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Chapter 1: Laser Radiation and its properties

- Laser technology is one of the most rapidly developing areas in modern technology.
- When the laser was invented, in 1960, it was classified as a ***solution in search of a problem***, and today laser technology is applied in many different areas such as: medicine, communication, daily use, military, and industry.
- To explain how the laser can be applied in such diverse areas, we need to understand the **basic physical principles of the operation of a laser**.
- In principle, the laser is a device which transforms energy from other forms into **electromagnetic radiation**. This is a very general definition, but it helps to understand the basic physics of the laser.

The energy put into the laser can be in any form such as:

➤ electromagnetic radiation, electrical energy, chemical energy, etc.
Energy is always emitted from the laser as electromagnetic radiation
(which includes light beams).

➤ From this light output, the laser got part of its name:
LASER = Light Amplification by Stimulated Emission of Radiation.

The terms used in this definition will be explained later in this course.

➤ We will try to obtain a **qualitative picture of the quantum nature of the laser**, based on some basic principles which came from the advanced mathematical tools.



Things you need to know:

Before studying about lasers, you must be familiar with **basic terms** used to describe electromagnetic waves:

- Wavelength
- Frequency (ν)
- Period (T)
- **Velocity of light (c)**
- **Index of refraction (n)**

We will **briefly review** these terms (in chapters 1&2), but it is much better if the reader will be familiar with:

- Some terms from **geometric optics** such as: refraction, reflection, thin lenses etc.
- Some terms from "**Modern Physics**" such as photons, Models of atoms, etc.

1.1 Electromagnetic Radiation

The electromagnetic radiation out of the laser can be in any part of the spectrum, including the [visible spectrum](#), the [Ultra-Violet \(UV\) spectrum](#), the [Infra-Red \(IR\) spectrum](#), and beyond In the following pages the **properties of Electromagnetic radiation** is briefly described:

1.1.1 [Electromagnetic Radiation in vacuum](#).

1.1.2 [Electromagnetic Radiation in Matter](#).

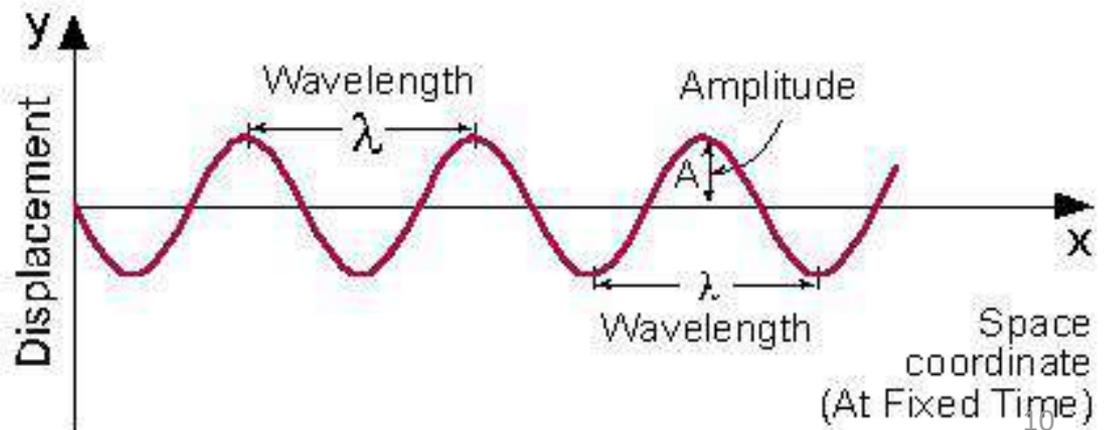
1.1.1 Electromagnetic Radiation in vacuum

- **Electromagnetic Radiation** is a **transverse wave**, advancing in vacuum at a constant speed which is called: **velocity of light**.

All electromagnetic waves have the same velocity in vacuum, and its value is approximately:

- **$C = 3 \cdot 10^8$ [m/sec]**

One of the most important parameters of a wave is its **wavelength**.



- **Wavelength (λ)** is the distance between two adjacent points on the wave, which have the same **phase**. As an example (see figure 1.1) the distance between two adjacent peaks of the wave.
- In a parallel way it is possible to define a wave by its **frequency**.
- **Frequency (ν)** is defined by the **number of times that the wave oscillates per second** (The number of periods of oscillations per second).
- Between these two parameters the relation is:

$$C = \lambda * \nu$$

From the physics point of view, **all electromagnetic waves are equal (have the same properties) except for their wavelength (or frequency)**.

As an example: the speed of light is the same for visible light, radio waves, or x-rays.

Wave Description

A **wave** can be described in two standard forms:

1. Displacement as a function of space when time is held constant.
2. Displacement as a function of time at a specific place in space.

1- Displacement as a function of space, when time is "frozen" (held constant), as described in figure 1.1. In this description, the minimum distance between two adjacent points with the same phase is wavelength (λ). Note that the horizontal (x) axis is space coordinate !

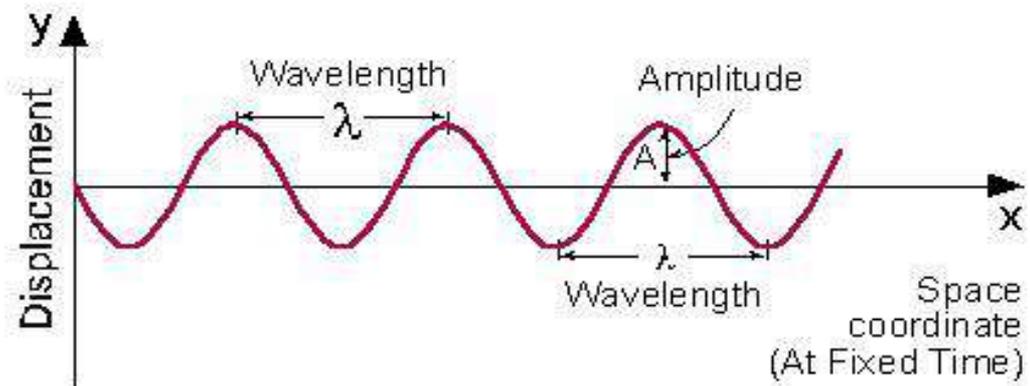


Fig 1.1: Displacement as a function of space coordinate (at fixed time)

A = Amplitude = Maximum displacement from equilibrium.

2-Displacement as a function of time

in a specific place in space, as described in figure 1.2. In this description, **the minimum distance between two adjacent points with the same phase is period (T)**. Note that the horizontal (x) axis is time coordinate !

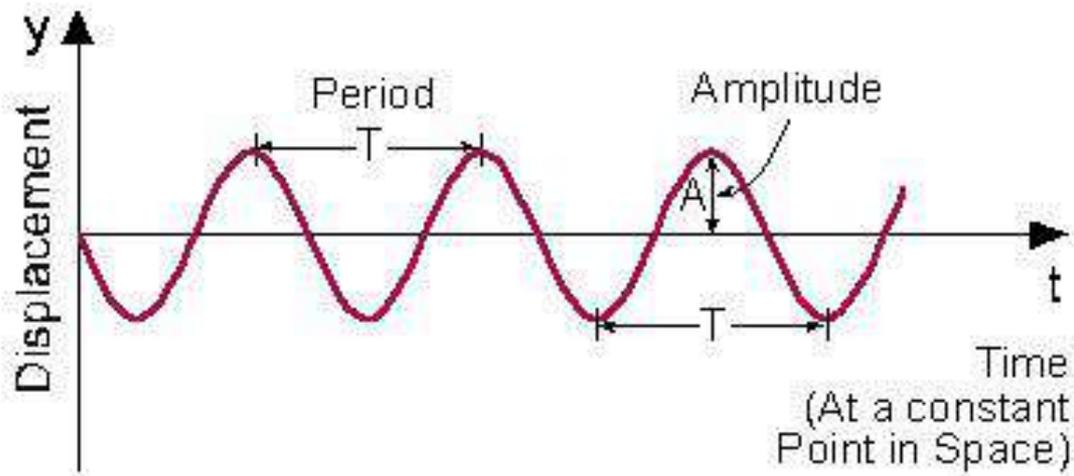
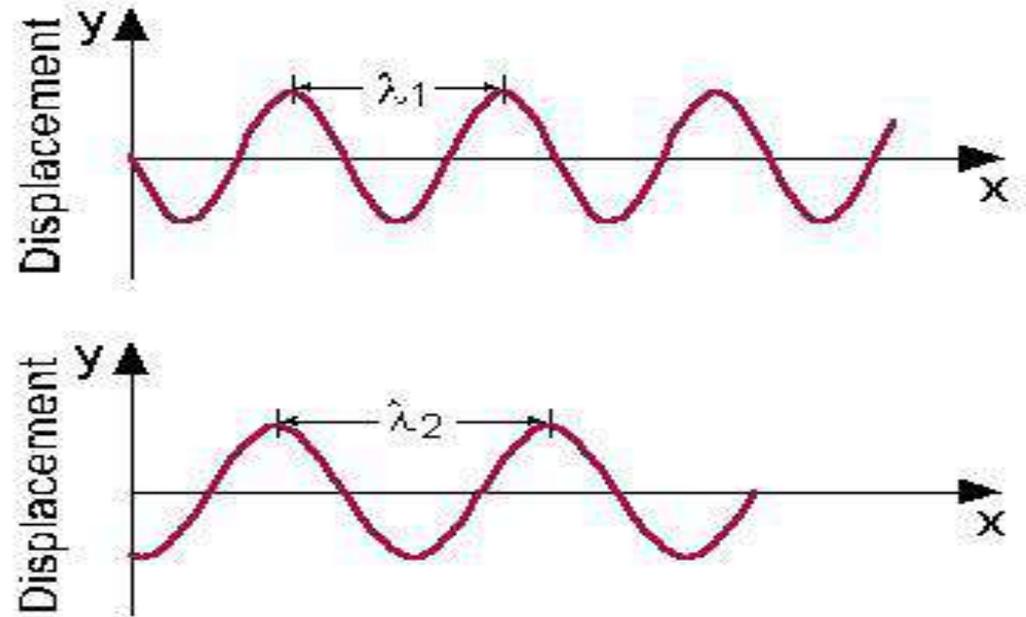


Figure 1.2: Displacement as a function of time (at a fixed point in space)

Wavelengths Comparison

Figure 1.3 describes how two different waves (with different wavelengths) look at a specific moment in time. Each of these waves can be uniquely described by its wavelength.



For electromagnetic waves, this wavelength is related to the type of radiation of the wave.

Figure 1.3: Short wavelength (λ_1) compared to longer wavelength (λ_2)

➤ The electromagnetic spectrum

Figure 1.4 describes the **electromagnetic spectrum**.

Each part of the spectrum has a **common name**, and its range of wavelengths, frequencies and energies. The borders between the ranges are not sharp and clear, but are defined according to the applications of radiation in that portion of the spectrum.

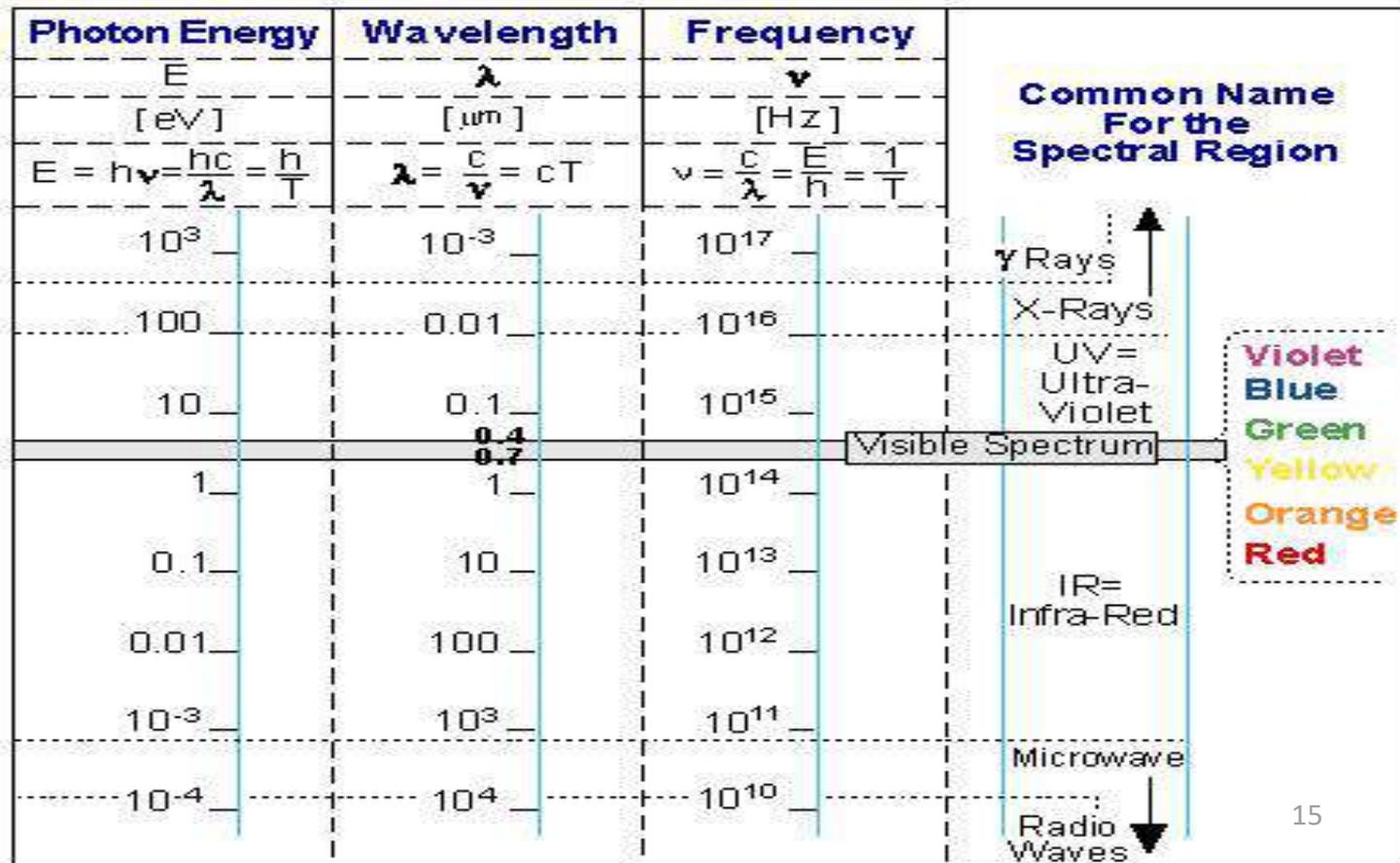


Figure 1.4: The Electromagnetic Spectrum

The most important ideas summarized in figure 1.4 are:

1. Electromagnetic waves span over many orders of magnitude in wavelength (or frequency).
2. **The frequency of the electromagnetic radiation is inversely proportional to the wavelength.**
3. The visible spectrum is a very small part of the electromagnetic spectrum.
4. Photon energy increases as the wavelength decreases. The shorter the wavelength, the more energetic are its photons.

Examples for electromagnetic waves are:

1. **Radio-waves** which have wavelength of the order of meters, so they need big antennas (The dimensions of an antenna are of the same order of magnitude as the wave).
2. **Microwaves** which have wavelength of the order of centimetres. As an example: in a **microwave oven**, these wavelengths can not be transmitted through the protecting metal grid in the door, while the **visible spectrum** which have much shorter wavelength allow us to see what is cooking inside the microwave oven through the protecting grid.
3. **x-Rays** which are used in medicine for taking pictures of the bone structure inside the body.
4. **Gamma Rays** which are so energetic, that they cause ionization, and are classified as ionizing radiation.

The **discrete aspects of electromagnetic radiation** is the result of Einstein's work at the beginning of the 20th century.

1.1.2 Electromagnetic Radiation in Matter

- (We shall use the words "electromagnetic radiation" and "light" as synonyms)
- **Light Velocity in Matter**
When electromagnetic radiation passes through matter with **index of refraction n** , its **velocity (v)** is less than the **velocity of light in vacuum (c)**, and given by the equation:

$$v = c / n$$

This equation is used as a **definition of the index of refraction (n)**:

- **$n = (\text{speed of light in vacuum}) / (\text{speed of light in matter})$**
 $= c / v$

Gases, including air, are usually considered as having index of refraction equal to vacuum **$n_0=1$** .

The values of the index of refraction of most materials transparent in the visible spectrum is between 1.4-1.8, while those of materials transparent in the [Infra-Red \(IR\) spectrum](#) are higher, and are 2- 4 .

➤ Wavelength in Matter:

We saw that the velocity of light in matter is slower than in vacuum. This slower velocity is associated with reduced wavelength: $\lambda = \lambda_0/n$, while the **frequency remains the same** (see figure 1.5).

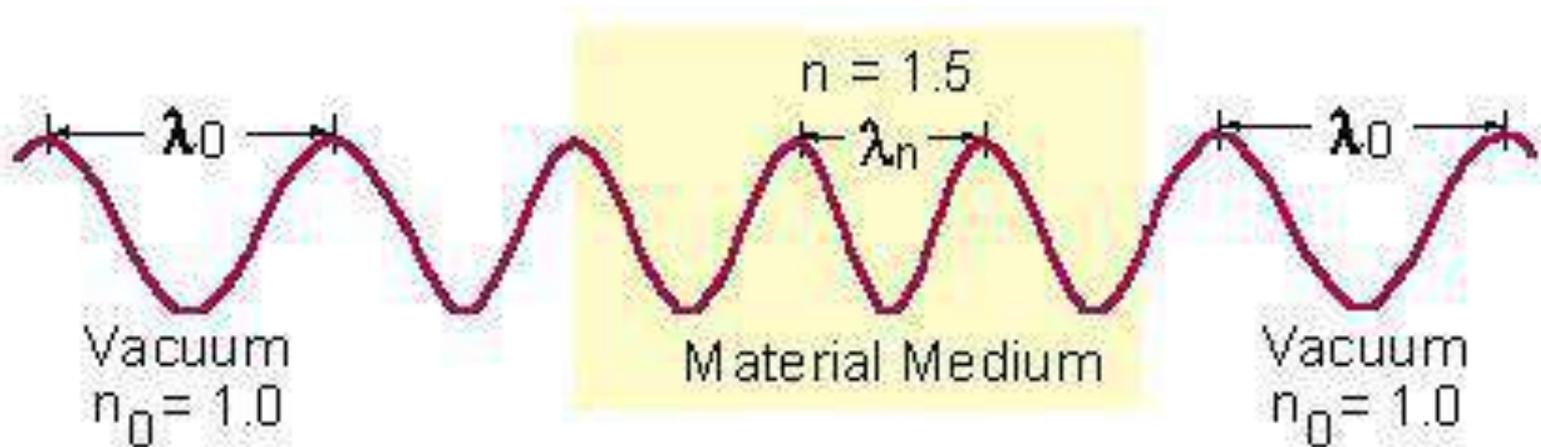


Figure 1.5: Change of wavelength in matter

➤ Refraction of Light Beam - Snell Law:

- Reducing the velocity of light in matter, and reducing its wavelength, causes **refraction of the beam of light**.

While crossing the border between two different materials, the light changes its direction of propagation according to the **Snell Equation**:

$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2)$$

- **Example 1.1:** The velocity of Red light ($\lambda_0 = 0.6\text{mm}$) in a certain medium is $1.5 \cdot 10^8$ [m/s].
What is the wavelength of this light in this material?

Solution to example 1.1:

First find the index of refraction:

$$n = \frac{c}{v} = \frac{3 \cdot 10^8 \cdot \frac{\text{m}}{\text{s}}}{1.5 \cdot 10^8 \cdot \frac{\text{m}}{\text{s}}} = 2.0$$

Using **n**, calculate the wavelength in the material:

$$\lambda_n = \frac{\lambda_0}{n} = \frac{0.6 \cdot \mu\text{m}}{2.0} = 0.3 \cdot \mu\text{m}$$

Conclusion: The wavelength of Red light in a material with an index of refraction of 2.0, is 0.3 [mm]

1.2 Properties of Laser Radiation

- **"Ordinary light"** (from the sun or lamps) is composed of many different wavelengths, radiating in all directions, and there is no phase relation between the different waves out of the source.
- **Laser radiation** is characterized by certain properties which are not present in other electromagnetic radiation:

1.2.1 Monochromaticity.

1.2.2 Directionality.

1.2.3 Coherence.

1.2.1 Monochromaticity

- **Monochromaticity means "One colour".**

To understand this term, examine "**white light**" which is the colour interpreted in the mind when we see all colours together.

When "white light" is transmitted through a prism, it is divided into the different colours which are in it, as seen in figure 1.6.

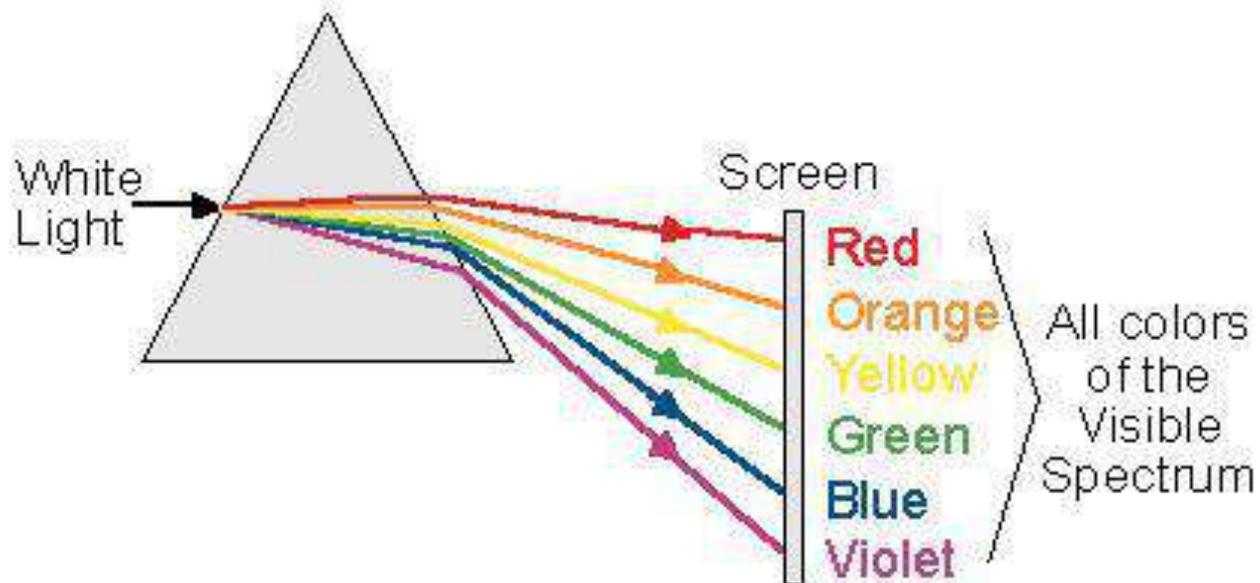


Figure 1.6: White light passing through a prism

– The Meaning of "One Colour"

In the theoretical sense "**One Colour**", which is called "**spectral line**", means **one wavelength** (λ_0).

A graph of light intensity versus wavelength for ideal "one colour" is shown on the right side of figure 1.7. The right side of figure 1.7 shows an artistic description of realistic "one colour". It has a peak of its value of "the colour", but include a spread around the central peak. In reality, every spectral line has a **finite spectral width** ($\Delta\lambda$) around its **central wavelength** (λ_0), a can be seen in the left side of figure 1.7.

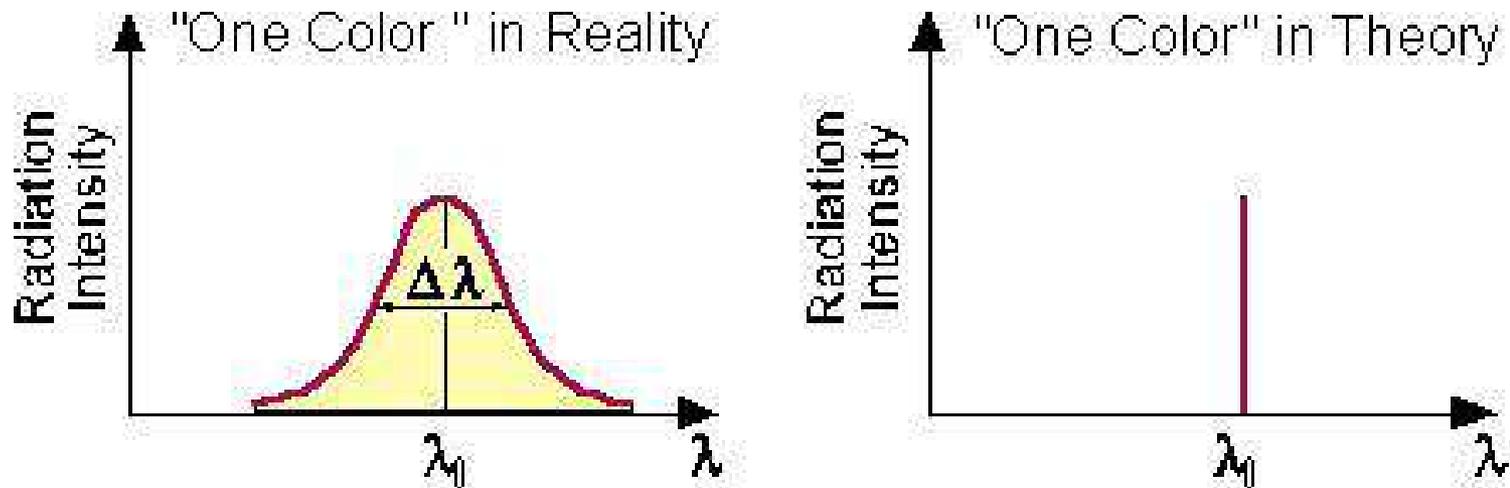


Figure 1.7: Bandwidth of laser radiation in Theory and in Reality

1.2.2. Directionality

- Radiation comes out of the laser in a certain direction, and spreads at a defined **divergence angle (θ)** (see fig. 1.8, and example 1.2). **This angular spreading of a laser beam is very small compared to other sources of electromagnetic radiation, and described by a small divergence angle (of the order of milli-radians).**
- In [chapter 7](#), laser radiation characteristics are discussed in more detail, and different methods for measuring beam divergence are described.
In figure 1.8, a **comparison** is made between the radiation out of a laser, and the radiation out of a standard lamp.

