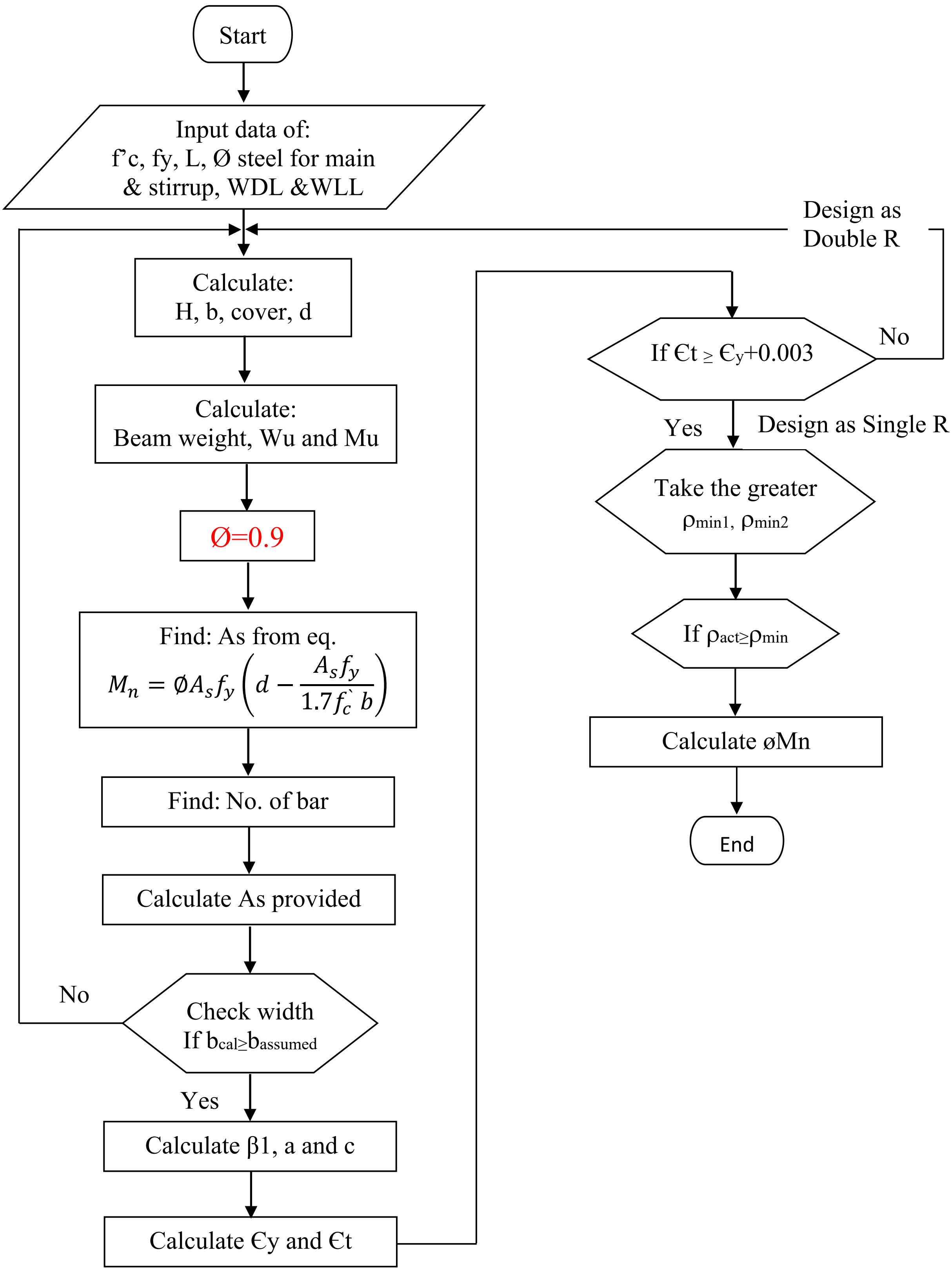
Determine d and d` for the beam shown, then check the width of the beam. Take stirrups *Ø10mm and* $d_{agg.}$=20mm.



2-12 Flow Chart for Analysis of Rectangular Beam



2-13 Flow Chart for Design of Rectangular Beam

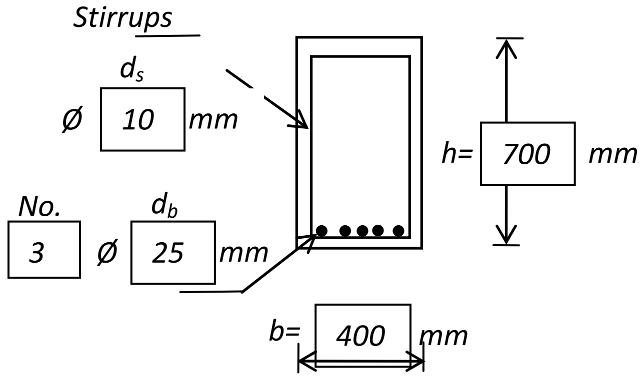


Excel Sheet:

Design and Analysis of Rectangular beams:

f_c\`= 28 MPa

f_y= 420 MPa



Es = 200 000MPa

* *Check* f_c\` $\geq$ 17.5Ma

=*if* (f_c\` >=17.5; " " ; " the value of $f_c$\` *must be more than 17.5MPa")*

* *Check* f_y $\leq$ 690Ma

= *if* (f_y <= 690; " " ; " the value of f_y *must be less than 690MPa")*

$d= h - 40 - d_s - d_b /2 =$

$\epsilon y= f_y / Es =$

$A_s= No. \times \pi \times d_b^2 / 4$

$\rho_{act.} = A_s / (b\ d)$

$\rho_{min1}= 1.4 / f_y$

$\rho_{min2} = 0.35 \sqrt{f_c`} / f_y$

$\rho_{min} = if (\rho_{min1} >= \rho_{min2} ; \rho_{min1} ; \rho_{min2})$

* Check steel content:

=If (ρ act. >= ρ min ; " The section is ok" ; " Decrease the depth of the beam ")

$\beta_1 = if\ (f_c` <= 28; 0.85 ; if\ (f_c` <= 56 ; 0.85 - 0.05(f_c` - 28) / 7 ; 0.65))$

$a= As\ fy / (0.85 f_c`\ b)$

$c = a / \beta_1$

$\epsilon_t = (d-c) \times 0.003 / c$

* Check the ductility of the section

=if(ϵ_t < = ϵ_t +0.003;*" The beam is ductile " ; " The beam is brittle, Kindly increase the depth of the beam")*

$M_n= A_s f_y (d- a /2) /1000000$

$M_u = Ø M_n$