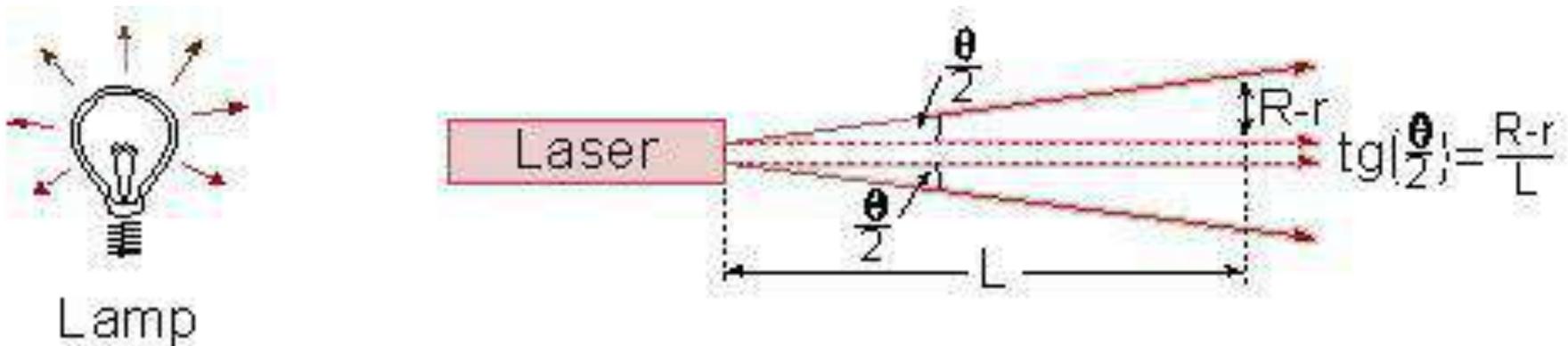


Fig.1.8: comparison between the light out of a laser, and the light out of an incandescent lamp

Divergence Angle

- **Divergence Angle** is the **full angle** of opening of the beam. (Some books use half of this angle as divergence angle). The relation between radians and degrees is given by:

$$360^{\circ} = 2\pi \text{ Radians}$$

$$1 \text{ Radian} = 57.3^{\circ}$$

$$1 \text{ milli-Radian} = 1 \text{ m.rad} = 0.057^{\circ}$$

Using the relation between minutes and degrees: $1^{\circ} = 60'$, we get:

- **1 m.rad = $0.057 * 60' = 3.5'$ Sec.**
- Since laser radiation divergence is of the order of **m.radians**, **the beam is almost parallel**, and **laser radiation can be send over long distances**.

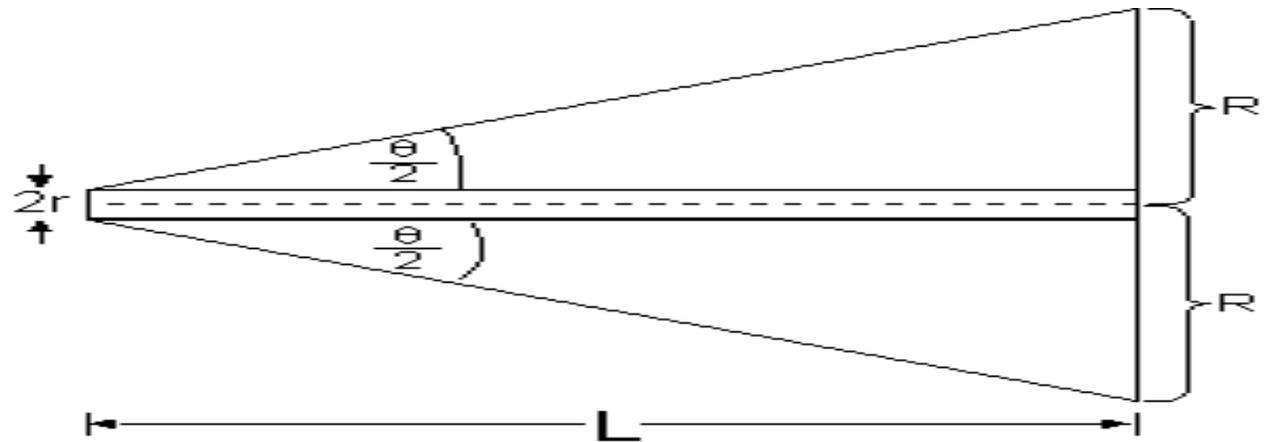
We shall see in [chapter 8](#), how a laser beam was sent to the moon, and returned to Earth to measure the distance between Earth and the moon with accuracy of tens of centimetres.

➤ Spot Size Measurement:

R = Radius of the illuminated spot at a distance L from the laser (see figure below).

If the spot size measurement is done **near the laser** (where the spot is small), then the size of the beam at the output of the laser needs to be taken into account:

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R - r}{L} \approx \frac{\Theta}{2}$$



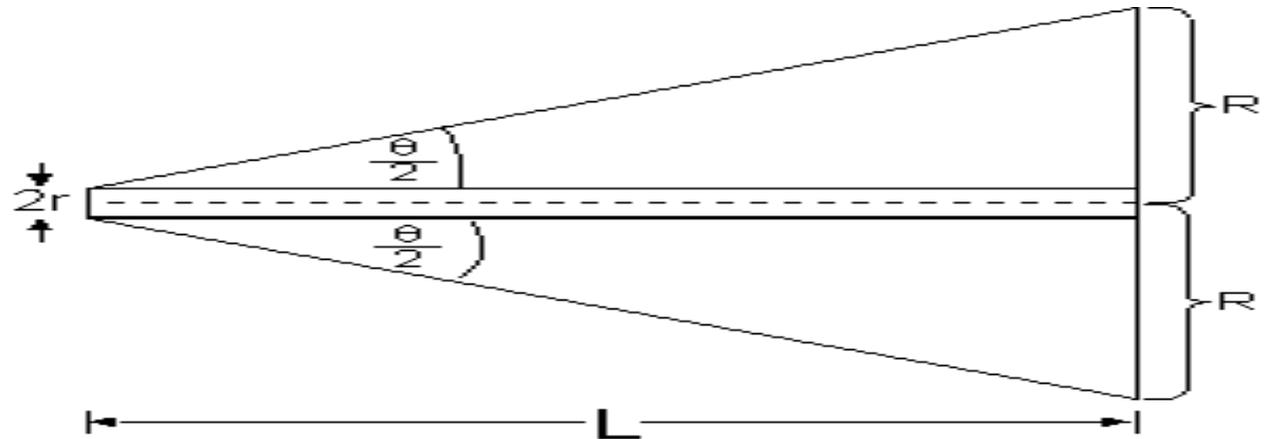
Because the **laser radiation has a very small divergence**, the small angle approximation can be used. Thus, we have set the tangent of the angle equal to the angle.

➤ Spot Size Measurement:

On a screen, the laser produce a spot. The diameter of this spot ($2R$) determines the **spot size**.

When the measurement is done **very far from the laser**, the spot size ($2R$) is big compared to the beam size at the laser output ($2r$), and it is accurate enough to measure the spot diameter and divide it by the distance, to find the **beam divergence**.

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R - r}{L} \approx \frac{\Theta}{2}$$



- Example: A laser with beam divergence of 1 m. radian creates a spot of about 10 [mm] at a distance of 10 [m].

The laser power measured over a defined unit surface area is called **Power Density**.

Looking at figure 1.8, it is clear that **from a laser it is possible to achieve higher power density than from conventional sources** (see example 1.2).

- **This is the reason why a 5 [m.W] laser radiation is considered dangerous, and the light out of a 100 W incandescent lamp is not !!!**

- **Example 1.2: Numerical Calculation of Power Density** Calculate the **power density** of radiation per unit area at a distance of 2 meters, from an incandescent lamp rated 100 [W], compared to a Helium-Neon laser of 1 [mW]. The laser beam diameter at the laser output is 2 [mm], and its divergence is 1 [m.rad].

- **Solution:**

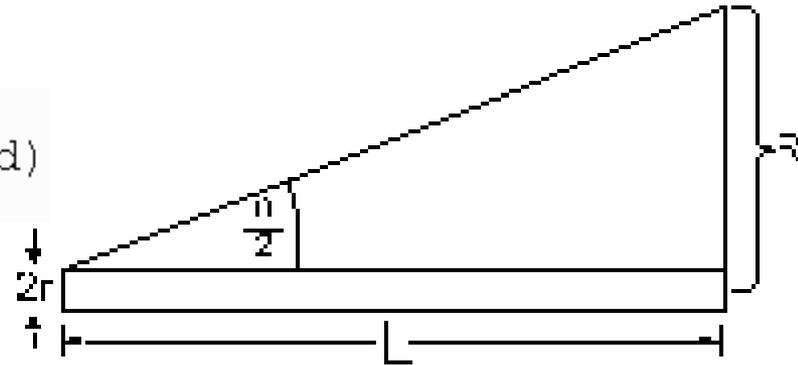
Light from incandescent lamp is radiated to all directions, so it is distributed on a surface of a sphere with a radius of 2 [m]. The surface area is: πR^2 , so the power density at a distance of 2 m is:

$$\frac{100 \cdot \text{W}}{\pi \cdot 200^2 \cdot \text{cm}^2} = 0.2 \cdot \frac{\text{mW}}{\text{cm}^2}$$

Compared to the incandescent lamp, the laser beam diameter at a distance of 2 [m] increased to 4 [mm] (see drawing below):

$$\tan\left(\frac{\Theta}{2}\right) = \frac{R - r}{L} = \frac{\Theta}{2} \quad (\text{in radians})$$

$$R = r + L \cdot \tan\left(\frac{\Theta}{2}\right) = 1 \cdot \text{mm} + 2000 \cdot \text{tg}(0.5 \cdot \text{mrad})$$



$$R = 2.1 \text{ [mm]} = 0.2 \text{ [cm]}$$

The power density of the laser radiation is:

$$\frac{1 \cdot \text{mW}}{\pi \cdot 0.2^2 \cdot \text{cm}^2} = 8 \cdot \frac{\text{mW}}{\text{cm}^2}$$

!!! When calculating radiation power in the visible spectrum (used for illumination), the low efficiency of the incandescent lamp must be considered (A 100 [W] lamp emits only 1-3 [W] of visible radiation, and all the rest is in the infrared spectrum).

At a distance of 2 [m] from the radiation source, the power density of the laser radiation is 40 times higher than from the lamp, although the power from the lamp is many times greater than original power of the laser.

1.2.3 Coherence

- Since **electromagnetic radiation** is a wave phenomena, every electromagnetic wave can be described as a sum (**superposition**) of sine waves as a function of time. From **wave theory** we know that every wave is described by a **wave function**:

$$y = A \cos(\omega t + \phi)$$

A = **Amplitude**.

$\omega = 2\pi\nu$ = **Angular Frequency**.

ϕ = **Initial Phase** of the wave (Describe the starting point in time of the oscillation).

$(\omega t + \phi)$ = **Phase** of the wave.

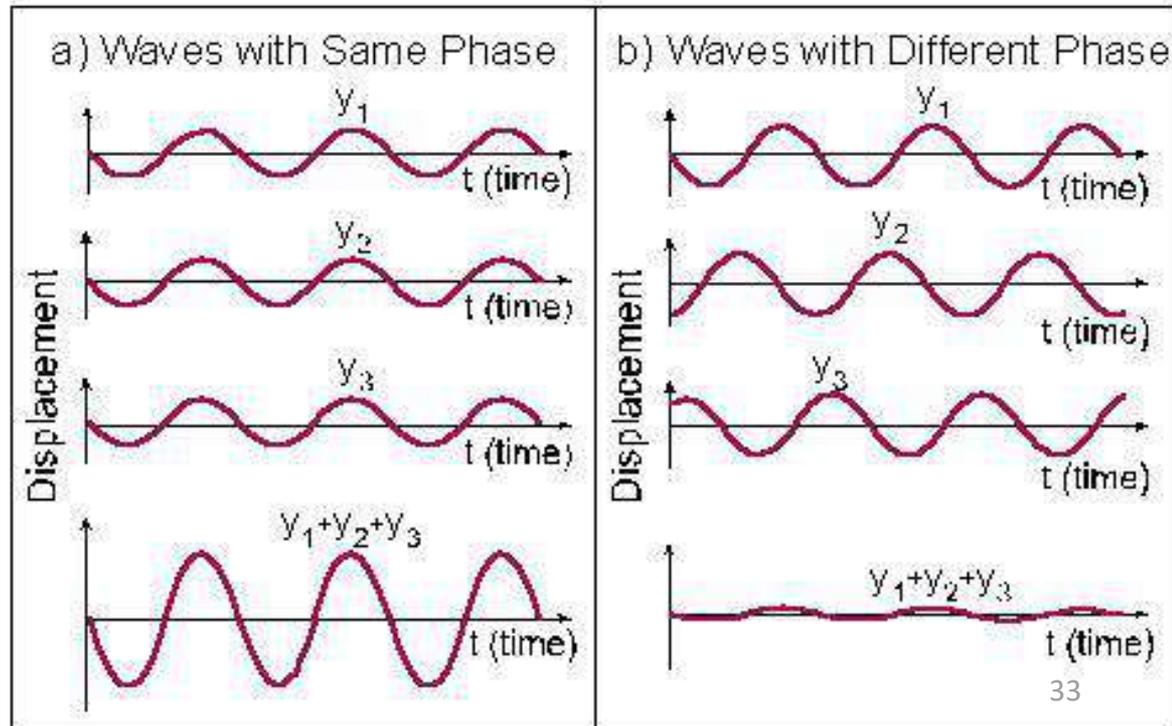
Superposition of Waves

- **Coherent waves** are waves that maintain the relative phase between them.

Figure 1.9 describes, using the same time base, 3 waves marked y_1 , y_2 , y_3 , and their superposition. In figure 1.9a, the waves are **coherent**, like the waves out of a laser. In figure 1.9b, the waves have the same wavelength, but are not coherent with each other.

Light from an incandescent lamp is composed of waves at many wavelengths, and each wave appears randomly with no systematic relation between its phase and that of the other wave.

Laser radiation is composed of waves at the same wavelength, which start at the same time and keep their relative phase as they advance. By adding (superposition) the wave amplitudes of the different waves, higher peaks are measured for laser radiation.



- **Example 1.3:** Can waves with different wavelength be coherent?
- **Solution to example 1.3:**
Waves with different wavelengths can have the same phase in one point in space (or even in some points), but they can not keep this phase difference as can be seen in figure 1.10.

As can be seen from this example, **coherence depends on monochromaticity.** Coherence is important for applications like **interference** and **diffraction**, and the entire process of **holography**.

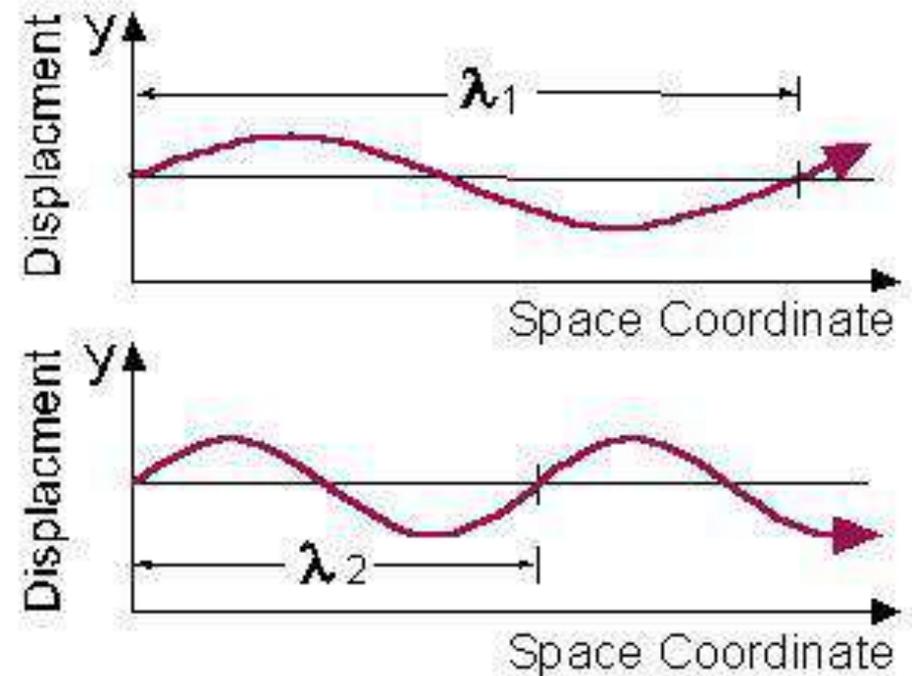


Figure 1-10: Waves with different wavelengths

Summary: Laser Radiation Properties

1. **Very small divergence of the beam.** The beam is almost a **parallel beam** and move in **one direction in space** - Directionality.
2. High degree of monochromaticity. The radiation is almost one wavelength, as can be measured by the **very narrow spectral width**.
3. Coherence.

The combination of these properties gives the laser radiation many advantages, like achieving **very high power densities**, not available from other sources.

cesses,

matter -

Chapter 2

Lasing Processes

of these

2.1 Bohr model of the atom.

Lasing action is a process that occurs in matter. Since matter is composed of atoms, we need to understand (a little) about the structure of the atom, and its energy states. We shall start with the **semi-classical model**, as suggested in 1913 by **Niels Bohr**, and called: **The Bohr model of the atom**.

According to this model, every atom is composed of a very **massive nucleus** with a **positive electric charge (Ze)**,

around it electrons are moving in specific paths.

Z = Number of protons in the nucleus,

e = Elementary charge of the electrons:

Figure 2.1 illustrates a simple, but adequate,

picture of the atom, the Bohr model:

Every "**allowed orbit**" of the electron around the nucleus, is connected to a specific energy level.

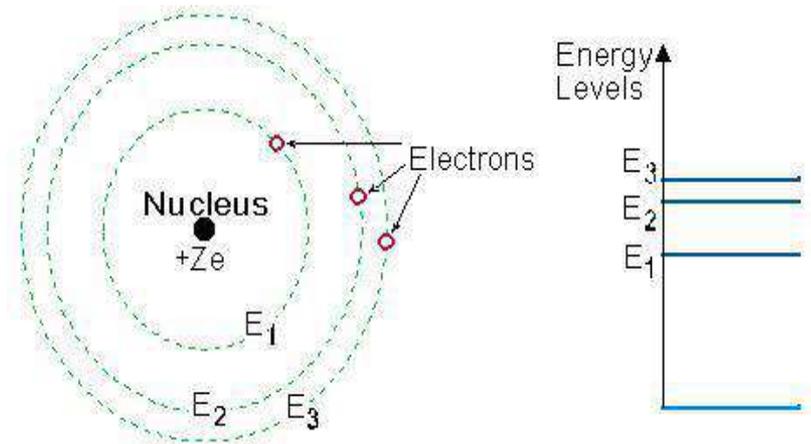


Fig 2-1: Bohr picture of the Atom

The energy level is higher as the distance of the "orbit" from the nucleus increases.

Since for each atom there are only certain "allowed orbits",

only certain discrete energy levels exist, and are named: **E₁, E₂, E₃**, etc..

Energy States (Levels)

Every atom or molecule in nature has a specific structure for its energy levels.

The lowest energy level is called the **ground state**.

As long as no energy is added to the atom, the electron will remain in the ground state

When the atom receives energy (electrical energy, optical energy, or any form of energy), this energy is transferred to the electron, and raises it to a higher energy level (in our model further away from the nucleus). The atom is then considered to be in an **excited state**. The electron can stay only at the specific energy states (levels) which are unique for each specific atom.

The **electron can not be in between these "allowed energy states"**, but it can "jump" from one energy level to another, while receiving or emitting specific amounts of energy.

These specific amounts of energy are equal to the **difference between energy levels within the atom**.

Each amount of energy is called a "**Quantum**" of energy

(The name "**Quantum Theory**" comes from these discrete amounts of energy).

Energy transfer to and from the atom

Energy transfer to and from the atom can be performed in two different ways:

1. Collisions with other atoms, and the transfer of kinetic energy as a result of the collision. This kinetic energy is transferred into internal energy of the atom.

2. Absorption and emission of electromagnetic radiation.

Since we are now interested in the **lasing process** we shall concentrate on the second mechanism of energy transfer to and from the atom

(The first excitation mechanism is used in certain lasers, like Helium-Neon, as a way to put energy into the laser, and will be discussed in chapter 6 about the different kinds of lasers).

2.2 Photons and the energy diagrams

Electromagnetic radiation has, in addition to its wave nature (described in [Chapter 1.1](#)),

some aspects of "**particle like behavior**". In certain cases, the electromagnetic radiation behaves as an ensemble of discrete units of energy that have momentum. These discrete units (quanta) of electromagnetic radiation are called "**Photons**". The relation between the **amount of energy (E)** carried by the photon, and its **frequency (ν)**, is determined by the formula (first given by Einstein):

$$E = h\nu = \hbar\omega$$

This formula shows that **the frequency of the radiation (ν), uniquely determines the energy of each photon in this radiation.**

Or with wavelength:

$$E = h * c/\lambda$$

This formula shows that **the energy of each photon is inversely proportional to its wavelength.**

Summary

- The interactions between electromagnetic radiation and matter cause **changes in the energy states of the electrons in matter**.
- Electrons can be transferred from one energy level to another, while **absorbing or emitting a certain amount of energy**. This amount of energy is equal to the **energy difference between these two energy levels ($E_2 - E_1$)**.

We shall see later in this chapter that:

- When this energy is absorbed or emitted in a form of **electromagnetic radiation**, the energy difference between these two energy levels ($E_2 - E_1$) determines uniquely the frequency (ν) of the electromagnetic radiation:

$$\Delta E = E_2 - E_1 = h\nu = \hbar\omega$$

Example 2.1: Visible Spectrum

The visible spectrum wavelength range is: 0.4 - 0.7 [μm] (400-700 [nm]).

The **wavelength of the violet light is the shortest**, and **the wavelength of the red light is the longest**. Calculate:

- What is the **frequency range of the visible spectrum**.
- What is the amount of the photon's energy associated with the violet light, compared to the photon energy of the red light.

Solution to example 2.1:

The frequency of violet light:

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}}}{0.4 \cdot 10^{-6} \cdot \text{m}} = 7.5 \cdot 10^{14} \cdot \frac{1}{\text{sec}}$$

The frequency of red light:

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}}}{0.7 \cdot 10^{-6} \cdot \text{m}} = 4.3 \cdot 10^{14} \cdot \frac{1}{\text{sec}}$$

The difference in frequencies:

$$\Delta\nu = \nu_1 - \nu_2 = 7.5 \cdot 10^{14} - 4.3 \cdot 10^{14} = 3.2 \cdot 10^{14} \cdot \frac{1}{\text{sec}}$$

The **energy of a violet photon:**

$$E_1 = h \cdot \nu_1 = (6.626 \cdot 10^{-34} \cdot \text{J} \cdot \text{sec}) \cdot \left(7.5 \cdot 10^{14} \cdot \frac{1}{\text{sec}} \right)$$

$$E_1 = 5 \cdot 10^{-19} \cdot \text{Joule}$$

The **energy of a red photon:**

$$E_2 = h \cdot \nu_2 = (6.626 \cdot 10^{-34} \cdot \text{J} \cdot \text{sec}) \cdot \left(4.3 \cdot 10^{14} \cdot \frac{1}{\text{sec}} \right)$$

$$E_2 = 2.85 \cdot 10^{-19} \cdot \text{Joule}$$

The difference in energies between the violet photon and the red photon is :

$$]^{19-10} \cdot 2.15 \text{ J}$$

This example shows how much more energy the violet photon have compared to the red photon .

Question 2.1:

Calculate in units of Nanometer, the wavelength of light emitted by the transition from energy level E_3 to energy level E_2 in a 3 level system in which:

$$E_1 = 0 \text{ [eV]}$$

$$E_2 = 1.1 \text{ [eV]}$$

$$E_3 = 3.5 \text{ [eV]}$$

2.3 Absorption of electromagnetic Radiation

We saw that the process of **photon absorption** by the atom is a process of raising the atom (electron) from a lower energy level into a higher energy level (excited state), by an amount of energy which is equivalent to the energy of the absorbed photon.

Our discussion involved a **microscopic system** in which one photon interacts with one atom. In a **macroscopic system**, when electromagnetic radiation passes through matter, part of it is transmitted, and part is absorbed by the atoms.

The **intensity (I)** of the transmitted radiation through a thickness (x) of homogeneous material, is described by the experimental equation of exponential absorption (**Lambert Law**):

$$I = I_0 \exp(-\alpha x)$$

I_0 = Intensity of incoming radiation.

α = Absorption coefficient of the material.

The thicker the material (bigger x), the lower the transmitted beam.

The **transmission (T)** of this material is described by the relation between the transmitted intensity (I) to the incident intensity (I_0):

$$T = I / I_0$$

From the last two equations we get the Transmission:

$$T = \exp(-\alpha x)$$

the units of the absorption coefficient (α) are: $[\text{cm}^{-1}]$

Every material is transparent differently to different wavelengths, so **the absorption coefficient (α) is a function of the wavelength: $\alpha(\lambda)$.**

This fact is very important (as we shall see) to understand the interaction of electromagnetic radiation with matter, in the variety of applications of the laser

Example 2.2: Absorption Coefficient (α)

Calculate the absorption coefficient (α) of materials which transmit 50% of the intensity of the incident radiation on a 10 [mm] width, to the other side.

Solution to example 2.2:

Using the exponential absorption law:

$$\alpha = 1/x * \ln(1/T) = 1/1 * \ln(1/0.5) = 0.69 [\text{cm}^{-1}]$$

Results from the exponential absorption law:

- For every material, **absorption depends on the width of the material**. The thicker the material, less radiation will be transmitted through.
- For a certain width (x) of the material, absorption depends only on the **absorption coefficient (α), which is characteristic of each material**.

2.4 Spontaneous emission of electromagnetic Radiation

One of the basic physical principles (which is the basis of a subject in physics called Thermodynamics) is that:

Every system in nature "prefers" to be in the lowest energy state.

This state is called the **Ground state**. As an example, we mentioned this principle in the [Bohr model of the atom](#).

When energy is applied to a system, The atoms in the material are **excited**, and **raised to a higher energy level**.

(The terms "excited atoms", "excited states", and "excited electrons" are used here with no distinction)

These electrons will remain in the excited state for a certain period of time, and then will return to lower energy states while emitting energy in the exact amount of the difference between the energy levels (ΔE).

If this package of energy is transmitted as electromagnetic energy, it is called [photon](#).

The emission of the individual photon is random, being done individually by each excited atom, with no relation to photons emitted by other atoms.

When photons are randomly emitted from different atoms at different times, the process is called [Spontaneous Emission](#). Since this emission is independent of external influence, there is **no preferred direction for different photons, and there is no phase relation between photons emitted by different atoms.**

2.4.1 Decay Rate

Energy loss by an excited atom can be performed in two basic ways:

- **Non-radiative decay** - by transferring the energy to mechanical vibrations of neighboring atoms. The rate for this type of decay is γ_{nr} . The macroscopic effect of these vibrations is what we call heat (or rise in temperature).

- **Radiative decay** - by spontaneous emission at a rate γ_{rad} .

The **total decay rate** of a certain level is the sum of the decay rates of the two processes:

$$\gamma = \gamma_{nr} + \gamma_{rad}$$

Orders of magnitude for decay lifetime for atoms:

Non radiative decay: picoseconds - microseconds (10^{-12} - 10^{-6} [sec]) - very fast.

Radiative decay: microseconds - milliseconds (10^{-6} - 10^{-3} [sec])

2.5 Thermodynamic (Thermal) Equilibrium

From thermodynamics we know that a collection of atoms, at a temperature T [$^{\circ}\text{K}$], in thermodynamic equilibrium with its surrounding, is distributed so that at each energy level there is on the average a certain number of atoms.

The number of atoms (N_i) at specific energy level (E_i) is called **Population Number**.

The **Boltzmann equation** determines the relation between the population number of a specific energy level and the temperature:

$$N_i = \text{const} * \exp (-E_i/kT)$$

N_i = Population Number = number of atoms per unit volume at certain energy level E_i .

k = Boltzmann constant: $k = 1.38 * 10^{23}$ [Joule/ $^{\circ}\text{K}$].

E_i = Energy of level i .

Const = proportionality constant. It is not important when we consider population of one level compared to the population of another level as we shall see shortly.

T = Temperature in degrees Kelvin [$^{\circ}\text{K}$] (Absolute Temperature).

The Boltzmann equation shows the dependence of the population number (N_i) on the energy level (E_i) at a temperature T .

From this equation we see that:

1. **The higher the temperature, the higher the population number.**
2. **The higher the energy level, the lower the population number.**

Relative Population (N_2/N_1)

The **relative population (N_2/N_1)** of two energy levels E_2 compared to E_1 is:

$$\begin{aligned} N_2/N_1 &= \text{const} * \exp(-E_2/kT) / \text{const} * \exp(-E_1/kT) \\ &= \exp(-(E_2-E_1)/kT) \end{aligned}$$

The proportionality constant (const) is canceled by division of the two population numbers.

Conclusions:

1. The **relation between two population numbers (N_2/N_1)** does not depend on the values of the energy levels E_1 and E_2 , but only on the difference between them: $E_2 - E_1$.
2. For a certain energy difference, the higher the temperature, the bigger the relative population.
3. The relative population can be between 0 and 1.
4. **In a thermodynamic equilibrium**, the population number of higher energy level is always less than the population number of a lower energy level.
5. **The lower the energy difference between the energy levels, the less is the difference between the population numbers of these two levels .**

Physically, the electrons inside the atom prefer to be at the lowest energy level possible because they emit photons when they transition from a higher energy level to a lower energy level.

Population at Thermodynamic Equilibrium

Figure 2.2 shows the population of each energy level at thermodynamic equilibrium .

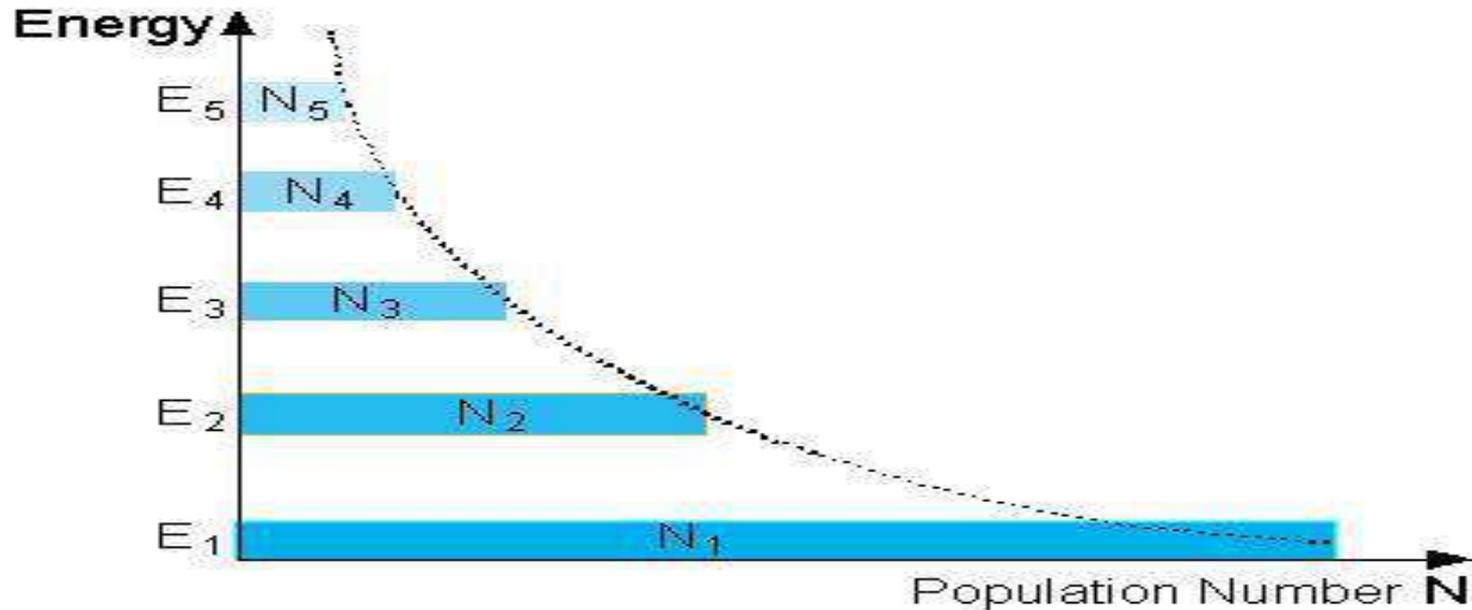


Figure "noitalupoP lamron" ta srebmuN noitalupoP :2.2

Question :2.3

The difference between population numbers

Prove that the difference in population numbers (N_2 and N_1) between energy levels E_2 and E_1 is given by :

$$N_2 - N_1 = \frac{N_1 \nu}{\nu - \nu_0} \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

$\nu = \nu_0 \exp\left(-\frac{E_2 - E_1}{kT}\right)$ is the frequency which corresponds to the energy difference between the two levels E_2 and E_1 .

Example 2.3:

Calculate the ratio of the population numbers (N_1, N_2) for the two energy levels E_2 and E_1 when the material is at room temperature (300°K), and the difference between the energy levels is 0.5 [eV] . What is the wavelength (λ) of a photon which will be emitted in the transition from E_2 to E_1 ?

Solution to example 2.3:

When substituting the numbers in the equation, we get:

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B \cdot T}\right) = \exp\left[-\frac{(0.5 \cdot \text{eV}) \cdot \left(1.6 \cdot 10^{-19} \cdot \frac{\text{J}}{\text{eV}}\right)}{\left(1.38 \cdot 10^{-23} \cdot \frac{\text{J}}{\text{K}}\right) \cdot (300\text{K})}\right]$$
$$= 4 * 10^{-9}$$

This means that **at room temperature, for every 1,000,000,000 atoms at the ground level (E_1), there are 4 atoms in the excited state (E_2) !!!**

To calculate the **wavelength**:

$$\lambda = \frac{h \cdot c}{\Delta E} = \frac{(6.626 \cdot 10^{-34} \cdot \text{J} \cdot \text{sec}) \cdot \left(3 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}}\right)}{(0.5 \cdot \text{eV}) \cdot \left(1.6 \cdot 10^{-19} \cdot \frac{\text{J}}{\text{eV}}\right)} = 2.48 \cdot \mu\text{m}$$

This wavelength is in the Near Infra-Red (NIR) spectrum.

Question 2.4:

A material is in thermodynamic equilibrium at room temperature (300°K).

The wavelength of the photon emitted in the transition between two levels is $0.5\ [\mu\text{m}]$ (visible radiation).

Calculate the ratio of the population numbers for these energy levels.

2.6 Population Inversion

We saw that in a thermodynamic equilibrium Boltzmann equation shows us that :

$$N_1 > N_2 > N_3$$

Thus, the population numbers of higher energy levels are smaller than the population numbers of lower ones.

This situation is called "**Normal Population**" (as described in Figure 2.3a). In a situation of normal population a photon incident on the material will be absorbed, and raise an atom to a higher level.

By putting energy into a system of atoms, we can achieve a situation of "**Population Inversion**". In population inversion, at least one of the higher energy levels has more atoms than a lower energy level.

An example is described in Figure 2.3b. In this situation there are more atoms (N_3) in an higher energy level (E_3), than the number of atoms (N_2) in a lower energy level (E_2).

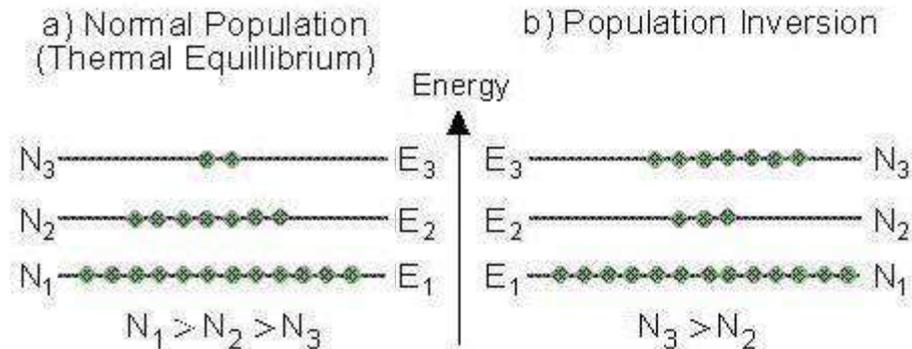


Figure 2.3: "Normal Population" compared to "Population Inversion".

2.7 Stimulated Emission

Atoms stay in an excited level only for a short time (about 10^{-8} [sec]), and then they return to a lower energy level by spontaneous emission.

Every energy level has a **characteristic average lifetime**.

According to the quantum theory, **the transition from one energy level to another is described by statistical probability**.

The probability of transition from higher energy level to a lower one is inversely proportional to the lifetime of the higher energy level.

In reality, the probability for different transitions is a characteristic of each transition, according to selection rules .

When the transition probability is low for a specific transition, the lifetime of this energy level is longer (about 10^{-3} [sec]), and this level becomes a "**meta-stable**" level. In this meta-stable level a large population of atoms can assembled. As we shall see, this level can be a candidate for lasing process.

When the population number of a higher energy level is bigger than the population number of a lower energy level, a condition of "**population inversion**" is established.

If a population inversion exists between two energy levels, then the probability is high for an incoming photon to **stimulate** an excited atom to return to a lower state, while emitting another photon of light. The probability for this process depend on the match between the energy of the incoming photon and the energy difference between these two levels.

Properties of stimulated emission

The photon which is emitted in the stimulated emission process is identical to the incoming photon. They both have:

- 1- Identical wavelengths (and thus, frequencies) monochromaticity.**
- 2. Identical directions in space - Directionality.**
- 3. Identical phase - Coherence**

These are the properties of laser radiation.

Remember that **two photons with the same wavelength (frequency) have the same energy:**

$$E = h\nu = hc/\lambda$$

The incoming photon does not change at all as a result of the stimulated emission process.

As a result of the stimulated emission process, we have **two identical photons created from one photon** and one excited state. Thus we have amplification in the sense that the number of photons has increased.

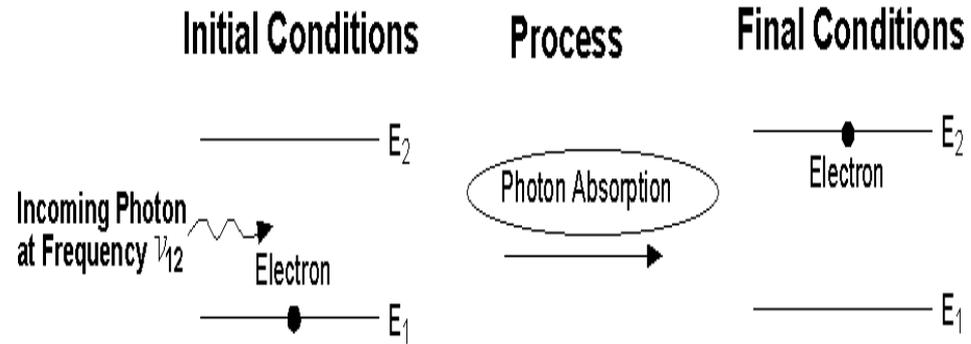
This is the process that was explained in the introduction:

Light Amplification by Stimulated emission of Radiation = LASER

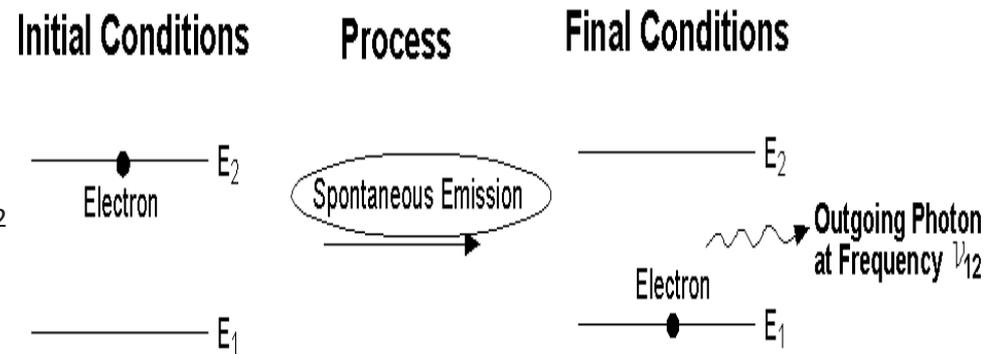
Possible Processes Between Photons and Atoms

Figure 2.4 summarizes the three possible processes between photons and atoms: absorption, spontaneous emission, and stimulated emission.

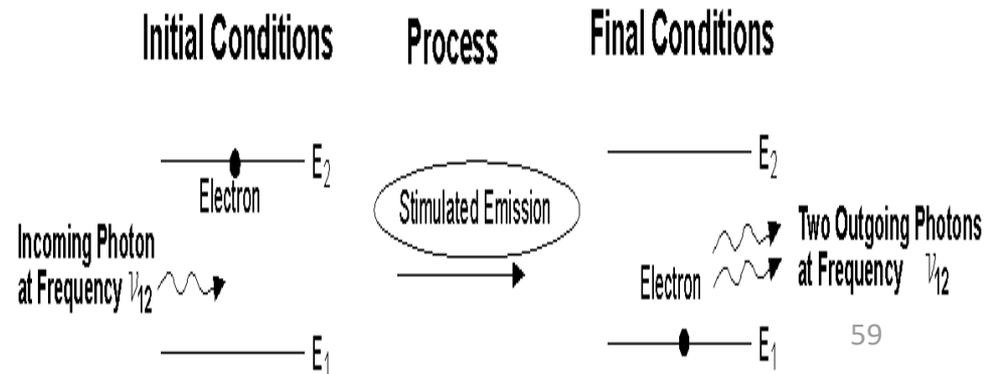
Photon Absorption: A photon with frequency ν_{12} hits an atom at rest (left), and excites it to higher energy level (E_2) while the photon is absorbed.



Spontaneous emission of a photon: An atom in an excited state (left) emits a photon with frequency ν_{12} and goes to a lower energy level (E_1).



Stimulated emission of a photon: A photon with frequency ν_{12} hit an excited atom (left), and cause emission of two photons with frequency ν_{12} while the atom goes to a lower energy level (E_1).



2.8 Rate Equations for spontaneous emission

(Advanced level only, need to know first order differential equations).

For simplicity we shall assume:

1. The material is composed of many identical atoms.
2. Each atom has only two energy levels: E_1 and E_2 .
3. The only decay mechanism of energy level E_2 is spontaneous emission

At time t , N_1 atoms are in energy level E_1 , and N_2 atoms in energy level E_2

The rate at which the excited atom population $N_2(t)$ decays from the higher energy level (E_2) to the lower energy level (E_1), by **spontaneous** emission is given by the **decay coefficient** g_{21} multiplied by the instantaneous population number of this level $N_2(t)$ as seen by the first order differential equation:

$$d N_2(t)/dt = - g_{21} N_2(t) = - N_2(t)/ \tau_2$$

This equation defines the **lifetime** τ_2 of energy level E_2 :

$$\tau_2 = 1/ g_{21}$$

The solution to the rate (differential) equation is:

$$N_2(t) = N_2(0) \exp(-g_{21}t) = N_2(0) \exp(-t/ \tau_2)$$

Conclusion:

If at specific moment ($t=0$) the number of atoms in the excited state E_2 is $N_2(0)$, then when we leave the system **without external influence**, the number of atoms in the excited state will **decay exponentially** according to the equation

$$N_2(t) = N_2(0) \exp(-g_{21}t) = N_2(0) \exp(-t/\tau_2).$$

by attention :

in spontaneous emission, the population number (N_1) of the lower energy level (E_1) is unimportant

2.9 Stimulated Transitions:

The strength of an optical signal (The number of photons) is described by:

- **Intensity (I)**, which means the ratio of measured power over a specific surface area (since **power** is a measure of the amount of energy over time, then Intensity is a measure of the amount of energy over time over surface area).
- **Energy density (n(t))** = The **number of photons per unit volume**
= The energy of the electromagnetic radiation in units of **hν**

An **Optical signal** is an oscillating electromagnetic field, and an **atom** can be described as an electric dipole.

When an optical signal of the right frequency (**hν** equal to the energy difference between the **energy levels** $E_2 - E_1$) is approaching an atom, both the atoms at the lower **energy level** (E_1) and the atoms at upper **energy level** (E_2) will start to oscillate.

That is why there are **two forced processes**: **absorption** and **stimulated emission**.

2.9.1 Absorption Rate Equations

The incoming optical signal (photons) causes the atoms to "jump" from the lower energy level (E_1) to the upper energy level (E_2).

The **absorption rate** is proportional to the product of the density $n(t)$ of the incoming photons (the number of photons in a unit volume) with the number of atoms ($N_1(t)$) in the lower energy level (E_1):

$$d N_1(t)/dt = K n(t) N_1(t)$$

Each photon excites one atom to the higher energy level.

K = Proportionality constant - A measure of the relative strength of the response of the atom to the incoming radiation in this specific transition.

2.9.2 Stimulated Emission Rate Equations

The incoming optical signal (photons) causes the atoms in the **upper** energy level (E_2) to oscillate (forced oscillations), and make a transition to the lower energy level (E_1).

In this process two **photons are emitted together**: The incoming photon and the photon from the energy transition

$$h\nu = E_2 - E_1.$$

The **stimulated emission rate** is proportional to the product of the energy density $n(t)$ of the incoming photons (the number of photons in a unit volume) with the number of atoms ($N_2(t)$) in the upper energy level (E_2):

$$d N_2(t)/dt = K n(t) N_2(t)$$

2.9.3 The Proportionality Constant (K)

From quantum considerations we get that the **proportionality constant (K) for stimulated emission and (stimulated) absorption are identical.**

This constant depends on the frequency (ν) of the incoming photon.

The value of **K** is maximum when the frequency of the incoming photon is equal to the transition frequency ν_{21} .

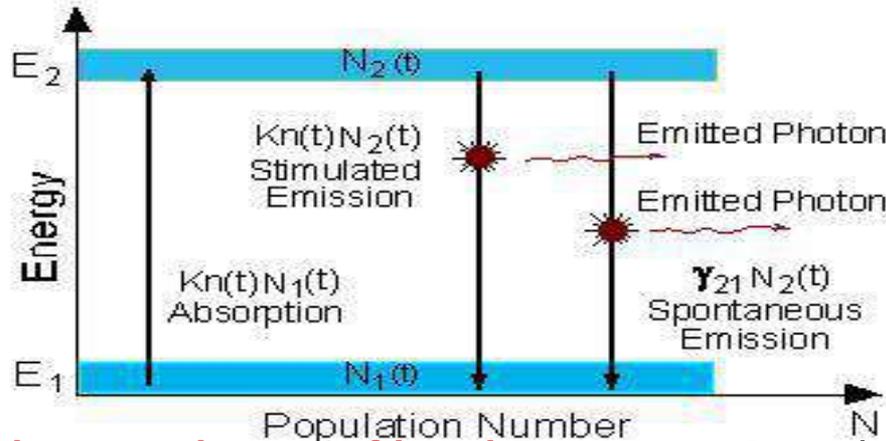
The farther apart from the transition frequency, the less the value of the proportionality constant, up to zero.

Every transition has a **linewidth ($\Delta\nu$)** around the transition frequency. This linewidth shows the frequency range in which transitions can occur (will be seen in [chapter 5](#)).

If the frequency of the incoming photon is not in the range $\nu_{21} + (\Delta\nu)$, then the value of **K is zero**

Diagram of energy Level Population

We shall summarize all the transitions in the **diagram of energy level population** (figure 2.5).



The **rate equation for the population of level E_2** summarizes the spontaneous emission and the two stimulated emissions (transition), for the simple case of a two level system:

$$\begin{aligned} \frac{dN_2(t)}{dt}_{\text{tot}} &= \frac{dN_2(t)}{dt}_{\text{absorp}} + \frac{dN_2(t)}{dt}_{\text{stimul}} + \frac{dN_2(t)}{dt}_{\text{spontan}} \\ &= +Kn(t)[N_1(t) - N_2(t)] - g_{21}N_2(t) \\ &= \frac{dN_1(t)}{dt}_{\text{total}} \end{aligned}$$

We should remember that the spontaneous emission and the stimulated emission occur at the same time, and are independent of each other, so their emission rates can be added.

The process of stimulated emission is a result of resonance response of the atom to the forcing signal, so they oscillate at the same frequency and are coherent in space and time (with the same phase and amplitude).

On the other hand, the spontaneous emission is into all directions in space, and each photon is randomly emitted.

2.10 Amplification

We saw in the rate equation that:

- Two processes decrease the population number of the excited level: **Spontaneous emission and stimulated emission**.
- One process increases the population number of the excited state - **absorption**.

Since the same amount of energy ($h\nu$) is involved in every transition up or down, than the rate at which energy is absorbed in a unit volume of atoms is given by **the transition rate times the unit of energy:**

$$dU_a / dt = K n(t) [N_1(t) - N_2(t)] * h\nu$$

U_a = energy density in stimulated transitions.

Question 2.6:

Compare this equation to the rate equation. What is the difference? explain why.

The **incoming energy** is:

$$U_{\text{signal}}(t) = n(t) * h\nu$$

The absorbed energy is taken out of the incoming signal, so the **rate of loss of energy from the incoming signal** is:

$$\begin{aligned} dU_{\text{signal}}(t)/dt &= -K * n(t) * [N_1(t) - N_2(t)] * h\nu \\ &= -K * [N_1(t) - N_2(t)] * U_{\text{signal}}(t) \end{aligned}$$

The same equation can be written for the **photon density**:

$$dn(t)/dt = -K * [N_1(t) - N_2(t)] * n(t)$$

Question 2.7:

According to what we learned so far about the conditions for lasing:

Is it possible to operate a laser on a two level system (E_1 and E_2)?

Can a **metastable level** exist in a two level system?

Absorption or Amplification:

From the equation of rate of loss of energy from the incoming signal, we see that **the sign of the difference in the population number ($\Delta N = N_1(t) - N_2(t)$)** determines if the energy density of the incoming signal will increase or decrease as a function of time.

Consider **two possible situations**:

1. When a collection of atoms is in a **normal population (Thermal equilibrium)**, The population number of the lower energy level (E_1) is higher than the population number of the higher energy level (E_2): $N_1(t) > N_2(t)$.

In this situation **only absorption is possible**, and as a result the atoms will receive energy from the incoming signal which will decrease in magnitude.

2. When a collection of atoms is in a **population inversion**, The sign in front of the parenthesis is minus, and the signal will increase - **Amplification !**

Energy from the system of atoms will be transferred to the incoming signal, and amplify it at a rate which is proportional to the difference in population numbers and to the intensity of the incoming signal.

Conclusion:

If a material is in a thermal equilibrium, only absorption can occur, and no amplification.

To produce **amplification**, the material must be in a **population inversion** in which more atoms are pumped to an excited state as compared to a lower state.

After understanding the amplification process at the **microscopic level** of atoms, we shall connect this process to the **macroscopic system**, as we did in the process of absorption of electromagnetic radiation.

In the process of **absorption** in the laser medium, the **absorption coefficient (α)** depends on the material, and on the difference in population numbers ($\Delta N = N_1(t) - N_2(t)$) between the energy levels E_1 and E_2 , as:

$$\alpha = K(N_1 - N_2)$$

The **proportionality constant (K)** depends on the material and the wavelength of the laser radiation.

As long as $N_1(t) > N_2(t)$, α is positive, and the process is **absorption**.

In the "**population inversion**" situation, $N_1(t) - N_2(t) < 0$, so that $N_2(t) > N_1(t)$, then **α is negative**. According to Lambert law: $I = I_0 e^{-\alpha x}$, the factor $(-\alpha x)$ is positive, which means that **the intensity at the output (I) is bigger than the intensity at the input (I_0)**, thus **amplification (gain of energy)**.

In case of **amplification**, α is called **Gain Coefficient**.

Attention

The **probability of the stimulated transition** by the incoming radiation is identical for the two processes (**absorption and stimulated emission**). The direction in which more processes will occur depends on the population of the energy levels at that moment.

Example 2.4: Dependence of amplification on the length of the laser A laser is 15 [cm] long. For a certain wavelength, the amplification of the laser is 1.5. Calculate the amplification of this laser if the length of the active medium is 30 [cm].

Solution to Example 2.4:

using the definition of amplification:

$$\text{Amplification} = \frac{I}{I_0} = e^{-\alpha \cdot x}$$

substitute the known parameters:

$$1.5 = e^{-\alpha \cdot 15}$$

we get:

$$\alpha = -0.027 \text{ [cm}^{-1}\text{]}$$

using the new length in the amplification definition:

$$\text{Amplification} = \frac{I}{I_0} = e^{-\alpha \cdot x} = e^{-(-0.027 \cdot \text{cm}^{-1} \cdot 30 \cdot \text{cm})} = 2.25$$

Conclusion:

As the length of the active medium increases, the amplification increases.

Attention!

There is a limit to this conclusion. In the above discussion **absorption in the active medium was not taken into account.**

Later we shall see how to calculate both absorption and amplification.

An example for such mathematical calculations are:

- Radioactive decay chain.
- Chain of water tanks in which the water is flowing from the higher to the lowest through all the tanks.

Result:

If the absorption rate is proportional to N_1 , and the emission rate is proportional to N_2 , with the same proportionality factor, then **the number of photons in the output laser beam depends on $N_1 - N_2$.**

2.11 Three Level Laser

A schematic energy level diagram of a laser with three energy levels is shown in figure 2.6. The two energy levels between which lasing occur are: the **lower laser energy level (E_1)**, and the **upper laser energy level (E_2)**.

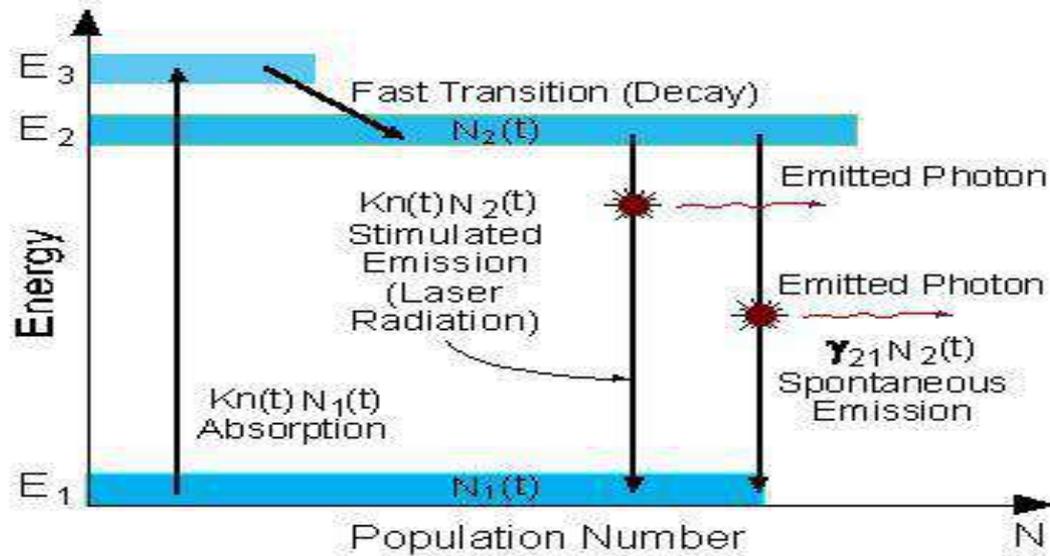


Figure 2.6: Energy level diagram in a three level laser

To simplify the explanation, we neglect spontaneous emission.

To achieve lasing, energy must be pumped into the system to create population inversion. So that more atoms will be in energy level E_2 than in the ground level (E_1).

Atoms are pumped from the ground state (E_1) to energy level E_3 . They stay there for an average time of 10^{-8} [sec], and decay (usually with a non-radiative transition) to the meta-stable energy level E_2 .

Since the lifetime of the meta-stable energy level (E_2) is relatively long (of the order of 10^{-3} [sec]), many atoms remain in this level.

If the pumping is strong enough, then after pumping more than 50% of the atoms will be in energy level E_2 , a population inversion exists, and lasing can occur

Question 2.8:

The condition of high pumping, limits the operation of a three level laser to pulsed operation. Why is continuous operation impossible in a three level laser?

2.12 Four Level Laser

The schematic energy level diagram of a four level laser is shown in figure 2.7. Compared to the equivalent diagram of a three level laser, there is an **extra energy level** above the ground state. This extra energy level has a **very short lifetime**.

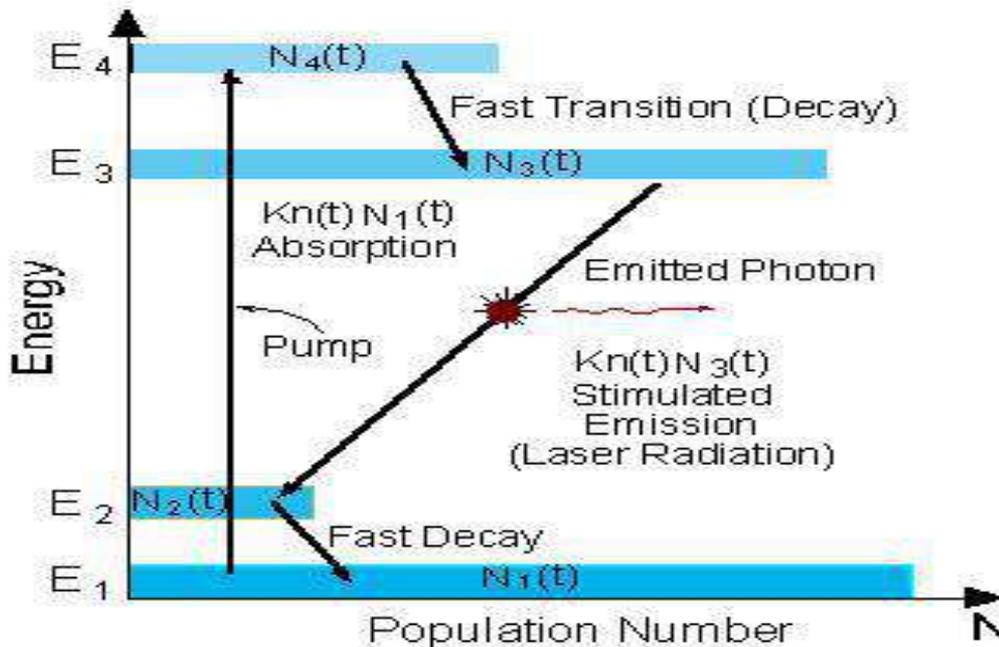


Figure 2.7: Energy level diagram in a four level laser

The pumping operation of a four level laser is similar to the pumping of a three level laser. This is done by a rapid population of the upper laser level (E_3), through the higher energy level (E_4).

The advantage of the four level laser is the low population of the lower laser energy level (E_2).

To create population inversion, there is no need to pump more than 50% of the atoms to the upper laser level. The population of the lower laser level ($N_2(t)$) is decaying rapidly to the ground state, so practically it is empty. Thus, a continuous operation of the four level laser is possible even if 99% of the atoms remain in the ground state (!)

Advantages of four level lasers Compared to three level lasers:

- The lasing threshold of a four level laser is lower.
- The efficiency is higher.
- Required pumping rate is lower.
- Continuous operation is possible.

Summary:

- In a three level laser the lower laser level is the ground state.
- In a four level laser the lower laser level is above the ground state

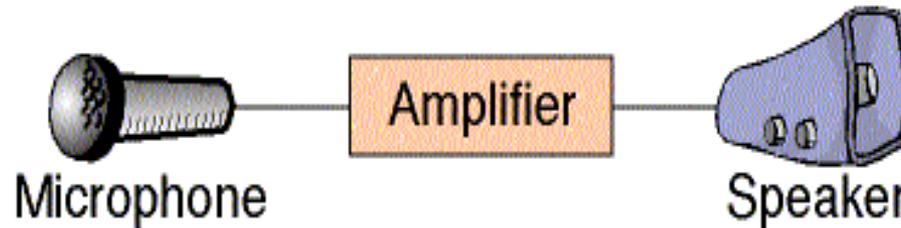
Chapter 3: The Laser System

The laser is a system that is similar to an **electronic oscillator**.

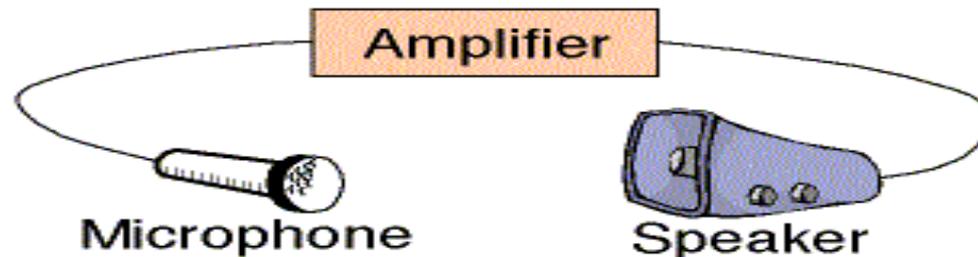
An **Oscillator** is a system that produces oscillations without an external driving mechanism.

To demonstrate an oscillator, we can use the familiar acoustic analog:

A sound amplification system has a **microphone**, **amplifier** and **speaker**.



When the microphone is placed in front of the speaker, a **closed circuit** is formed, and a whistle is heard out of the speaker.



The whistle is created spontaneously, without any external source.

Explanation: The speaker's internal noise is detected by the microphone, amplified and the amplified signal is again collected by the microphone. This **positive feedback** continues until a loud whistle is heard.

The Laser System (continuous)

Every oscillator has 3 main parts (as seen in figure 3.1):

1. Amplifier.
2. Power source.
3. Resonator (Positive feedback).

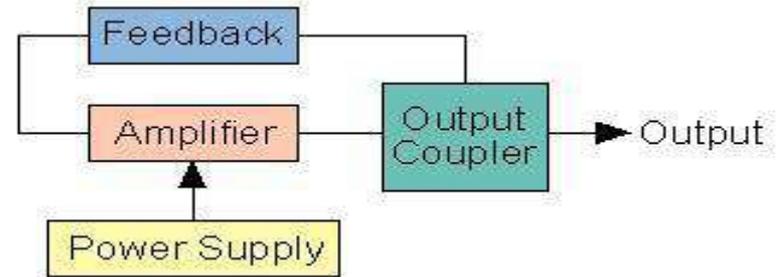


Figure 3.1: Electronic Oscillator

In analogy to the electronic amplifier, **the laser** can be described as composed of three structural units:

1. Active medium, which serves as an optical amplifier.
2. Pumping Source or Excitation mechanism.
3. Resonator or Optical feedback.

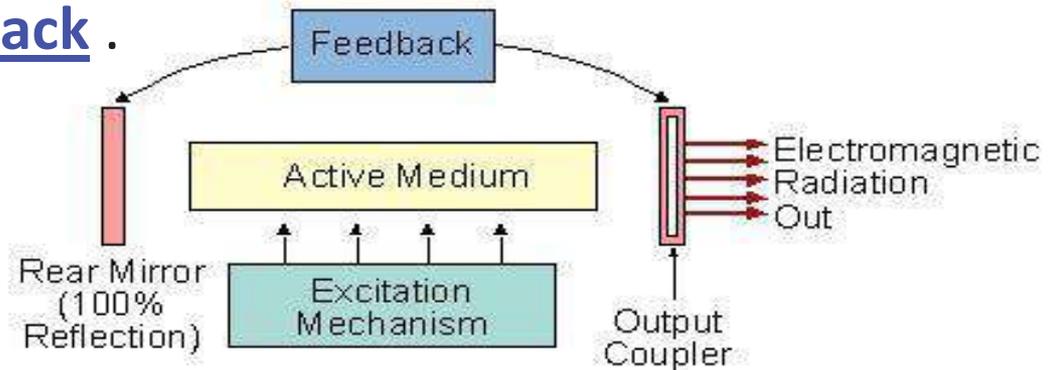


Figure 3.2: The Basic Laser System

3.1 The laser active medium:

- The **active medium** is a collection of atoms or molecules, which can be excited into a **population inversion** situation, and can have electromagnetic radiation extracted out of it by stimulated emission.
- **The active medium can be in any state of matter: solid, liquid, gas or plasma.**
- **The active medium determines the possible wavelengths that can be emitted from the laser.** These wavelengths are determined by the specific transitions between the laser energy levels in this material.

The list of materials that lase under certain laboratory conditions include hundreds of substances, and the number increases with time.

The basic physics of the laser is similar for all types of lasers, and we will use the term "Active Medium" and assume that it is composed of "**atoms**". In reality, the active medium can be atoms, molecules, ions, or semiconductors, according to the laser type.

3.2 The excitation mechanism or pumping sources:

The **excitation mechanism** is the source of energy that raises the atoms in the active medium into their excited state, thus creating population inversion.

In accordance to the law of conservation of energy, the electromagnetic radiation out of the laser is always less than the input energy by the excitation mechanism. There are lasers with efficiency of much less than 1% (!), while others with efficiencies approaching a 100%.

There are **few types of excitation mechanisms**:

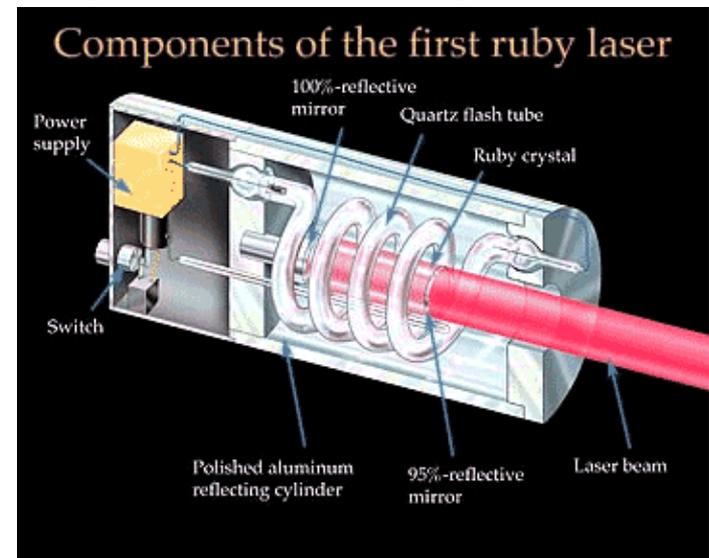
a) Optical pumping - Excitation by photons:

In lasers with solid or liquid active medium, it is common to supply the excitation energy in a form of electromagnetic radiation (photons) which are absorbed in the active medium.

In this method a beam of light with a proper wave – length and intensity is employed by directing it through the material in question. Through which the atoms will be excited to the higher state.



In this method the atoms may be excited directly to the third or fourth level as in the solid state laser



The electromagnetic radiation source can be of different kinds:

- **Flash lamps**, which are build from a quartz tube filled with gas at low pressure. Usually **Xenon gas**, **Krypton** or **Helium** are used.
- **Another laser** or **any other light source**.

b) Electrical excitation of a gas:

- When the active medium is in the gas state, the best excitation is by electrical discharge of the gas (Fig.3.3)

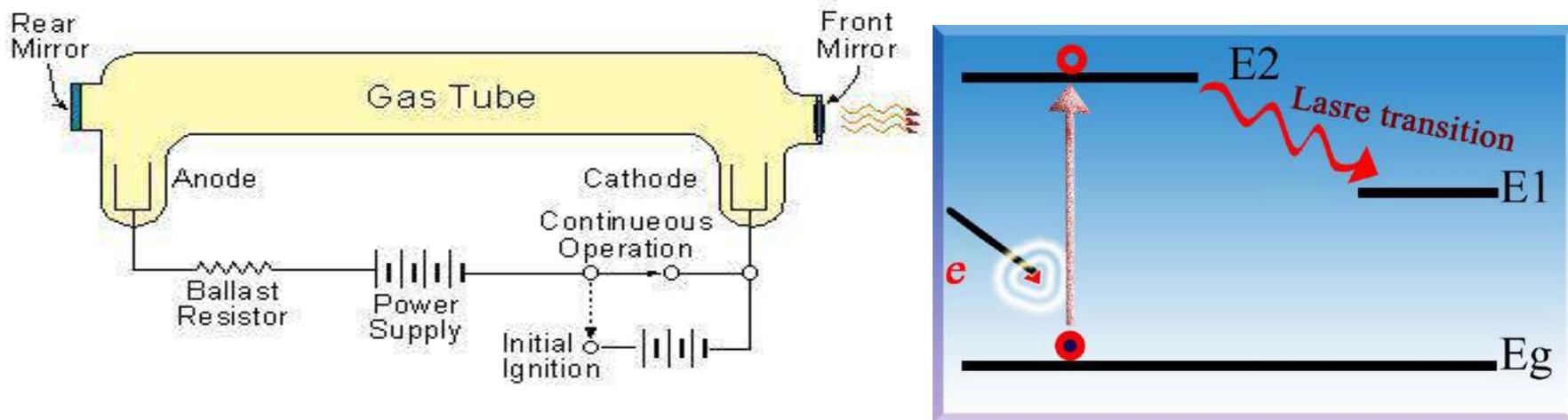


Figure 3.3: Electrical Excitation of a Gas Laser

The gas in the tube is electrically neutral, and as long as no external energy is applied, most of the molecules are in the ground state.

When the **high electrical voltage** is applied, electrons are released from the cathode and accelerated toward the anode. On their way, these **electrons collide with the gas molecules and transfer energy to them**. Thus, the gas molecules are raised to excited state. as it is in the following equation:

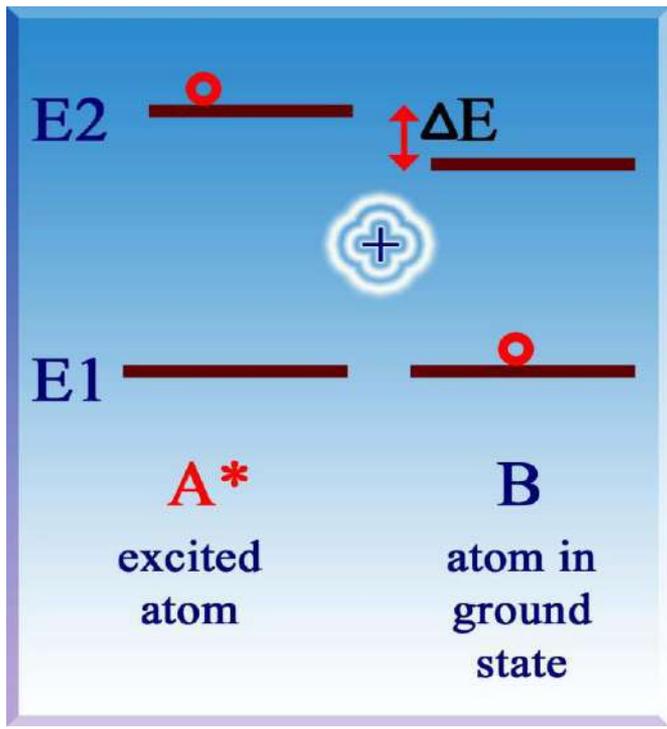
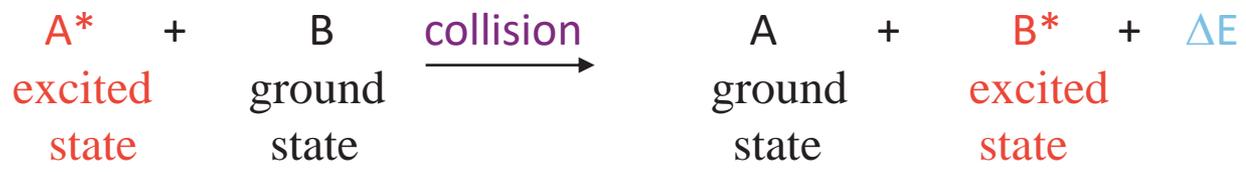


This is called **FIRST KIND COLLISION**.

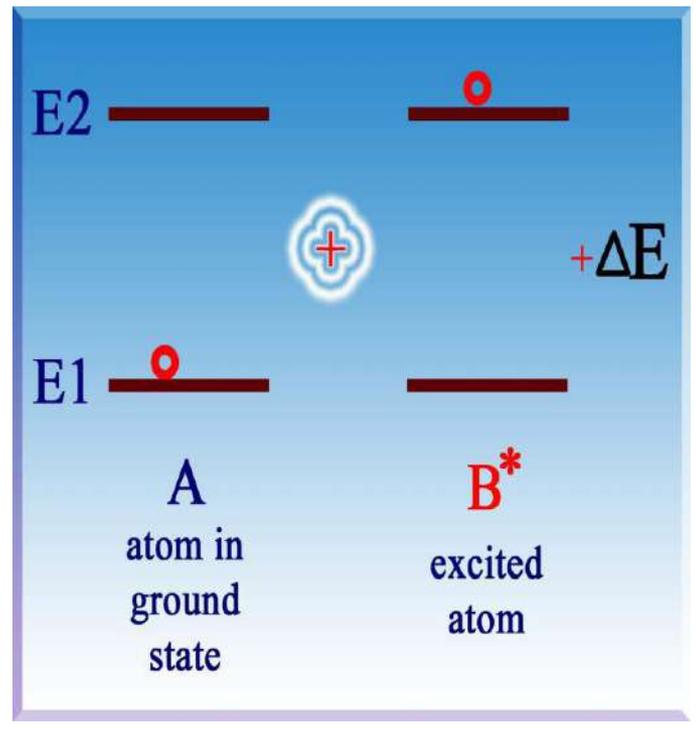
- Since the right conditions for direct excitation of the lasing gas are difficult to find, a variation of this method is used:

* Collision of the second kind:

In this method two kinds of atoms participate one of the theme is originally in an excited state (possibly Meta-stable state) and the other in the ground state, after the collision the first kind decays to the ground state resulting excitation of the second kind



after collision



Example 3.1: Helium-Neon Laser

Figure 3.4 show the energy level diagram of Helium-Neon laser, with the possible transitions. The mass of the Helium atom is about one-fifth of the mass of the Neon atom.

The amount of Helium in the tube is about 6 times the amount of Neon. Thus Helium atoms have more chance to receive energy from the accelerated electrons, and transfer into the excited energy levels E_3 and E_5 .

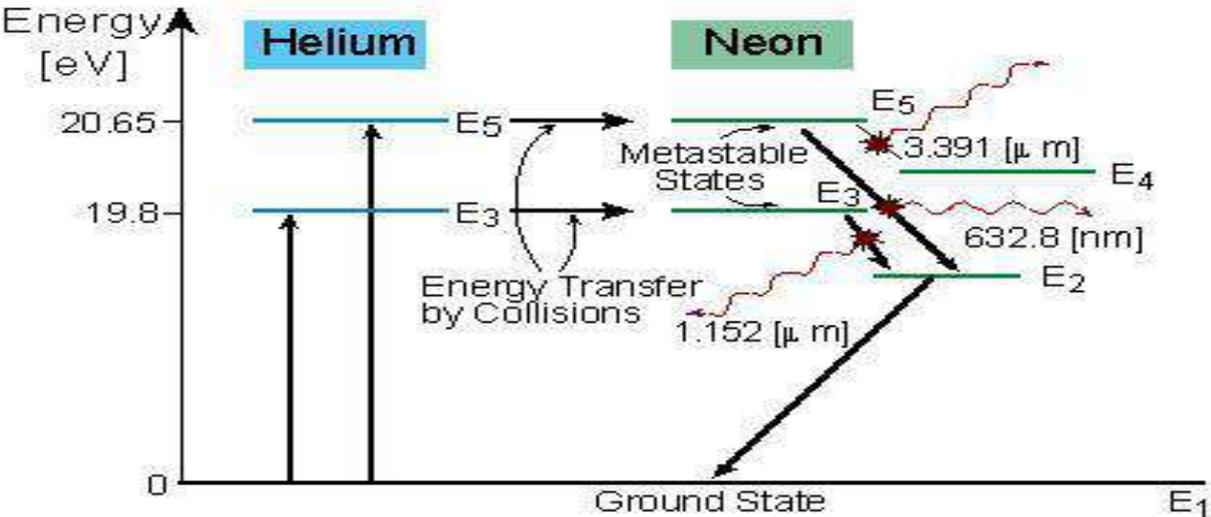


Figure 3.4: Energy Level Diagram of Helium-Neon Laser

Neon atom have two excited energy levels (E_3 and E_5) which are very close to the excited energy levels of the Helium atom. The excited Helium atoms transfer their excitation energy to the Neon atoms by collisions - **Resonance excitation**.

Energy from the He-Ne laser is emitted at wavelengths which correspond to the energy difference between the levels:

$$E_5 - E_4 = > \lambda_1 = 3.391 \text{ [}\mu\text{m]}$$

$$E_5 - E_2 = > \lambda_2 = 0.632 \text{ [}\mu\text{m]}$$

$$E_3 - E_2 = > \lambda_3 = 1.152 \text{ [}\mu\text{m]}$$

Later we shall understand how to choose the specific required wavelength at the output of the laser.

c) *Chemical Excitation* :

It is a method used during reaction between chemical materials, and as a result the reaction producing a new chemical material in the excited state.

Exp. The **Hydrogen Fluoride lasers**.



(as will be describe in [Chapter 6.1.9](#))

d) Electric current in diode lasers:

When **Forward Biased Voltage** is applied across the p-n junction, the population of the energy bands changes.

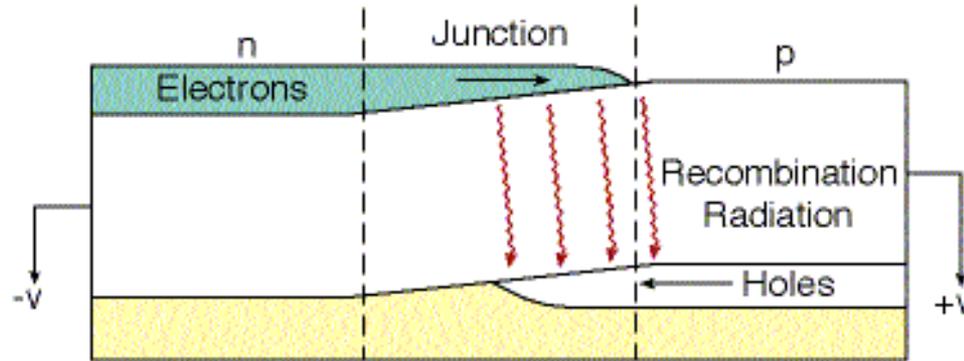


Figure 6.23: Energy band of a p-n junction which is attached to a forward bias voltage.

When an electron from the conduction band in the "n" side is injected through the junction to an empty "hole" in the valence band on the "p" side, a process of **Recombination** of (electron + hole) takes place. As a result of this recombination process, energy is released in the form of laser radiation.

(we will see in [chapter 6.3](#))

3.3 Resonator or Feedback Mechanism :

The **feedback mechanism** returns part of the coherent laser radiation which was created inside the active medium, back.

Usually the feedback is done by **using mirrors at both ends of the active medium**. These mirrors are aligned so that the radiation is moving back and forth between them. In this way an **optical cavity** is created (as explained in details in chapter 4).

Usually one mirror is 100% reflecting, so all the radiation coming toward the mirror is reflected back to the active medium. The other mirror is partially reflecting (10%-99%), according to the laser type.

The part of the radiation which is not reflected back into the optical cavity, is transmitted out, and it is the **laser output**.

The feedback allows each photon to pass many times through the active medium, so enough amplification will result.

Because of the feedback mechanism, only photons which move between the mirror remain in the active medium, which give the **directionality of the output beam**

. 3.4 Output Coupler

Output coupler is a main to transmit electromagnetic radiation out of the laser.

The standard output coupler uses a **partially reflecting mirror** as explained in section 3.3. The part of the beam which is not reflected back into the active medium, is transmitted out.

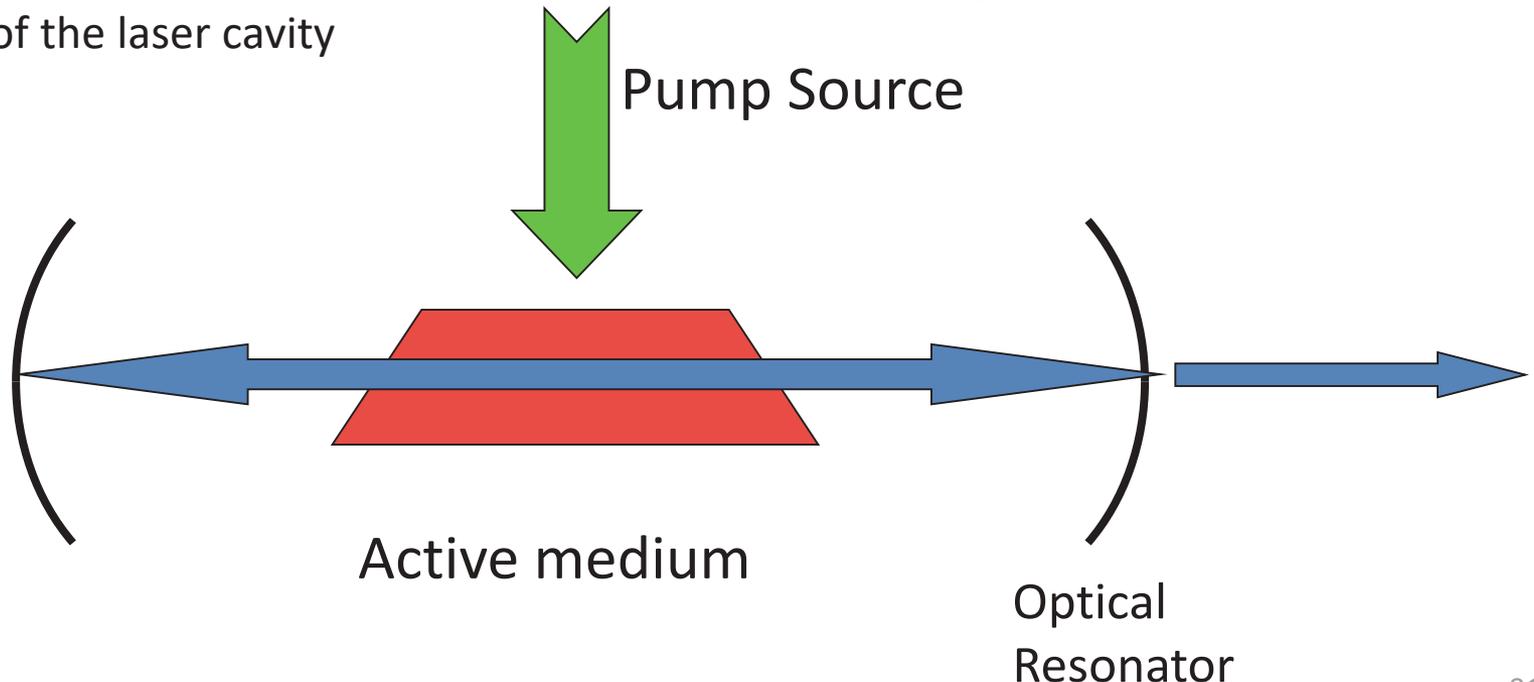
In a **continuous wave laser** (in which the radiation is emitted continuously), most of the radiation is reflected back into the cavity, and only a **few percent** is transmitted out.

In certain pulsed lasers most of the radiation within the cavity at a specific moment is transmitted out in a pulse ([see chapter 7.4](#)).

3.5 Operation of a Laser System - Interactive Applets:

In the following set of interactive demos, the **basic principles of operation of a laser system** are described.

- At first, the **active medium** is not excited, no emission is observed, as can be seen below:
- When the **pump energy** is turned on, energy flows into the system, but the energy is not enough to create **population inversion**. In this case only **spontaneous emission** can be seen.
- When the pump energy increases, a **population inversion** occurs, and we can see both spontaneous and **stimulated emission**.
- When mirrors are added, they form an **optical cavity** around the active medium. The **stimulated emission becomes dominant in a specific direction** (the optical axis of the laser).
- When the amount of reflectivity of one of the mirrors is changed, part of the laser radiation is emitted out of the laser cavity



Chapter 4:

Optical Cavity and laser Modes

Before explaining the details of each of the laser components, few terms must be understood:

- **Active medium** and its importance for the lasing process.
- **Population inversion** in the active medium, and the conditions for making it (Excitation of the active medium with an external source of energy).
- **Stimulated emission** and light amplification caused by it.
- **Laser system** and its components (**chapter 3**).

Chapter 4 explains the **conditions which determine the radiation modes** created in common lasers.

First, a few concepts from wave theory are needed:

- What are **standing waves**.
- What are the conditions for creating standing waves.
- How standing waves in a laser cavity are determined by the laser design.

Second, the properties of the optical signal which is amplified while passing back and forth through the active medium are discussed.

Third, **longitudinal modes** are created in the laser cavity. Their importance and methods for controlling them are discussed.

Forth, the distribution of energy along the cross section of the beam, which determine the **transverse modes**, is explained.

At the end of chapter 4 the **common optical cavities** are described and the way to test their stability.

4.1 Standing Waves

As known from the **wave theory**, when two waves with similar amplitudes and same frequency are moving in the same path at opposite directions, their **interference** creates an oscillating wave which appears as if it is fixed in space - a **standing wave**.

In figure 4.1 a pictorial demonstration is given for the creation of a standing wave during one complete cycle (T).

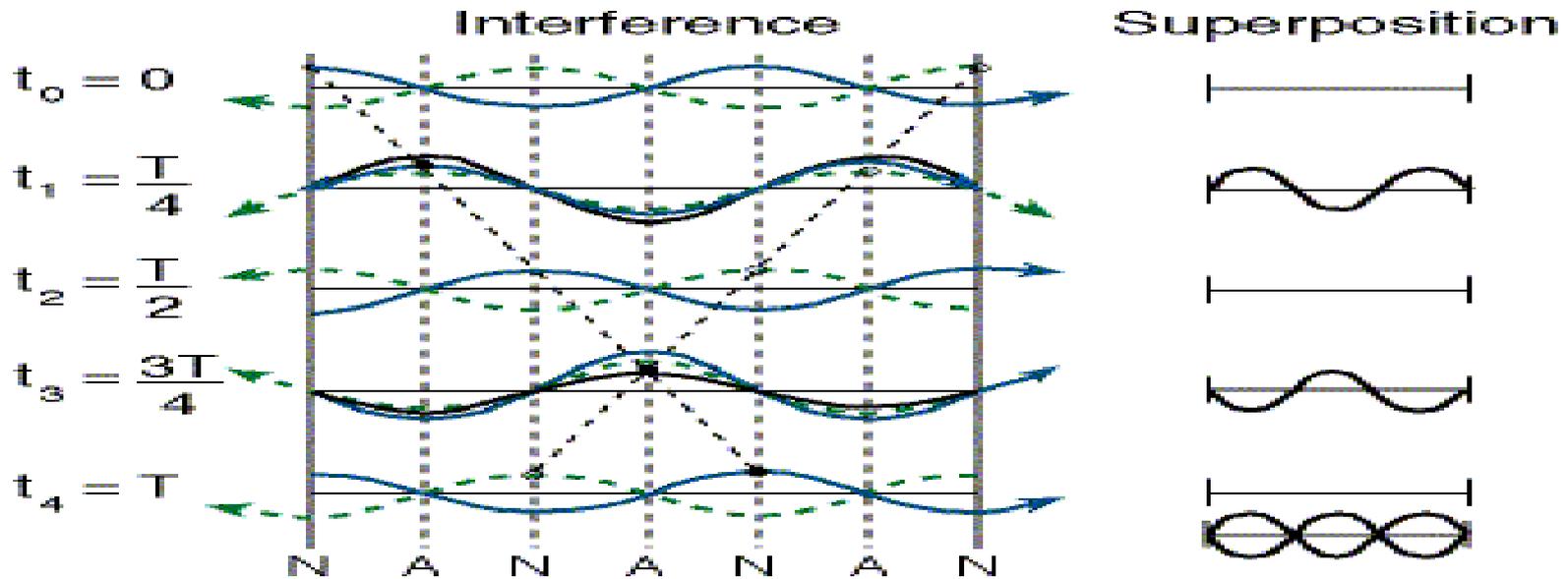


Figure 4.1: Creating standing wave from two waves moving in opposite directions.

In figure 4.1 the wave is described every $1/4$ cycle.

The **black spot** shows how the disturbance of the peak of the wave advances during a complete period (T).

The **solid line** describes a wave moving to the left.

On the right side of figure 4.1 **the superposition of the two waves** is shown.

Like a standing wave in a string attached to fixed points at both sides, the fixed points of a standing wave are called **Nodes**.

The distance between adjacent Nodes is half the wavelength of each of the interfering waves. Thus, it is the same as half a wavelength of the standing wave they create.

Standing waves in a laser

In a laser an **optical cavity** is created by **two mirrors at both ends of the laser**.

Laser mirrors

These mirrors serve two goals:

1.They increase the length of the active medium, by making the beam pass through it many times.

2.They determine the boundary conditions for the electromagnetic fields inside the laser cavity.

A cavity with two mirrors is called: **Fabry – Perot resonator**, and it will be discussed in [section 8.3](#).

The axis connecting the centers of these mirrors and perpendicular to them is called **Optical Axis** of the laser. **The laser beam is ejected out of the laser in the direction of the optical axis**.

An electromagnetic wave which move inside the laser cavity from right to left, is reflected by the left mirror, and move to the right until it is reflected from the right mirror, and so on.

Thus, **two waves of the same frequency and amplitude are moving in opposite directions**, which is the **condition for creating a standing wave**.

Conditions for Standing Waves

In order to create a **standing wave**, the wave must start with the same **phase at the mirror**.

Thus, the optical path from one mirror to the other and back must be an **integer multiplication of the wavelength**.

Since the **Length between the mirrors is constant (L)**, the suitable wavelengths, which create standing waves, must fulfill the condition:

$$\lambda_m = 2L/m$$

L = Length of the optical cavity.

m = Number of the mode, which is equal to the number of half wavelengths inside the optical cavity. The first mode contains half a wavelength, the second mode 2 halves (one) wavelength.

λ_m = Wavelength of mode m inside the laser cavity.

Conditions for Standing Waves (continuous)

The wavelength of the laser mode (λ_m) is measured inside the active medium

Wavelength in matter (λ_m) is equal to:

$$\lambda_m = \lambda_0/n$$

λ_0 = Wavelength of light in vacuum.

n = **Index of refraction** of the active medium.

c = Velocity of light in vacuum.

Since:

$$c = \lambda_0 \nu = n \lambda_m \nu_m$$

The **frequency** of the longitudinal mode is:

$$\nu_m = \frac{c}{n \lambda_m}$$

Inserting λ_m into the last equation:

$$\nu_m = m \cdot \left(\frac{c}{2 \cdot n \cdot L} \right)$$

Basic Longitudinal Mode

The last equation of the **frequency of longitudinal modes** is:

$$\nu_m = m \cdot \left(\frac{c}{2 \cdot n \cdot L} \right)$$

The mathematical expression in parenthesis is the **first mode of oscillation** available for this optical cavity:

$$\nu_1 = \frac{c}{2 \cdot n \cdot L}$$

This mode is called **basic longitudinal mode**, and it has the basic frequency of the optical cavity.

Conclusion:

The frequency of each laser mode is equal to integer (mode number m) times the frequency of the basic longitudinal mode.

From this conclusion it is immediately seen that **the difference between frequencies of adjacent modes (mode spacing) is equal to the basic frequency of the cavity:**

$$\Delta\nu = c/(2nL)$$

Standing waves in a string

In figure 4.2 the first 5 standing waves are shown.

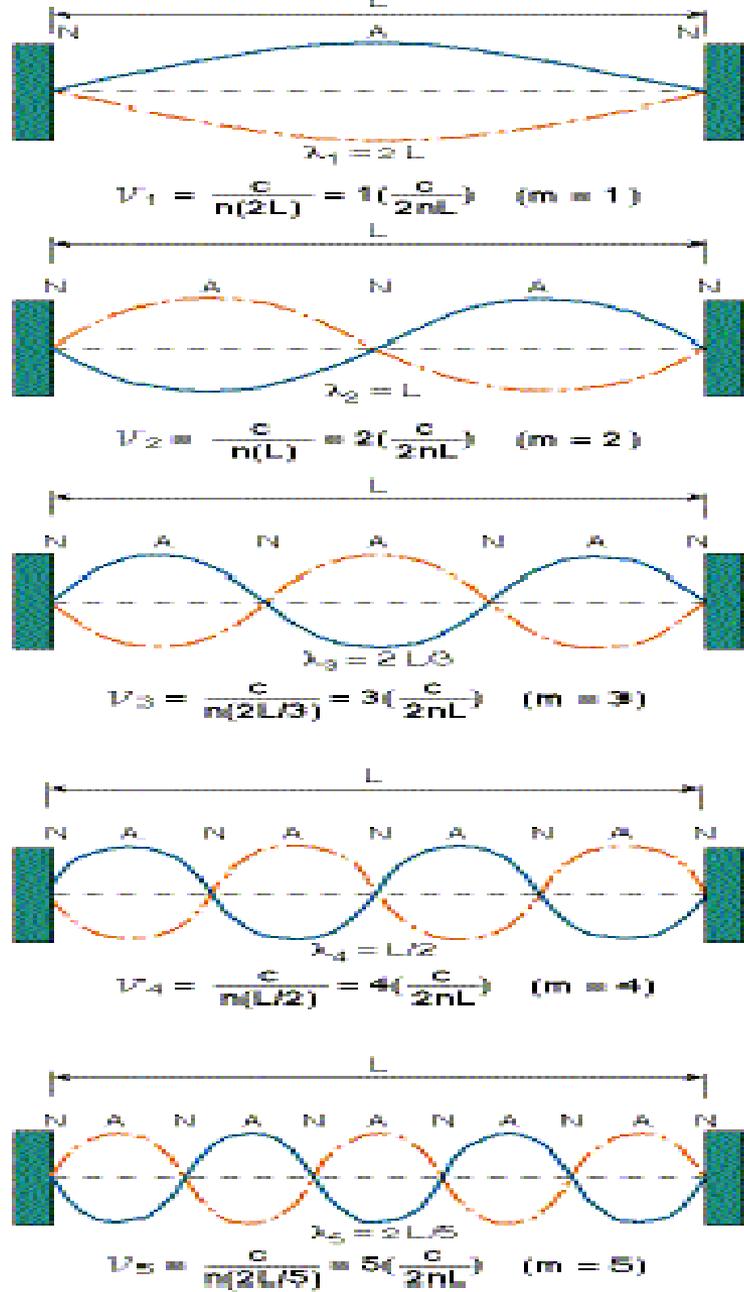


Figure 4.2:
Longitudinal modes
in an optical cavity of
length L.

A=Anti-Node N=Node

Wave Velocity = $\frac{c}{n}$

Frequency = $\nu = \frac{c}{n\lambda}$

$\Delta\nu \approx \frac{c}{2nL}$

Standing waves in a string (continuous)

These are equivalent to the **longitudinal laser modes** which are the modes along the optical axis of the laser.

L = Length of the optical cavity.

n = Index of refraction.

m = **Number of the mode**, which is equal to the number of half wavelengths inside the optical cavity.

λ_m = Wavelength of the **m** mode inside the laser cavity.

The necessary condition for these standing waves is a Node at each end (mirror)

Attention !

Until now it was assumed that the index of refraction (n) is constant along the optical cavity.

This assumption means that the length of the active medium is equal to the length of the optical cavity.

There are lasers in which the mirrors are not at the ends of the active medium, so L_1 is not equal to the length of the cavity (L).

In such case each section of the cavity is calculated separately, with its own index of refraction:

$$\Delta \nu_{MS} = \frac{c}{2 \cdot n_1 \cdot L_1 + 2 \cdot n_2 \cdot L_2}$$

Question 4.1

The length of the optical cavity of a Nd-YAG laser is 30 [cm].

The length of the **laser rod** which makes the **active medium** is 10 [cm].

The **index of refraction** of the laser rod is 1.823.

The rest of the cavity is **air** which have an index of refraction of 1.0.

Calculate the difference in frequencies between adjacent modes.

Example 4.1 Frequencies and wavelengths of possible modes in an optical cavity

The length of an optical cavity is 25 [cm].

The index of refraction is 1.0.

Calculate the frequencies ν_m and wavelengths λ_m of the following modes:

1. $m = 1$.
2. $m = 10$.
3. $m = 100$.
4. $m = 10^6$.

Solution to Example 4.1

$$\lambda_m = \frac{2L}{m}$$

$$\lambda_1 = 2 \cdot \frac{0.25}{1} = 0.5$$

$$\nu_m = m \cdot \left(\frac{c}{2L} \right)$$

$$\nu_1 = 6 \cdot 10^8 \text{ [Hz]}$$

Radio Wave

$$\lambda_{10} = 2 \cdot \frac{0.25}{10} = 0.05$$

$$\nu_2 = 6 \cdot 10^9 \text{ [Hz]}$$

Short Wave Communication

$$\lambda_{100} = 2 \cdot \frac{0.25}{100} = 5 \cdot 10^{-3}$$

$$\nu_3 = 6 \cdot 10^{10} \text{ [Hz]}$$

Microwaves

$$\lambda_{10^6} = 2 \cdot \frac{0.25}{10^{-7}} = 0.5 \cdot 10^{-6}$$

$$\nu_4 = 6 \cdot 10^{14} \text{ [Hz]}$$

Green Color

Number of Possible Modes

From example 4.1 we see that **for signals in the visible spectrum, the mode number m is very High, of the order of millions (!)**.

In example 4.2 it can be seen for Helium Neon laser which is common in science laboratories at school. Actually, it will be explained (in chapter 5) that not all possible modes according to the formula will appear in the laser beam, but there are more limiting conditions.

Example 4.2 Calculating the Number of Longitudinal Modes in He-Ne laser

The length of the optical cavity in He-Ne laser is 30 [cm].
The emitted wavelength is 0.6328 [μm].

Calculate:

1. The difference in frequency between adjacent longitudinal modes.
2. The number of the emitted longitudinal mode at this wavelength.
3. The laser frequency.

Solution to example 4.2:

Since He-Ne laser is a gas laser, the hidden information is the index of refraction of the active medium. It is quite accurate to approximate the index of refraction by 1.0.

1. The equation for difference in frequency is the same as for the basic mode:

$$\Delta\nu = c/(2nL) = 3*10^8 / (2*1.0*0.3) = 0.5*10^9 \text{ Hz} = 0.5 \text{ GHz}$$

2. From the equation for the wavelength of the m^{th} mode:

$$\lambda_m = 2L/m$$

$$m = 2L/\lambda_m = 2*0.3 / 0.6328*10^{-6} = 0.948*10^6$$

which means that **the laser operate at a frequency which is almost a million times the basic frequency of the cavity.**

3. The **laser frequency** can be calculated in two ways:

a) By multiplying the mode number from section 2 by the basic mode frequency:

$$\nu = m * \Delta\nu = (0.948*10^6)(0.5*10^9 \text{ Hz}) = 4.74*10^{14} \text{ Hz}$$

b) By direct calculation:

$$\nu = c/\lambda = 3*10^8 / 0.6328*10^{-6} = 4.74*10^{14} \text{ Hz}$$

Allowed Frequencies inside a Laser Cavity

In figure 4.3 a graphic representation is given for allowed frequencies inside a laser cavity.

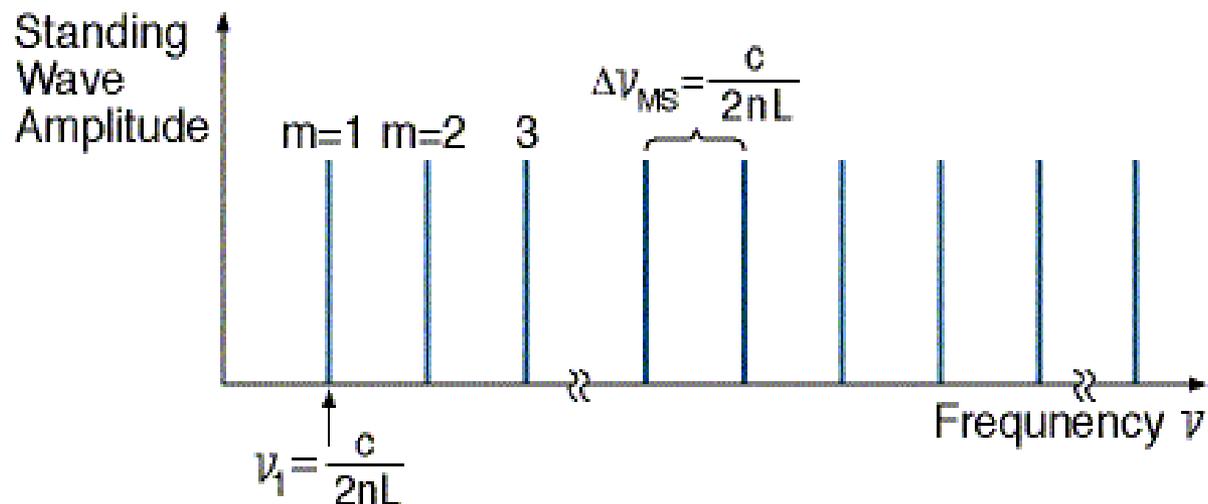


Figure 4.3: Allowed Longitudinal modes inside a Laser Cavity of length (L) and index of refraction (n).

In practice, the frequencies are not defined mathematically as single frequencies, but each have a **width of frequencies** around the possible modes, as will be explained [in chapter 5](#).

Not all these allowed frequencies will be emitted from the laser, since there are more limiting conditions which will be explained [in chapter 5](#)

Question 4.2: Nd Glass Laser

The length of the optical cavity of a Nd-Glass laser is 50 [cm].

The index of refraction is 1.5.

Calculate:

1. The basic frequency of the cavity.
2. The frequencies of the next 4 modes.
3. The difference between two adjacent modes.

Question 4.3: Ruby laser

The length of the optical cavity of a Ruby laser is 15 [cm].

The index of refraction is 1.76.

The mirrors are coated at the ends of the Ruby crystal (Rod).

Calculate the difference between two adjacent modes.

Compare it to the results you got in question 4.2, and 4.1. What conclusions can you deduce from this comparison?

Question 4.4: Ar⁺ Ion laser

The difference between adjacent modes in Ar⁺ Ion laser is 100 [MHz].

The mirrors are at the end of the laser tube.

Calculate:

1. The length of the laser cavity.
2. The mode number of the wavelength 488 [nm].
3. The change in difference between adjacent modes when the tube is shortened to half its length.

Summary: Longitudinal Modes in a Laser

- **Longitudinal modes** are standing waves along the optical axis of the laser.
- **Standing waves** are formed when two waves with the same frequency and amplitude are moving against each other.
- **Optical cavity** is created by two mirrors at both sides of the laser.
- The **standing waves** inside a laser are created when the electromagnetic radiation is forced to move back into the cavity from the mirrors.
- **The allowed frequencies inside an optical cavity are determined by the length of the cavity (L) and the index of refraction of the active medium.**
- Only those frequencies which create **nodes at both mirrors** are allowed. Thus, the cavity length must be an integer multiplication of half their wavelengths.
- **The allowed frequencies are spaced at constant interval, which is equal to the basic frequency of the cavity.**

4.2 Longitudinal Modes in a Laser

Section 4.1 explains that **only specific frequencies are possible inside the optical cavity of a laser**, according to **standing wave condition**.

From all these possible frequencies, only those that fulfill another condition will be emitted as laser lines:

Only those frequencies (modes) that have **amplification above certain minimum, to overcome absorption** (as explained in chapter 5), will be emitted out of the laser.

This minimum amplification is defined as **lasing threshold**.

The condition of minimum amplification means that the amplification is equal to losses, so that **in a round trip path inside the cavity** $G_L = 1$

Gain Curve of the Active Medium

In figure 4.4 the gain curve of the active medium, as a function of frequency is marked with the **lasing threshold** and **possible longitudinal modes of the laser**.

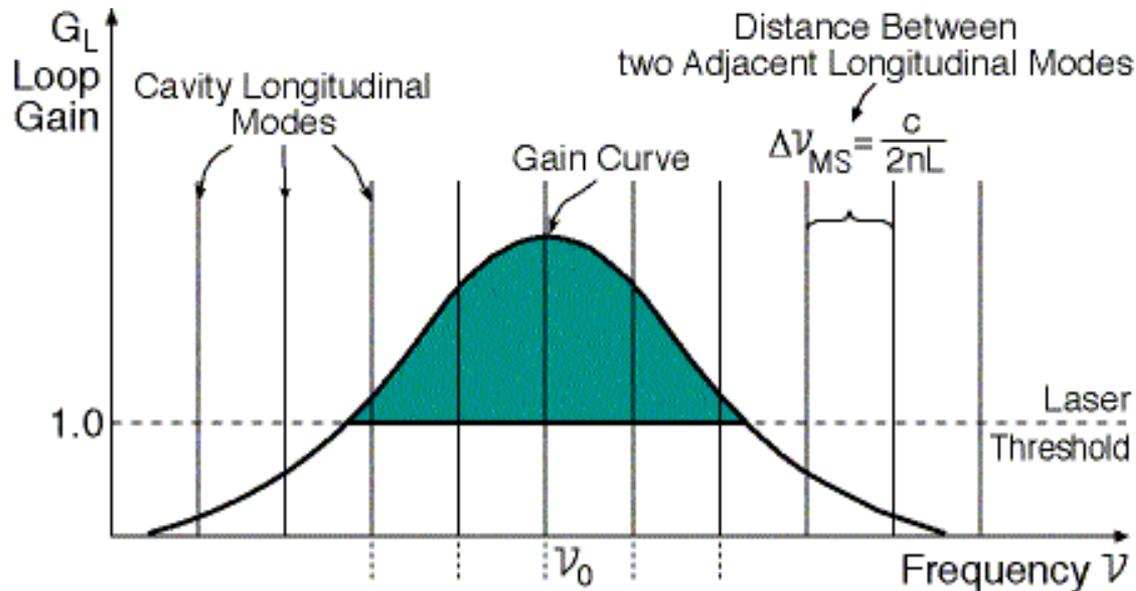


Figure 4.4: Gain Curve of a laser.

The height of each lasing line depends on the losses in a round trip inside the cavity, including the emitted radiation through the output coupler. The shape and properties of the gain curve is explained in [chapter 5](#).

The marked region under the curve and above the lasing threshold include the range where lasing can occur.

The height of the gain curve depend on the length of the active medium and its excitation.

The possible longitudinal modes of the laser are marked as perpendicular lines at equal distances from each other.

In section 4.1 the condition of standing waves for longitudinal modes was determined by the length of the cavity, and its index of refraction.

In figure 4.4 only 5 frequencies from those allowed inside the cavity, are above the lasing threshold. Thus only these 5 frequencies can exist at the output of this laser

4.2.1 The Number of Longitudinal Optical Modes

In figure 4.5 the spectral distribution of the lines emitted out of the laser described in figure 4.4 is shown.

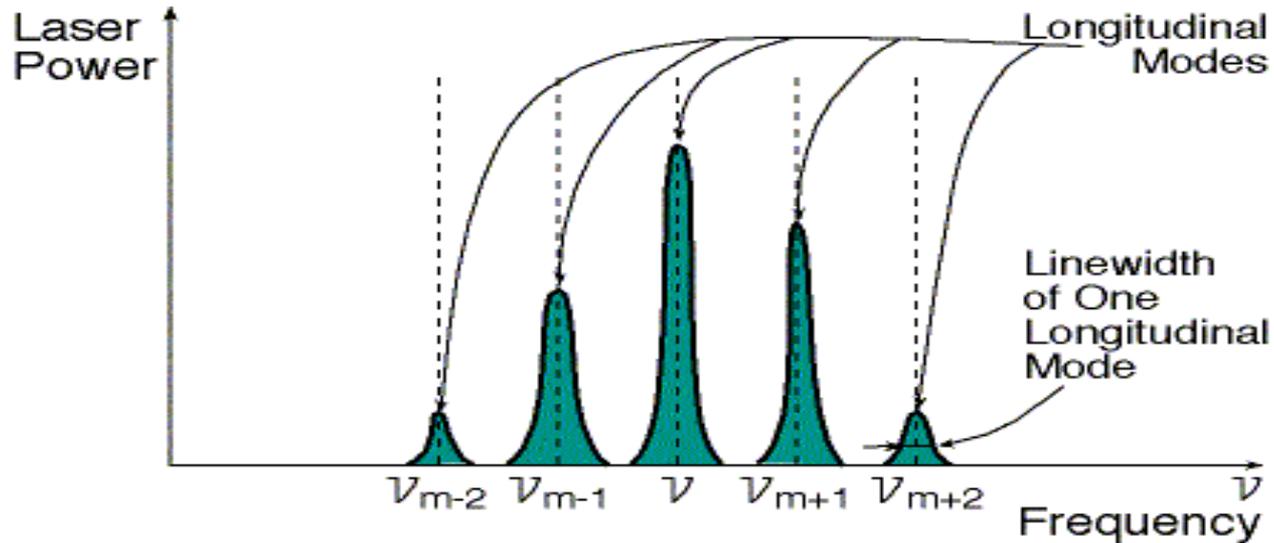


Figure 4.5: Spectral distribution of laser lines.

In this laser 5 frequencies are allowed at the output, and they are spaced at equal distances, which are equal to the mode spacing:

$$\Delta \nu_{MS} = \frac{c}{2nL}$$

The Fluorescence linewidth $\delta\nu_{LW}$ of a Laser (Broadening)

The **gain curve** is a plot of gain as a function of frequency.

The fluorescence linewidth $\delta\nu_{LW}$ of a Laser is the full width of the amplification curve at half the maximum height (FWHM).

This fluorescence linewidth determines the maximum width of all the emitted laser lines (coherent radiation at the output of the laser at all possible longitudinal modes). Detailed explanation can be found in [chapter 5](#).

The **approximate number of possible laser modes (N)** is given by the **width of the fluorescence line divided by the distance between adjacent modes:**

$$N = \delta\nu_{LW} / \Delta\nu_{MS}$$

MS = Mode Spacing.

LW = Line Width.

$\delta\nu_{LW}$ = **Fluorescence Linewidth.**

$\Delta\nu_{MS}$ = **Distance between adjacent longitudinal modes.**

$$\Delta\nu_{MS} = \frac{c}{2nL}$$

In [example 4.3](#) this approximation is used.

Example 4.3: Number of longitudinal laser modes

The length of the optical cavity in He-Ne laser is 55 [cm].
The fluorescence linewidth is 1.5 [GHz].
Find the approximate number of longitudinal laser modes.

Solution to example 4.3

The distance between adjacent longitudinal modes is:

$$\Delta \nu_{MS} = c / (2nL) = 3 \cdot 10^8 / (2 \cdot 1.0 \cdot 0.55) = 2.73 \cdot 10^8 = 0.273 \text{ GHz}$$

The approximate number of longitudinal laser modes:

$$N = \delta \nu_{LW} / \Delta \nu_{MS} = 1.5 \text{ GHz} / 0.273 \text{ GHz} = 5.5 \longrightarrow 5$$

There are 5 longitudinal laser modes in this laser, and it is the laser that was described in [figures 4.4](#) and [4.5](#).

Controlling the Number of Longitudinal Modes of the Laser

The way to control the number of longitudinal modes in a laser is to **control the length of the laser cavity**. This can be done in two ways:

1. Changing the length of the cavity by physically moving the mirrors to a new position.

Doubling the length of the cavity reduces to half the distance between adjacent longitudinal modes, thus doubling the number of possible laser modes under the fluorescence curve.

It is clear that a **single mode laser** can be made by reducing the length of the cavity, such that only one longitudinal mode will remain under the fluorescence curve with $G_L > 1$.

In such single mode laser the exact distance between the mirrors is critical, since if there will be no modes to fulfill the condition, no lasing will occur.

The disadvantage of this method is that **the short length of the cavity limits the power output of the laser.**

2. Adding an extra mirror inside the laser cavity.

Adding an extra mirror inside the laser cavity is described in figure 4.6.

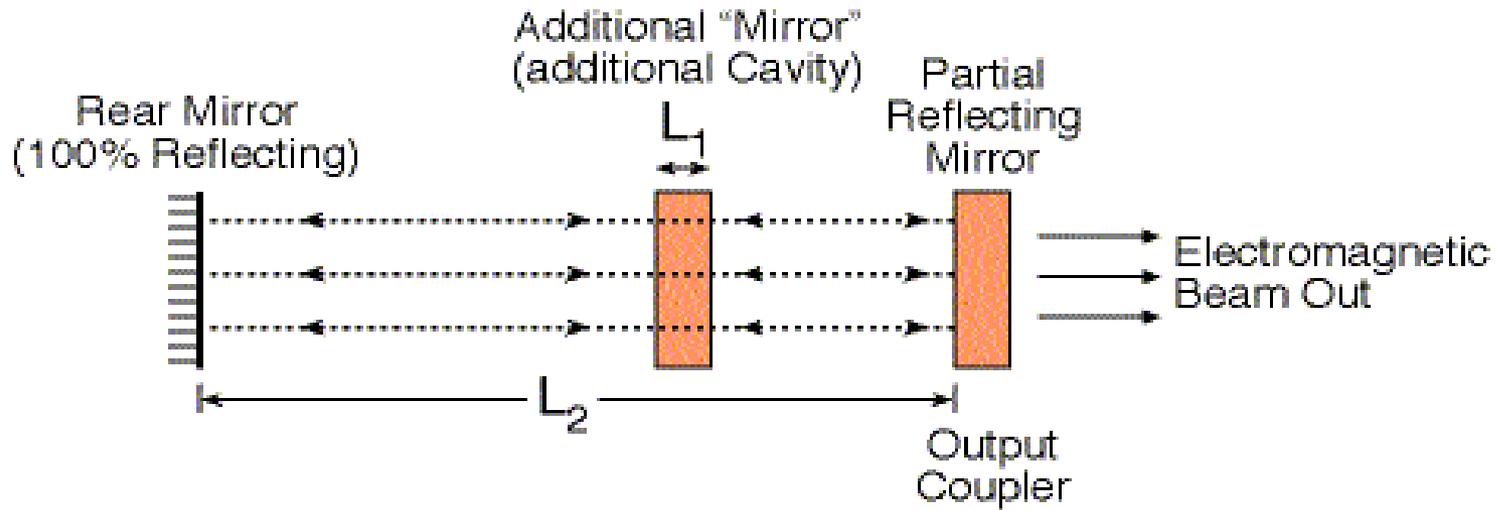


Figure 4.6: Operating a laser in single mode by 3 mirrors.

This method **determines two cavity lengths at the same time: L_1 , and L_2 .**

The length L_1 is chosen such that **only one longitudinal mode will be under the fluorescence curve of the laser.**

The laser as a system need to fulfill the conditions for both cavities.

This arrangement require strict stable positions for the mirrors, and is used wherever **high power in a single mode is needed,** especially in solid state lasers.

Etalon

Etalon is an additional short optical cavity inserted into the laser optical cavity.

Etalon is composed of **two parallel surfaces (mirrors)**.

The distance between the two surfaces is about the size of the gain curve of the laser. Thus, only one longitudinal mode is under the gain curve each time, as seen in figure 8.13.

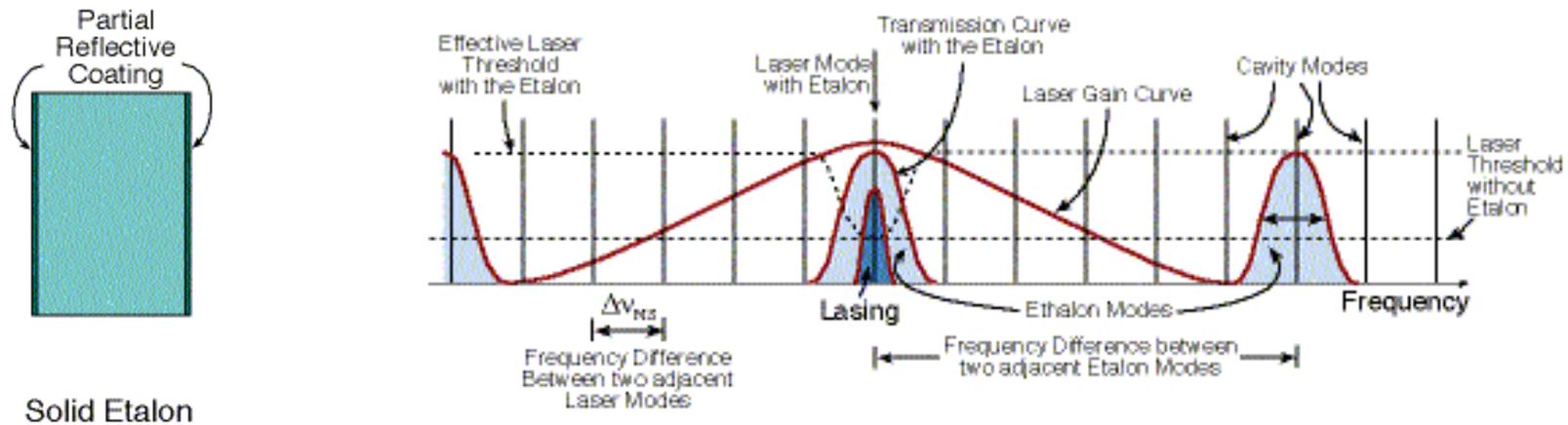


Figure 8.13: Choosing single Longitudinal Mode with an Etalon.

There are **two optical cavities inside the laser with etalon**.

Longitudinal mode can exist in the double cavity structure only if it **simultaneously fulfill the conditions of standing wave for both cavity lengths**.

The **linewidth of the etalon longitudinal mode** is determined by the **reflection of its surfaces**. The higher the reflection, the narrower the longitudinal mode of the etalon.

4.2.2 difference between Adjacent Longitudinal Modes.

Although most lasers operate in many longitudinal modes, we still see them as **monochromatic sources**.

Example 4.4 show how small is the distance between wavelengths of adjacent longitudinal modes.

Example 4.4: Distance between Wavelengths of Adjacent Longitudinal Modes

The length of He-Ne cavity is 50 [cm]. If the wavelength of the mode m is exactly 632.8 [nm], calculate the wavelength of mode $m+1$.

Solution ;

The difference in frequencies between adjacent longitudinal modes is:

$$\Delta \nu_{MS} = \frac{c}{2 \cdot n \cdot L} = \frac{3 \cdot 10^8 \frac{m}{s}}{2 \cdot 1 \cdot 0.5 m} = 3 \cdot 10^8 \text{ Hz} = 300 \text{ MHz}$$

$$\lambda_m = 0.6328000 \cdot 10^{-6} [m]$$

$$\nu_m = \frac{c}{\lambda_m} = \frac{3 \cdot 10^8 \frac{m}{s}}{6.328000 \cdot 10^{-7}} = 4.7408344 \cdot 10^{14} \text{ Hz}$$

$$\nu_{m+1} = \nu_m + \Delta \nu$$

$$\nu_{m+1} = 4.7408344 \cdot 10^{14} \text{ Hz} + 3 \cdot 10^8 \text{ Hz} = 4.7408374 \cdot 10^{14} \text{ Hz}$$

$$\lambda_{m+1} = \frac{c}{\nu_{m+1}} = 3 \cdot \frac{10^8}{4.7408374 \cdot 10^{14}} = 6.327996 \cdot 10^{-7} m$$

$\lambda_m = 632.8000 [nm]$, $\lambda_{m+1} = 632.7996 [nm]$

The difference in wavelengths between adjacent longitudinal modes is $4 \cdot 10^{-13} [m]$ (1).

4.2.3 The importance of Longitudinal Optical Modes at the Output of the Laser

The importance of Longitudinal modes of the laser is determined by the specific application of the laser.

In [chapter 8](#) the main applications of lasers are discussed, so only a few principles will be mentioned here:

1. In **most high power applications** for [material processing](#) or [medical surgery](#), the laser is used as a mean for **transferring the energy to the target**. Thus there is no importance for the longitudinal laser modes.

2. In applications where **interference** of electromagnetic radiation is important, such as [holography](#) or [interferometric measurements](#), the longitudinal modes are very important. In these applications, the **coherence length** of the radiation is the important property, and it is determined by the linewidth of the laser radiation (inversely proportional to it).

In these applications a single mode laser is used, and special techniques are used to reduce its linewidth, thus increasing its coherence length.

3. In [spectroscopic and photochemical applications](#), a very defined wavelength is required. This wavelength is achieved by operating the laser in **single mode**, and than controlling the length of the cavity , such that this mode will operate at exactly the required wavelength. The structure of longitudinal laser modes is critical for these applications.

4. When high power short pulses are needed, **mode locking** is used (see [section 7.4.2](#)). This process causes **constructive interference** between all the modes inside the laser cavity. The structure of longitudinal laser modes is important for these applications.

Summary

At the output of the laser, there are only certain frequencies of the electromagnetic radiation, which are called **longitudinal modes**:

- Those frequencies for which the active medium have **enough gain** to compensate for all the losses.
- Those frequencies which create **standing waves in the optical cavity of the laser**.

Usually in a laser some longitudinal modes are operated simultaneously, and the difference between them is determined by the length of the cavity and the index of refraction of the active medium.

The width of the fluorescence line is characterized by the active medium

Question 4.5: He-Ne laser

The width of the fluorescence line of He-Ne laser is 1.5 [GHz].

The length of the optical cavity is 75 [cm].

calculate:

- 1.The difference between adjacent longitudinal modes.
- 2.The approximate number of longitudinal modes.

Question 4.6: Nd YAG laser

The properties of Nd YAG laser are:

Wavelength: 1.06 [μm].

Cavity length: 42 [cm].

Length of the laser crystal: 125 [mm].

Index of refraction of the crystal: 1.823.

Width of the fluorescence line: 30 [GHz].

Output coupler transmission: 4%.

Losses inside the cavity: 0.5%.

Calculate:

- 1.The difference between adjacent longitudinal modes.
- 2.The approximate number of modes in the cavity.
- 3.The wavelength difference between adjacent longitudinal modes.
- 4.The wavelength difference between the mode with the maximum frequency, and the mode with the minimum frequency (the range of wavelengths emitted from the laser)

Laser Modes

Inside a laser cavity the **laser modes** are characterized by:

1. Frequency (ν), or wavelength (λ).

2. Longitudinal Mode - Intensity distribution measured **along the optical axis of the laser**.

3. Transverse Mode - Intensity distribution measured **over beam cross section** (perpendicular to the optical axis of the laser).

4.3 Transverse Modes of a Laser

In previous sections the distribution of intensity was examined along the optical axis of the laser.

Longitudinal modes were described as standing waves between the laser mirrors.

This section examine the **transverse distribution of intensity**, in cross section of the beam, perpendicular to the optical axis of the laser.

These **transverse modes** are created by the width of the cavity, which enables a few “diagonal” modes to develop inside the laser cavity.

A little misalignment of the laser mirrors causes different path length for different “rays” inside the cavity. Thus, the distribution of intensity is not the perfect Gaussian distribution to be explained shortly.

4.3.1 Shape of Transverse Electromagnetic Modes

In cross section, the laser radiation has specific distribution into regions with high intensity and regions with no radiation at all.

Figure 4.7 show the energy distribution of the first few transverse electromagnetic modes. **The dark areas** mark places where laser radiation hit.

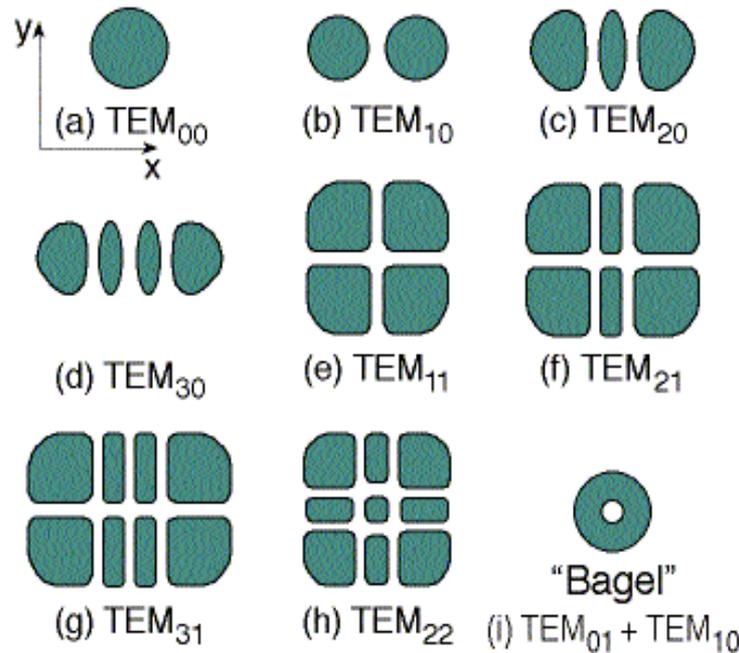


Figure 4.7: Transverse Electromagnetic Laser Modes.

When the laser output power is of the order of several Watts, the distribution of energy in the beam cross section can be measured by a short illumination of a stick of wood with the laser.

For low power lasers special screens which respond to the specific wavelength of the laser can be used. The screen change its properties in places exposed to the laser beam, an a similar picture to figure 4.7 is visible.

The shape of energy distribution in the beam cross section is called:

Transverse Electro-Magnetic Modes(TEM)

Transverse Electro-Magnetic Modes (TEM)

Transverse Electro-Magnetic Modes (TEM) describe the shape of energy distribution in the beam cross section.

Figure 4.7 show the energy distribution of the first few transverse electromagnetic modes. The dark areas mark places where laser radiation hit.

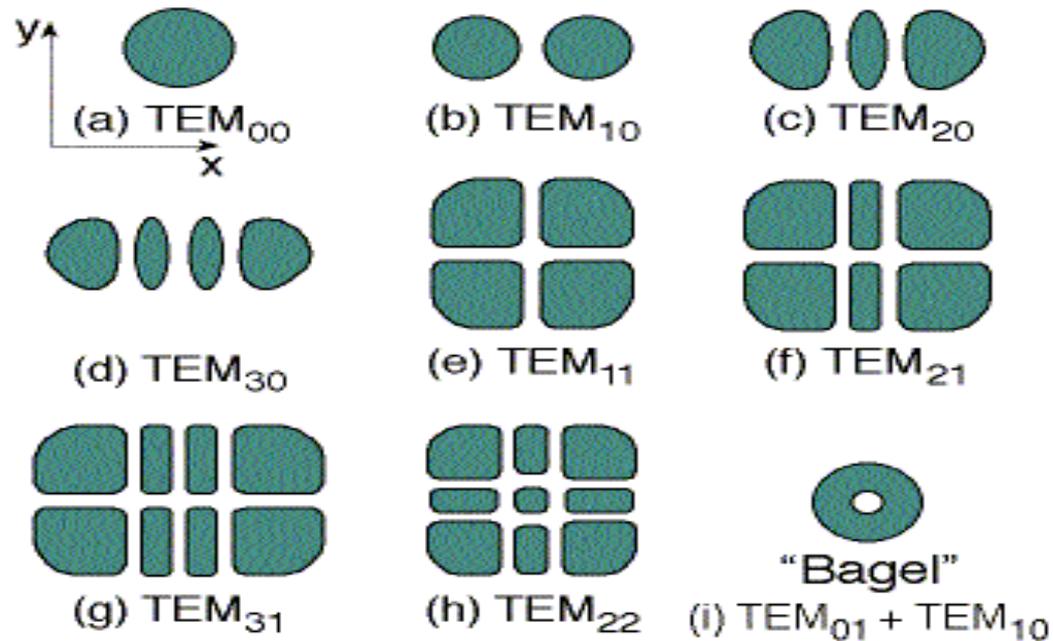


Figure 4.7: Transverse Electro-Magnetic Laser Modes.

Each transverse mode (TEM) is marked with two indexes: TEM_{mn} .

m, n , are integer numbers.

Assuming the beam advance in z direction (optical axis):

m = Number of points of zero illumination (between illuminated regions) along x axis.

n = Number of points of zero illumination (between illuminated regions) along y axis.

There is one transverse mode which does not fit this classification, and it has a special name (according to its shape) due to its importance: "Bagel". It is composed of TEM_{01} and TEM_{10} oscillating together (see figure 4.7).

Electric Field Distribution of TEM modes

Radiation intensity of the laser beam is a measure of the square of the electric field of the electromagnetic radiation.

The dark regions in figure 4.7 are areas with high electric field.

In figure 4.8 both intensity and electric field are shown for each of a few transverse modes

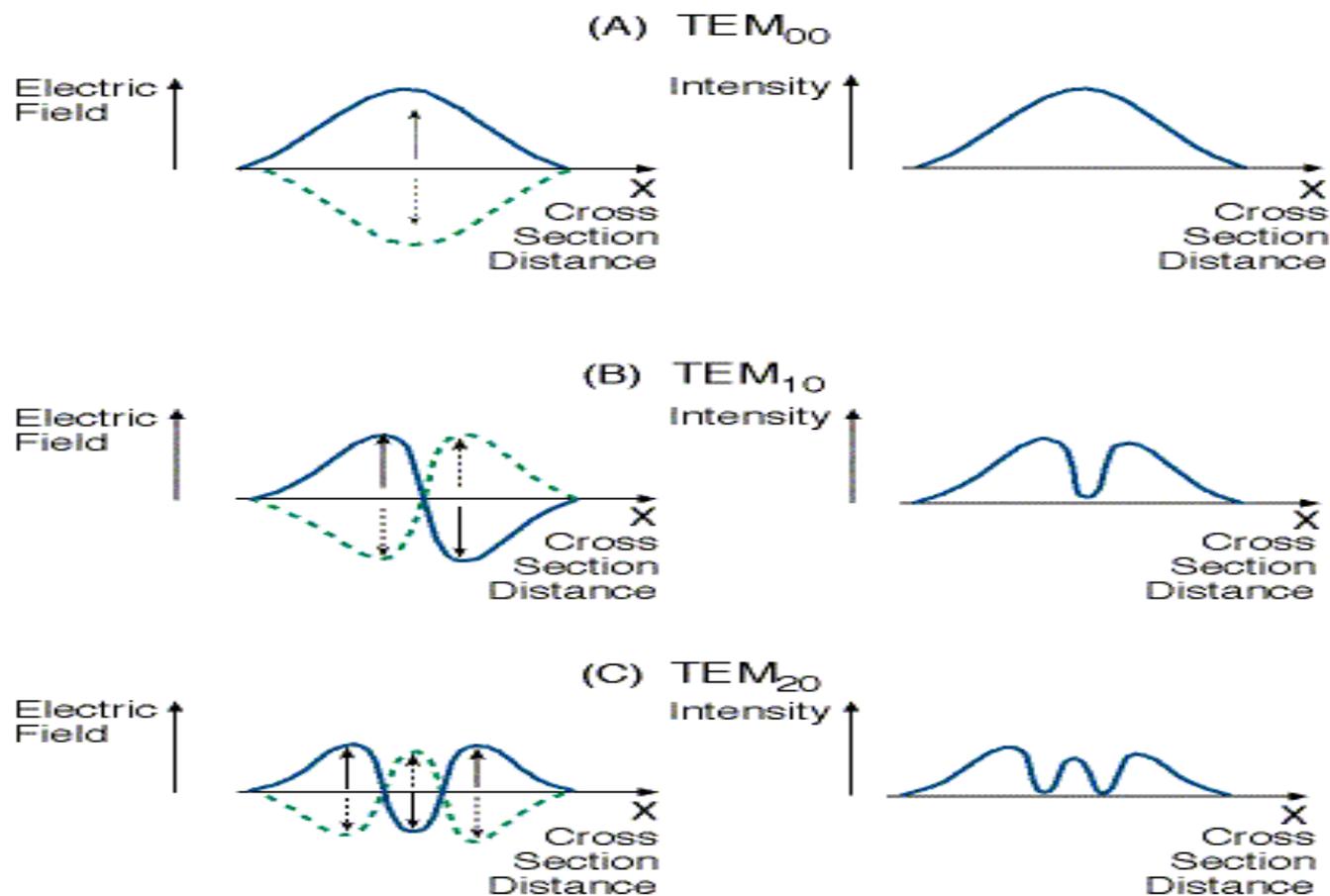


Figure 4.8: Intensity and Electric fields of few transverse modes.

4.3.2 Control of the Transverse Modes of the Laser

When a laser operates in several transverse modes, **the total intensity profile is a superposition of all existing transverse modes.**

Figure 4.9 describes the intensity distribution of 3 lower modes, and their superposition.

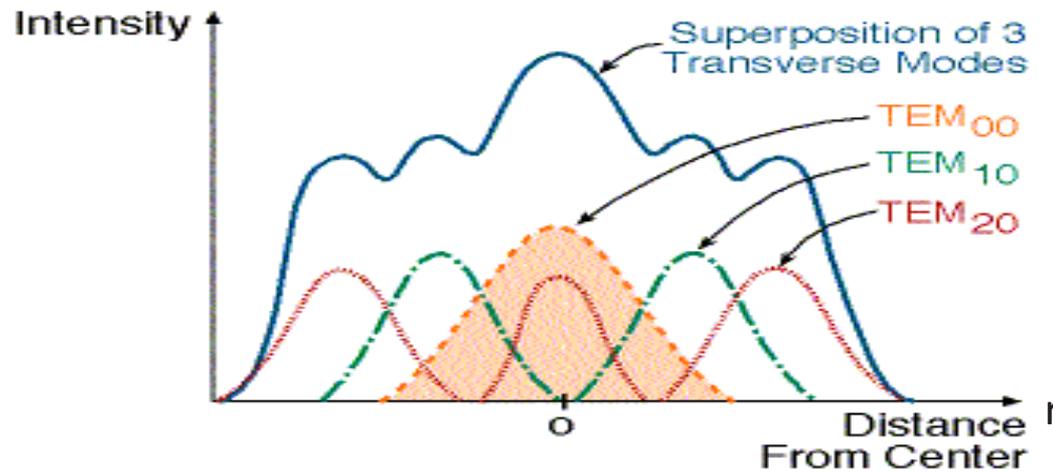


Figure 4.9: Laser radiation with a few transverse modes.

Figure 4.9 shows that the lower transverse mode TEM₀₀ have the smallest diameter compared to other modes.

This gives a hint **how to make a laser operate in a single basic transverse mode: Putting a pinhole with the proper diameter inside the optical cavity.**

By **choosing the pinhole diameter equal to the diameter of the lower mode**, only this mode can pass through the pinhole, and all higher modes are attenuated. Since radiation inside the optical cavity is moving many times, only the basic mode will be amplified, and appear in the output.

4.3.3 Characteristics of the Basic Transverse Mode (TEM₀₀) of the Laser

The intensity distribution of the basic transverse mode have the shape of a mathematical curve called **“Gaussian Mode”**.

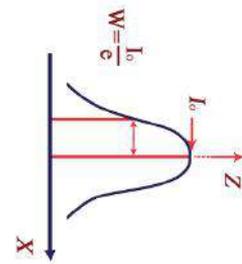
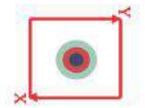
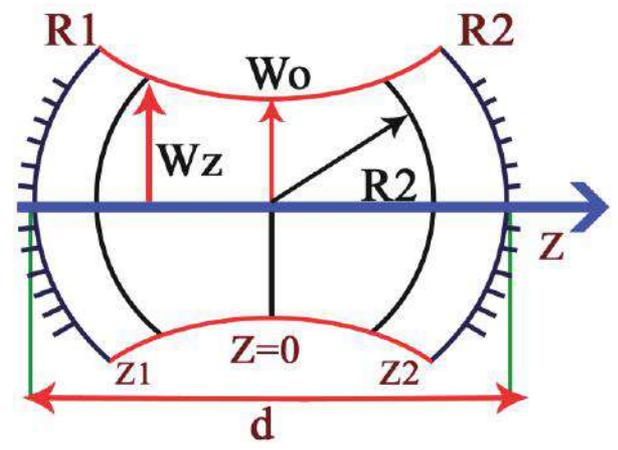
The basic transverse mode is the only mode in which all the radiation is in phase along the cross section of the beam ([see figure 4.8](#)).

The mathematical description of the distribution of Intensity as a function of the distance from the center is:

$$I(r) = I(0) \exp\left(\frac{-2r^2}{\omega_0^2}\right)$$

I_0 = Intensity at the center of the beam (maximum intensity).
 r or w = Radius of the Gaussian beam. This is the distance from the center, in which the intensity drops to $1/e^2$ of the maximum intensity.

ω_0 is called the spot size or the minimum waist, as a special case for a **symmetrical spherical cavity** the ω_0 is in the middle on the cavity.



The **total power in the Gaussian beam** is:

$$P = \pi \omega_0^2 I_0 / 2$$

It can be shown that the divergence angle of a Gaussian beam is:

$$\theta = 2 \lambda / \pi \omega_0$$

The **basic transverse mode** has properties which makes it very practical, and **laser manufacturers try very hard to build lasers which operate in single basic Gaussian mode**¹²⁹

Properties of Basic Gaussian Transverse Mode

- **Lowest divergence angle** than all other transverse modes.
- **Can be focused to the smallest spot** than all other transverse modes.
- Have the **maximum Spatial Coherence** compared to other transverse modes.
- The spatial distribution of a Gaussian beam **remains Gaussian** while the beam propagate through space.
- The imaging with a lens of a Gaussian beam, and other optical elements which the beam pass through, creates again a Gaussian beam.

4.4 Optical Cavity

In every laser cavity there are (at least) two mirrors at the end of the laser.

These mirrors are facing each other, and their centers are on the optical axis of the laser.

The distance between the mirrors determines the length of the optical cavity of the laser (L).

There are different shapes of mirrors, with different lengths between them.

A specific optical cavity is determined by the active medium used, the optical power in it, and the specific application.

The explanation here will summarize the **design principles of an optical cavity**:

- Important definitions.
- Losses inside optical cavity.
- Common optical cavities (4.4.1).
- Stability criterion of laser optical cavity (4.4.2).

Important Definitions for optical cavity:

Optical Cavity - Laser Cavity - The region between the end mirrors of the laser.

Optical Axis - The imaginary line connecting the centers of the end mirrors, and perpendicular to them. The optical axis is in the middle of the optical cavity.

Aperture - The beam diameter limiting factor inside the laser cavity.

Usually the aperture is determined by the diameter of the active medium, but in some lasers a pinhole is inserted into the laser cavity to limit the diameter of the beam. An example is the limiting aperture for achieving single mode operation of the laser ([as was explained in section 4.3.2](#)).

Losses inside an optical cavity

- **Misalignment of the laser mirrors** - When the cavity mirrors are not exactly aligned perpendicular to the laser axis, and parallel to each other (symmetric), the radiation inside the cavity will not be confined during its path between the mirrors.
- **Absorption, scattering and losses in optical elements** - Since optical elements are not ideal, each interaction with optical element inside the cavity cause some losses.
- **Diffraction Losses** - Every time a laser beam pass through a limiting aperture it diffract. It is not always possible to increase the aperture for reducing the diffraction. As an example, such increase will allow lasing in higher transverse modes which are not desired.
- **all the radiation missing from the output of the laser** (emitted through the output coupler).

The gain of the active medium must overcome these losses [as explained in section 5.2.](#)

4.4.1 Specific Laser Optical Cavities

Figure 4.10 describes the most common optical cavities.

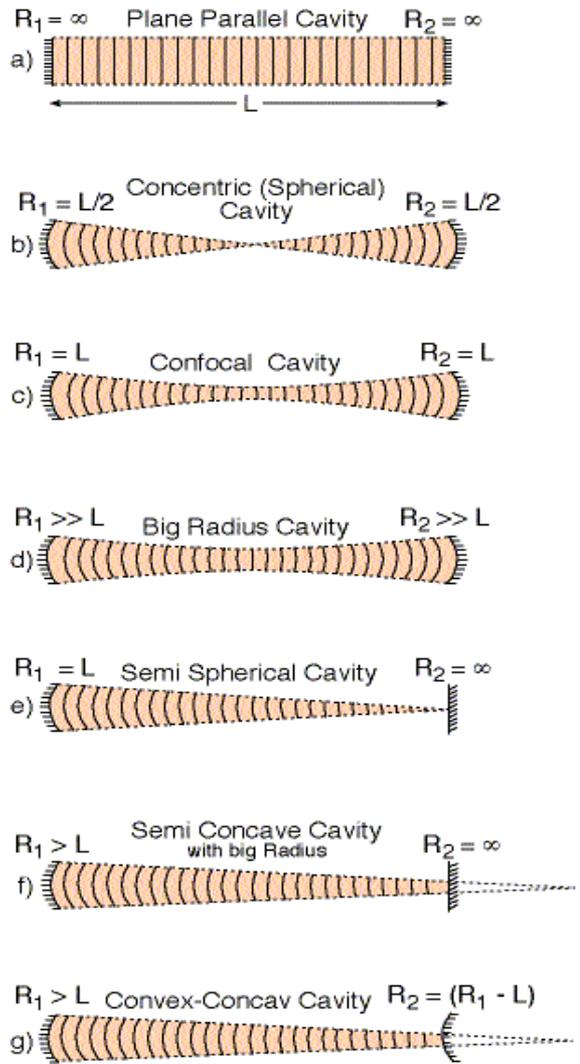


Figure 4.10: the most common optical cavities.

Each optical cavity has 2 end mirrors with radiuses of curvature R_1 and R_2 .

The **dark region in each of the optical cavities** mark the volume of the **active mode** in this specific cavity.

Regions inside the active medium which are not included inside the volume of the active mode do not participate in lasing.

Two parameters determine the structure of the optical cavity:

1. The volume of the laser mode inside the active medium.
2. The stability of the optical cavity.



In the following pages, each type of optical cavity is described:

1. Parallel Plane Cavity.
2. Concentric Circular Cavity.
3. Confocal Cavity
4. Slightly Concave (large radii).
5. Hemispherical Cavity.
6. Half Curve with longer than cavity radius of curvature.
7. Unstable resonator.

Plane Parallel Optical Cavity.

Figure 4.10a describes the Plane Parallel Optical Cavity.

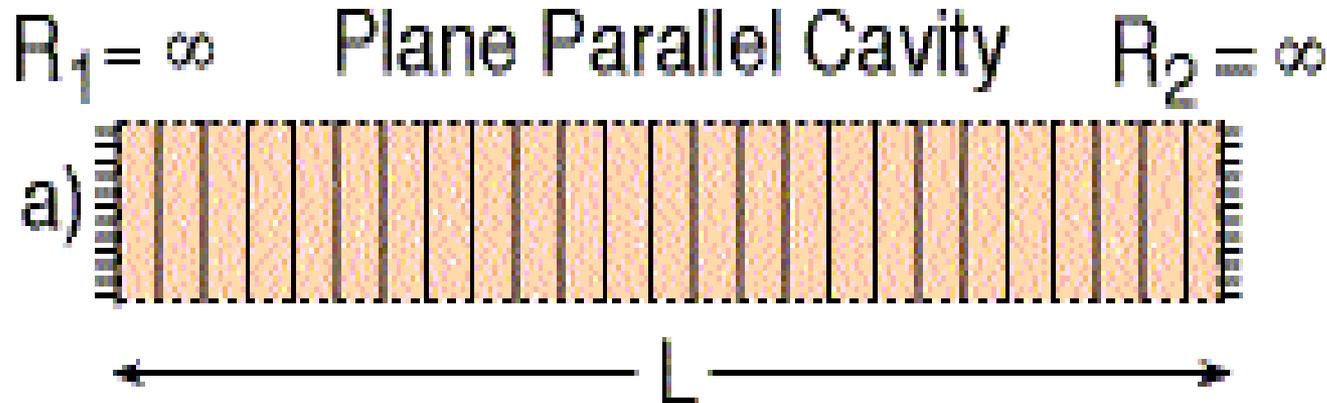


Figure 4.10a: Plane Parallel Optical Cavity.

At both ends there are two plan mirrors ($R_1 = \infty$, $R_2 = \infty$), parallel to each other, and perpendicular to the laser optical axis.

Advantages:

- **Optimal use of all the volume of the active medium.** Thus, used in pulsed lasers which need the maximum energy.
- **No focusing of the laser radiation inside the optical cavity.** In high power lasers such focusing can cause electric breakdown, or damage to the optical elements.

Disadvantages:

- High diffraction losses.
- Very high sensitivity to misalignment. Thus, **very difficult to operate.**

Concentric Circular Cavity (Spherical).

Figure 4.10b describes the circular concentric optical cavity.



Figure 4.10b: Concentric Circular Optical Cavity.

At both ends there are **two spherical mirrors with the same radiuses.**

The distance between the center of the mirrors is equal to twice the radius of curvature of each of them ($R_1 = R_2 = L/2$).

This arrangement cause **focusing of the beam at the center of the cavity.**

The properties of this cavity are the opposite of those of the plan parallel cavity:

Advantages:

- Very low sensitivity to misalignment. Thus, very easy to align.
- Low diffraction losses.

Disadvantages:

- **Limited use of the volume of the active medium.** Used in optical pumping of **continuous Dye lasers** ([see section 6.4](#)). In these lasers the liquid dye is flowing in the region of the beam focusing (The flow direction is perpendicular to the optical axis of the laser). Thus very high power density is used to pump the dye.
- **Maximum focusing of the laser radiation inside the optical cavity.** Such focusing can cause electric breakdown, or damage to the optical elements.

Confocal Cavity.

Figure 4.10c describes the Confocal cavity.

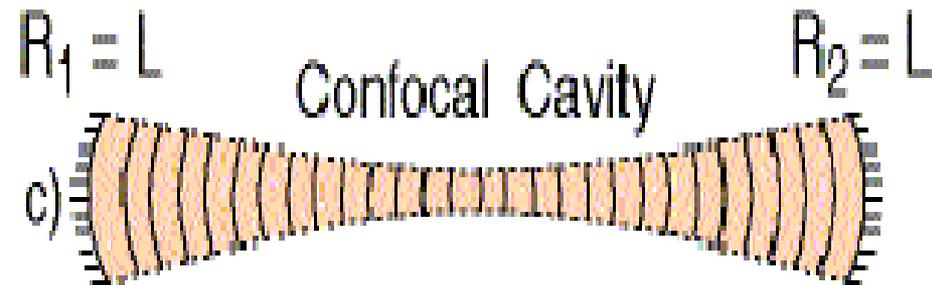


Figure 4.10c: Confocal Optical Cavity.

This cavity is a compromise between plan parallel and circular optical cavities.

At both ends there are two spherical mirrors with the same radiuses.

The distance between the center of the mirrors is equal to the radius of curvature of each of them ($R_1 = R_2 = L$).

This arrangement cause much less focusing of the beam at the center of the cavity.

Advantages:

- Little sensitivity to misalignment. Thus, easy to align.
- Low diffraction losses.
- No high focusing inside the cavity.
- **Medium use of the volume of the active medium.**

The main difference between the Confocal cavity and the spherical cavity is that in the Confocal cavity the **focal point of each mirror is at the center of the cavity**, while in spherical cavity the **center of curvature of the mirrors** is in the center of the cavity.

Slightly concave (large radii)

Figure 4.10d describes the Cavity with Radius of Curvature of the mirrors Longer than Cavity length.



Figure 4.10d: Cavity with Radius of Curvature of the mirrors Longer than Cavity length.

This cavity is a better than Confocal cavity which compromise between plan parallel and circular optical cavities.

At both ends there are two spherical mirrors with big radiuses of curvature (does not need to be the same).

The distance between the center of the mirrors is much less then the radius of curvature of each of them ($R_1, R_2 \gg L$).

This arrangement cause much less focusing of the beam at the center of the cavity.

Advantages:

- Medium sensitivity to misalignment.
- Medium diffraction losses.
- No high focusing of the beam inside the cavity.
- Good use of the volume of the active medium**

Hemispherical Cavity.

Figure 4.10e describes the Hemispherical Cavity.

The cavity is created by one plan mirror, and one spherical mirror with radius of curvature equal to the length of the cavity.



Figure 4.10e: Hemispherical Cavity.

This cavity is similar in properties to circular optical cavity (spherical), with the advantage of the low price of the plan mirror.

Most Helium-Neon lasers use this cavity which have low diffraction losses, and is relatively easy to align.

Advantages:

- Low sensitivity to misalignment.
- Low diffraction losses.

Half Curve with longer than cavity radius of curvature.

Figure 4.10f describes this Cavity.

The cavity is created by one plan mirror, and one spherical mirror with radius of curvature much larger than the length of the cavity.

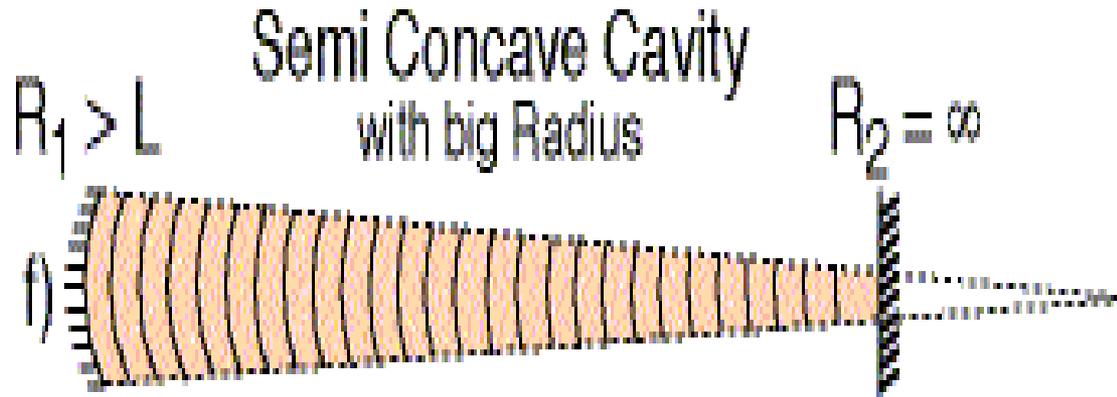


Figure 4.10f: Half Curve with longer than cavity radius of curvature.

This cavity is similar in properties to [Confocal cavity](#), with the advantage of the low price of the plan mirror.

Unstable resonator.

Figure 4.10g describes an example of Unstable Cavity.

An example for such cavity is created by convex concave arrangement of spherical mirrors.

$R_1 > L$ Convex-Concav Cavity $R_2 = (R_1 - L)$



Figure 4.10g: an example of Unstable Cavity.

The concave mirror is big and its radius of curvature is longer than the length of the cavity.

The convex mirror is small and its radius of curvature is small.

In such cavity no standing wave pattern is created inside the cavity.

The radiation does not move in the same path between the mirrors.

The radius of curvature of both mirrors meet at the same point.

Advantages:

- High volume of the modes inside the active medium (The entire volume).
- All the power inside the cavity is emitted out of the laser, not just a small fraction of it.

The laser radiation is emitted out of the laser around the edges of the small mirror.

This cavity is used in high power lasers, which can not use standard output coupler.

Disadvantages:

- The beam shape has a hole in the middle.

Geometric (g)Parameters of an Optical Cavity

First a **geometric parameter** is defined for each of the mirrors:

$$g_1 = 1 - L/R_1 \quad g_2 = 1 - L/R_2$$

A graphical representation of the geometric parameters is described in figure 4.12.

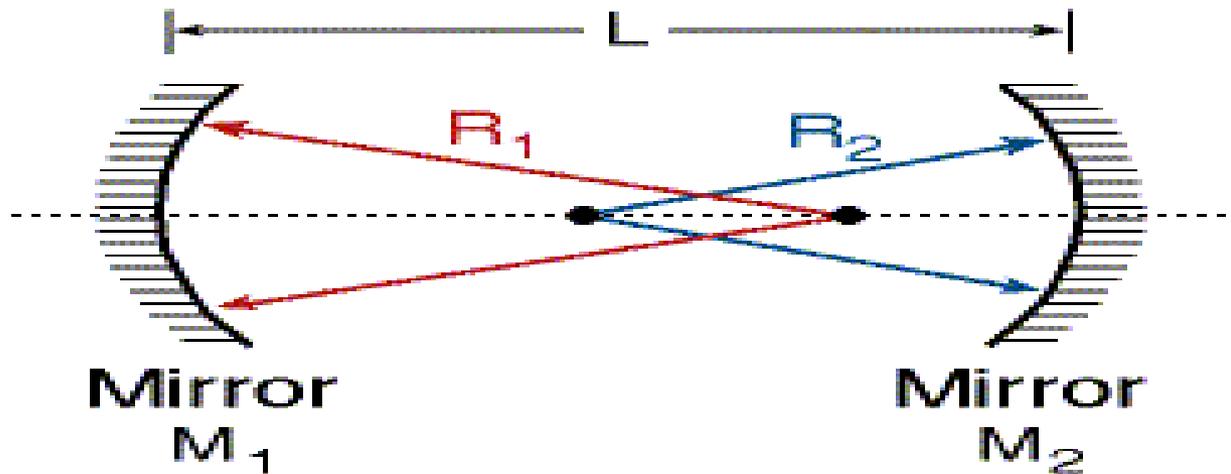


Figure 4.12: A graphical representation of the geometric parameters.

4.2 Stability Criterion of the cavity

A **stable cavity** is a cavity in which the radiation is captured inside the cavity, creating standing waves while the beam move between the mirrors.

The geometry of the cavity determines if the cavity is stable or not.

For determining stability of a cavity, a stability criterion need to be defined

A cavity is stable if:

$$0 \leq g_1 * g_2 \leq 1$$

Stability Diagram of an Optical Cavity

The stability criterion for laser cavity is:

$$0 < g_1 \cdot g_2 < 1$$

$$g_1 = 1 - L/R_1 \quad g_2 = 1 - L/R_2$$

In the stability diagram the geometric parameters of the mirrors are the axes x and y.

Figure 4.13 show the **stability diagram** of all laser cavities.

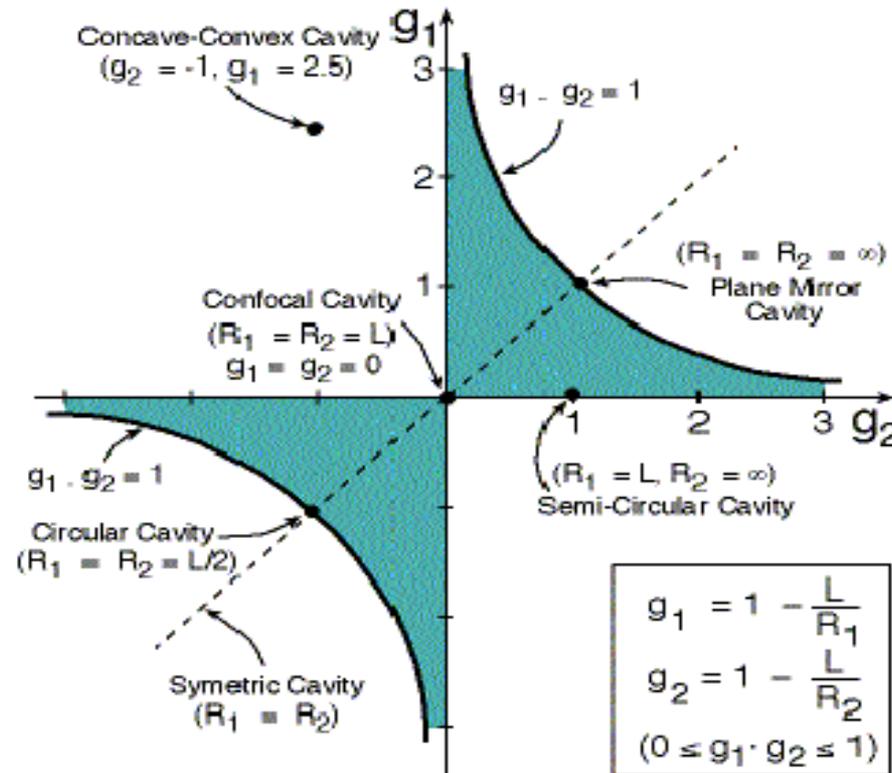


Figure 4.13: Stability Diagram of all laser cavities.

. In the stability diagram, in figure 4.13, **the dark region marks the area of stability.**

The stability region is surrounded by two hyperbolas defined by the stability criterion.

A few common cavities are marked on the stability diagram.
A cavity is stable if the center of curvature of one of the mirrors, or the position of the mirror itself, but not both, are between the second mirror and its center of curvature.

Pay special attention for cavities on the edges of the stability region !

For these cavities, the product $g_1 * g_2$ is either "0" or "1".

Example 4.4: Unstable Resonator

The laser cavity length is 1 [m].

At one end a concave mirror with radius of curvature of 1.5 [m].

At the other end a convex mirror with radius of curvature of 10 [cm].

Find if this cavity is stable.

Solution to Example 4.4:

$$R_1 = 1.5 \text{ [m]}.$$

As common in optics, a **convex mirror is marked with minus sign:**

$$R_2 = -0.1 \text{ [m]}$$

$$g_1 = 1 - L/R_1 = 1 - 1/1.5 = 0.333.$$

$$g_2 = 1 - L/R_2 = 1 + 1/0.1 = 11$$

The product:

$$g_1 * g_2 = 11 * 0.333 > 1$$

The product is greater than 1, so the cavity is unstable.

Question 4.7: He-Ne Laser

The exact wavelength out of He-Ne laser is 0.6328 [μm].

The distance between the mirrors is 30 [cm].

The linewidth of the laser is $1.5 \cdot 10^9$ [Hz].

Calculate:

1. What is the central wavelength of this laser line.
2. How many longitudinal laser modes are in this linewidth.

Summary of Chapter 4

Longitudinal Laser Modes: Longitudinal Optical Modes in a laser describe standing waves along the optical axis of the laser.

Standing waves are created when two waves with the same frequency and amplitude are interfering while moving in opposite directions.

Laser Cavity is made of **mirrors at the end of the active medium**. These mirrors reflect the electromagnetic radiation back to the cavity again and again, to create the standing waves.

The mirrors are nodes of the standing waves.

- The **frequency of the basic longitudinal laser mode** is: $\nu_1 = \frac{c}{2 \cdot n \cdot L}$
- **The frequency of m longitudinal laser mode** is: $\nu_m = m \cdot \left(\frac{c}{2 \cdot n \cdot L} \right)$

• Thus, the frequency of the **m** longitudinal laser mode is equal to **m** times the frequency of the basic longitudinal laser mode

- The **difference between adjacent longitudinal modes** is equal to the frequency of the basic longitudinal mode:

$$\Delta \nu_{\text{MS}} = \frac{c}{2 nL}$$

- The number of longitudinal modes is determined by the length of the cavity and its index of refraction.

Transverse Laser Modes:

The **basic transverse (TEM₀₀) mode** is a **Gaussian**:

- It has the lowest divergence.
- It can be focused to the smallest spot.
- Its Spatial coherence is the best of all the other modes.
- It stays with Gaussian distribution while passing through optical systems.

Stability Diagram:

The stability diagram describes the **geometrical parameters of the laser cavity**:

$$g_1 = 1 - L/R_1 \quad g_2 = 1 - L/R_2$$

The condition of stability:

$$0 \leq g_1 * g_2 \leq 1$$

An applet that show how the stability diagram depends on the cavity parameters can be reached by [clicking here Cavity Stability](#).

To check how the laser beam moves inside different type of cavities, and how the cavity parameters determine the divergence of the beam, [Click here: Beams in Laser Cavity](#).

Chapter 5: Laser Gain

The output power of the laser at specific moment is determined by two conflicting factors:

1. **Active medium gain** - which depends on:
 - a) **Population Inversion** ([see section 2.6](#)).
 - b) **Fluorescence line-shape** of the spontaneous emission that is related to the lasing transition (which will be explained in [section 5.1](#)).
2. **Losses in the laser**, which include:
 - a) **Reflections from end mirrors**.
 - b) **Radiation losses inside the active medium** - due to absorption and scattering.
 - c) **Diffraction losses** - Due to the finite size of the laser components.

It is clear that a **required condition for lasing** is:

In a round trip path of the radiation between the laser mirrors, the gain must exceed (or at least be equal to) the losses.

5.1 Fluorescence line shape of the laser

Laser action inside matter is possible only for those wavelengths for which this material have fluorescent emission.

Fluorescence line is described by plotting spontaneous emission radiation intensity as a function of frequency (or wavelength), for the specific lasing transition. Figure 5.1 describes a simplified energy level diagram for **Helium Neon laser** (other examples of He-Ne laser were shown in examples: [3.1](#), [4.2](#), [4.3](#), [4.4](#), and will be explained more in [section 6.1.1](#)).

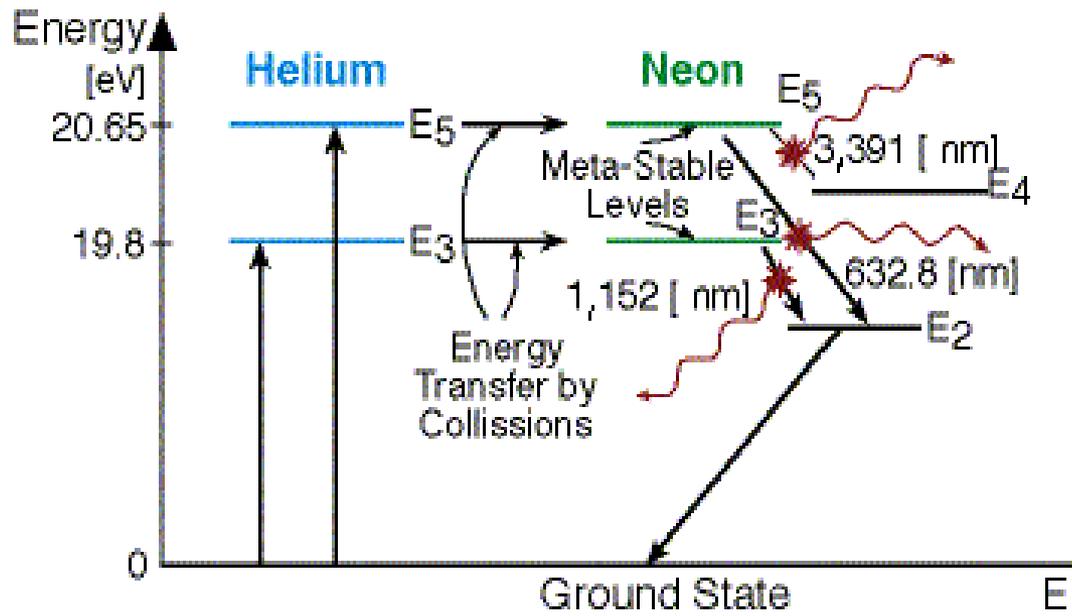


Figure 5.1: Energy Level Diagram for Helium Neon laser.

The main transition in the visible spectrum is from level E_5 to level E_2 , and the emission is at red wavelength of 632.8 [nm].

Fluorescence line-shape

Fluorescence line is described by plotting spontaneous emission radiation intensity as a function of frequency (or wavelength), for the specific lasing transition. [In figure 5.1](#) for the He-Ne laser it was for transition between E_5 and E_2 . **When the transition is between narrow levels, the fluorescence line is narrow.**

The narrower the fluorescence line the easier (less energy required) it is to achieve population inversion. The **ideal fluorescence line-shape** is a **spike pulse** with width approaching zero, as described in figure 5.2a.

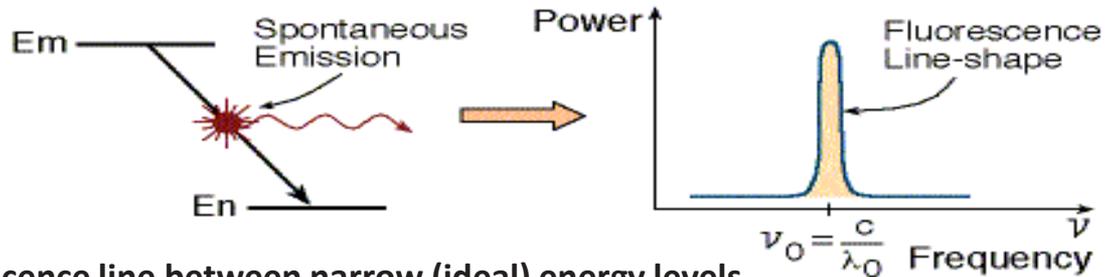


Figure 5.2a: Fluorescence line between narrow (ideal) energy levels.

In reality, each energy level have a finite width, as described in figure 5.2b. Thus, many transitions can occur between different regions in the upper lasing level to different regions in the lower laser level. All these transitions, plotted as a function of frequency, make the fluorescence line shape shown in [figure 5.3](#).

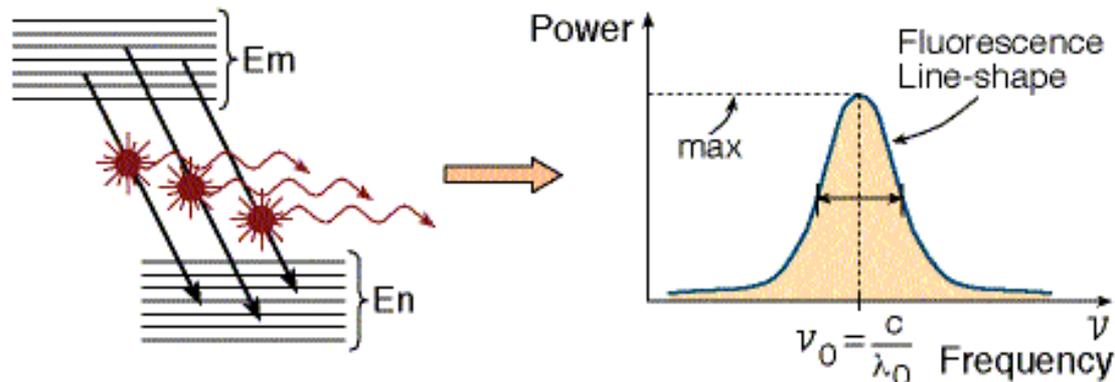


Figure 5.2b: Fluorescence line between wide (real) energy levels.

Fluorescence Linewidth

All possible spontaneous transition lines, plotted as a function of frequency, make the continuous fluorescence line shape shown in figure 5.3.

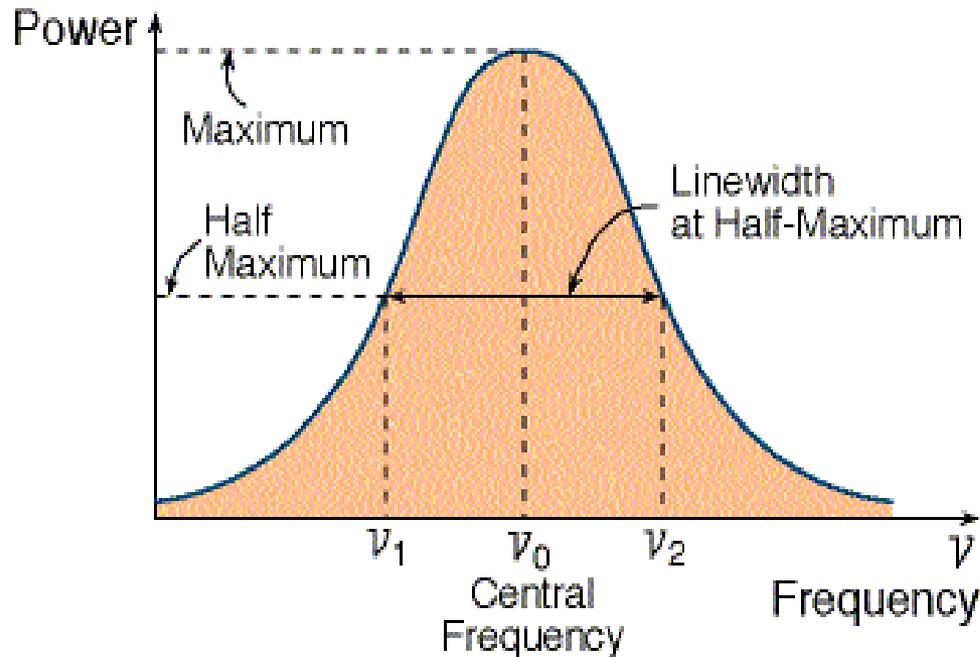


Figure 5.3: Fluorescence Line.

The width of the fluorescence line is called **Fluorescence linewidth**, and is the measure of the width of the fluorescence line at half its maximum height:

FWHM = **F**ull **W**idth at **H**alf **M**aximum.

Mathematical Expressions of fluorescence linewidth

Fluorescence linewidth is expressed by wavelengths, or frequencies, of two points on the spontaneous emission graph at **half the maximum height**.

$$\Delta\nu = \left| \nu_2 - \nu_1 \right| = \left| \frac{c}{\lambda_2} - \frac{c}{\lambda_1} \right| = \left| \frac{c\lambda_1 - c\lambda_2}{\lambda_1\lambda_2} \right| = \frac{c\Delta\lambda}{\lambda_1\lambda_2}$$

$$\Delta\nu = \frac{c\Delta\lambda}{\lambda_0^2}$$

In a similar way:

The linewidth ($\Delta\lambda$) is much smaller than each of the wavelengths ($\Delta\lambda \ll \lambda_1, \lambda_2$). Thus the **approximation**: $\lambda_1 * \lambda_2 = \lambda_0^2$ can be used.

λ_0 = Wavelength at the center of emission spectrum of the laser.

The result is:

$$\Delta\lambda = \left| \lambda_2 - \lambda_1 \right| = \left| \frac{c}{\nu_2} - \frac{c}{\nu_1} \right| = \left| \frac{c\nu_1 - c\nu_2}{\nu_1\nu_2} \right| = \frac{c\Delta\nu}{\nu_1\nu_2}$$

In [chapter 10](#) these mathematical relations will be used for determining the **coherence** of the laser.

Laser Gain Curve

There is a lot of similarity between the shape of the gain curve (figure 4.4) and the fluorescence line (figure 5.3). The reason is that **the active medium gain curve is directly proportional to the width of the fluorescence line of the spontaneous emission**.

When discussing linewidth, it is important to distinguish between the **linewidth of the laser**, and the **linewidth of specific longitudinal mode**, which can contain many longitudinal modes.

Figure 5.4 describes both the gain curve of the laser, and the longitudinal modes of the cavity

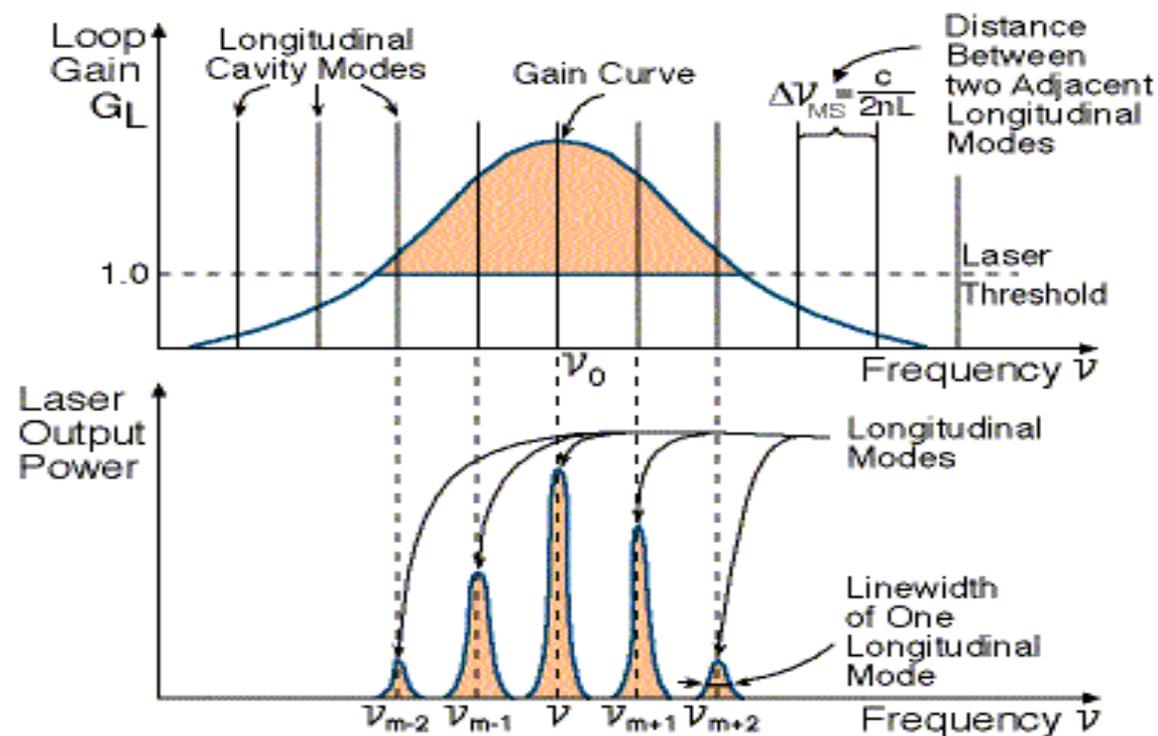
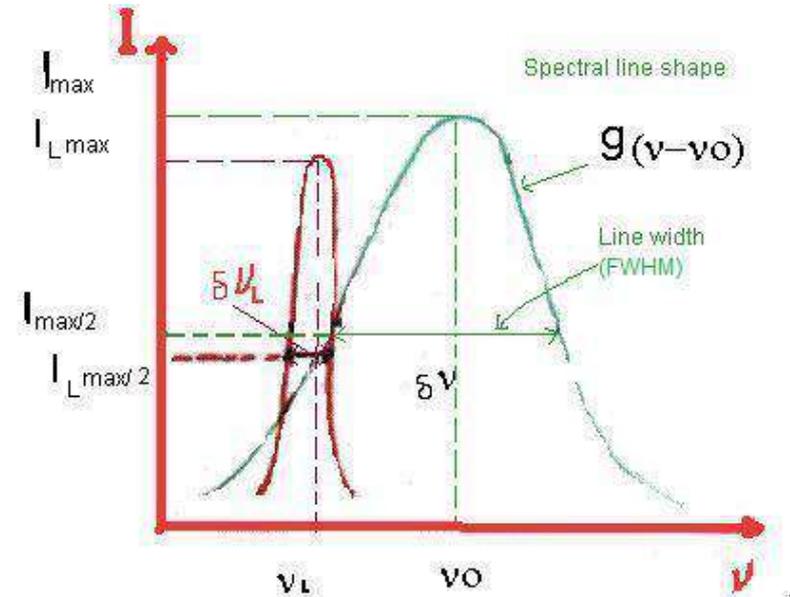


Figure 5.4: Laser Gain Curve, and the emitted linewidths.

Each of the longitudinal modes has its own linewidth, and emit certain intensity.

5-2 The Broadening of spectral lines:-

Radiation emitted from the active medium is not an absolutely monochromatic but has narrow frequency broadening spectrum compared to conventional light sources, the emission will look like the fig. i.e. it has a very small frequency spread because the interaction process continues during the laser action so there will be many mechanisms due to laser interaction and the result is a change in the frequency of radiation

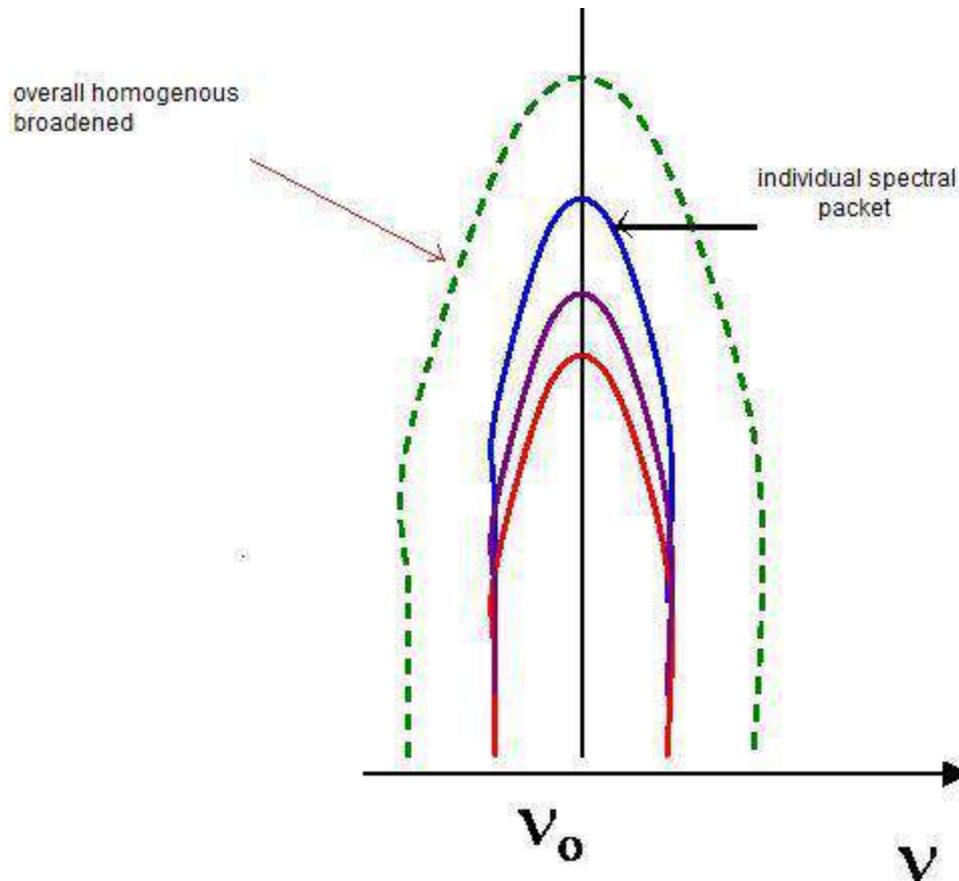


An understanding of the mechanism responsible for the broadening of the line width is necessary for the development of laser sources with sufficient spectral purity. Further, for detailed calculation of the interaction of radiation with atom it is necessary to have accurate knowledge of the line profile. The width and shape of spectral lines also provide information about temperature and density in the source.

– mainly they are of two kinds of Broadenings:
Homogenous and **Inhomogeneous** broadening

1.Homogenous broadening:-

Some broadening mechanisms broaden the response of each individual atom equally and hence, have the same effect on all the atoms in the ensemble. Every atom, therefore, has the same center frequency and the same atomic line-shape. This is homogeneous broadening. In which the action of the change is constant for all atom since they are relating to the natural phenomena inside the system. Among these are: (a) collision Broadening (b) natural Broadening



(a) Collision Broadening:-

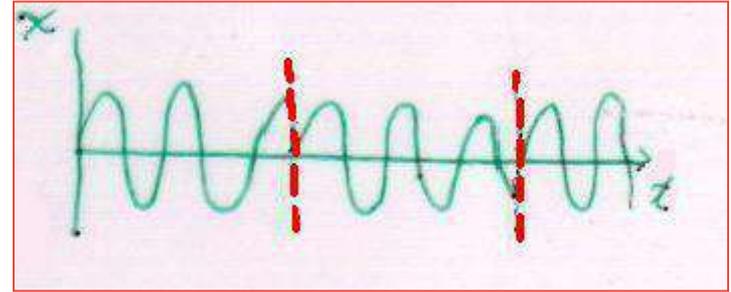
in this type of the Broadening we can take the net numbers of collision per unit time into consideration, and according to the **kinetic theory of gases**, these could be calculated statistically,

the main parameters are,

the **collision life time** (τ_c)

the **root mean square of the velocity** (v)

the **mean free path of the collision** (x).



At threshold $v = \frac{(8KT)^{1/2}}{\pi M}$ ----- (1)

Also

$$x = \frac{1}{\sqrt{2} \pi a^2 N}$$
 ----- (2)

And according to ideal gas eqn.

$$PV = nRT$$
 ----- (3)

So for one mole $n=1 \longrightarrow R=KN_o$

Substituting for **R** in eqn. (3) we get

$$PV = KN_o T \quad \text{And} \quad \frac{N_o}{V} = \frac{P}{KT} \quad \longrightarrow \quad \frac{N_o}{V} = N$$

K = Boltzmann's constant

T = Absolute temp

M = Mass of the molecule

N = no. of molecule p.u.v.

a = diameter of molecule

V = Volume

P = Pressure

N_o = Avogadro no.

R = gas's constant

Introducing the new value of N into eqn. (2) we have

$$x = \frac{KT}{\sqrt{2} \pi a^2 P} \quad \text{----- (5)}$$

Since $t = \frac{x}{v}$ so dividing eqn.(5) on eqn. (1) we get

$$\text{Collision life time } \tau_c = \frac{KT / \sqrt{2} \pi a^2 P}{(8KT / \pi M)^{1/2}} = \frac{(MKT)^{1/2}}{4\pi^{1/2} p a^2} \quad \text{----- (6)}$$

The change in the frequency of radiation due to τ_c is:-

$$\delta \nu_c = \frac{1}{\pi \tau_c} \longrightarrow \delta \nu_c = \frac{4 p a^2}{(\pi M K T)^{1/2}} \quad \text{----- (7)}$$

Numerical example:

1. At room temperature, the linewidth of CO₂ laser with gas pressure of 10 torr is 55 MHz
2. At room temperature, the linewidth of CO₂ laser with gas pressure of 100 torr is 500MHz
3. Above 100 [torr], the increase rate of broadening is about 6.5 [MHz] for each increase in pressure of 1 [torr].

(b) natural Broadening:-

When the radiation interacts with an atomic system, many process come into consideration of the radiation due to the change in the life time of the considered atomic state, since in general, both the upper and lower levels have finite lifetime, the uncertainty in the energy of both the levels must include the uncertainty in the frequency of emitted radiation.

The width of the line is given by

$$\delta \nu_{\text{nat}} = \delta_1 + \delta_2$$

Where δ_1, δ_2 are the widths of the two levels

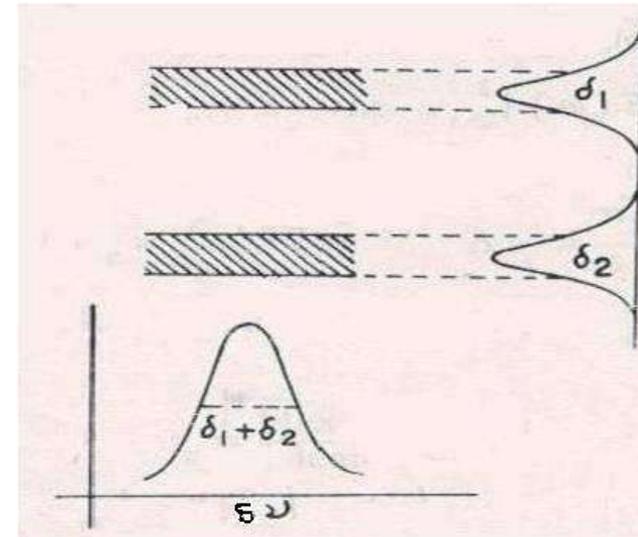
$$\therefore \delta \nu_{\text{nat}} = \frac{1}{2\pi t_{\text{spo}}} \quad \text{--- (8)} \quad t_{\text{spo}} \equiv \text{is spontaneous emission life-time}$$

Numerical examples:

$$\text{For } t_{\text{spn.}} = 10^{-8} \text{ [s]} \quad \Rightarrow \quad \delta \nu = 1.6 * 10^7 \text{ [Hz]}$$

$$\text{For } t_{\text{spn.}} = 10^{-4} \text{ [s]} \quad \Rightarrow \quad \delta \nu = 1.6 * 10^3 \text{ [Hz]}$$

The longer the specific energy level transition lifetime, the narrower is its linewidth.

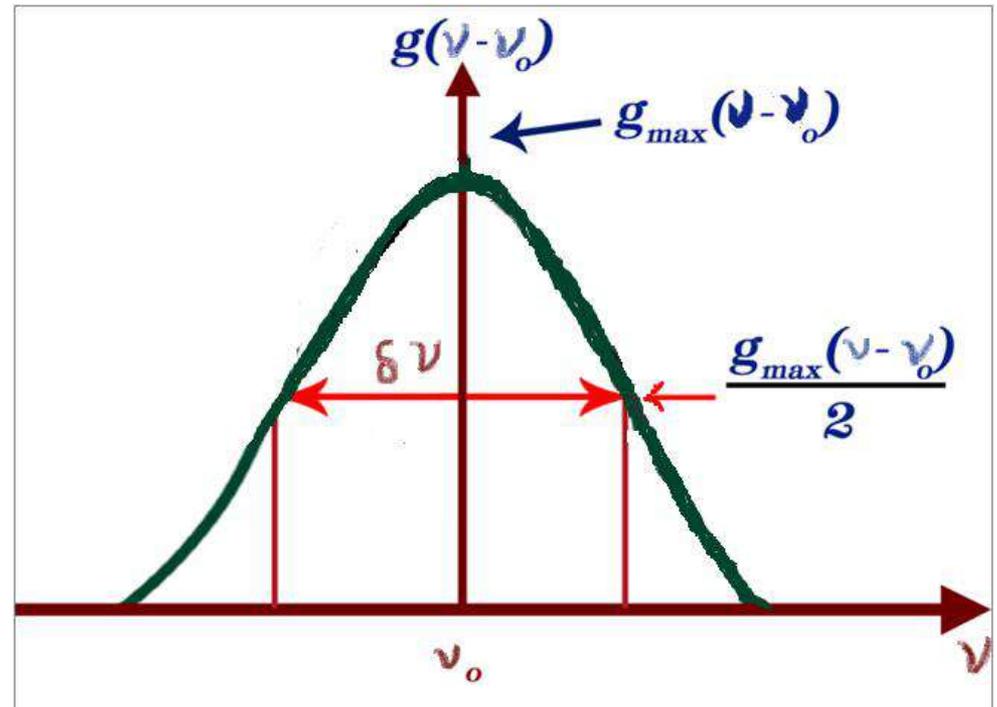


- The line width for this two broadened (collision and natural) distribution can be expressed as
- Lorentzian curve.

$$g(\nu - \nu_0) = \frac{1}{2\pi} \frac{\delta\nu}{[(\nu - \nu_0)^2 + \frac{(\delta\nu)^2}{2}]}$$

At $\nu = \nu_0$

$$g_{\max}(0) = \frac{1}{2\pi} \frac{\delta\nu}{\frac{(\delta\nu)^2}{2}} = \frac{0.637}{\delta\nu}$$

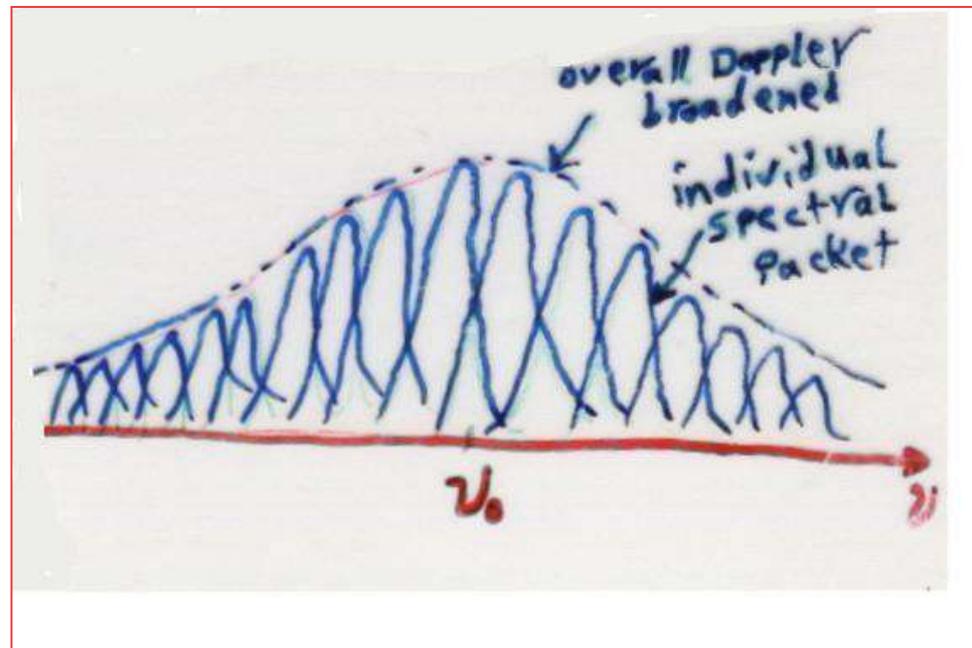


2. Inhomogeneous broadening:-

This type of broadening mechanism operates by shifting or spreading out the centre frequencies of individual atoms. The essential feature of this mechanism is that different atoms or groups of atoms, within the ensemble have slightly different resonance frequencies on the same transition. These broaden the overall response of the ensemble without broadening the response of the individual atoms.

In this type of broadening, each atom has its own type of the effect of the radiation and consequently has its own frequency.

Among the famous broadenings is **Doppler broadening**



-Doppler Broadening.

Doppler shift is a well known phenomena in wave motion.
It occurs when the source is in relative motion to the receiver.

The frequency detected is shifted by an amount determined by the **relative velocity between the source and the receiver.**

Since **gas molecules are in constant motion in random directions**, each molecule emit light while it is moving relative to the laser axis in a different direction. These distribution of frequency shifts cause the broadening of the laser linewidth.

Since any atom or molecule has a number of frequencies, we choose only the component which is parallel to the direction of inter action propagation.

the result of atomic spectral lines could be deduced through out what is called **Maxwell -Boltzmann distribution function of velocity and frequency.**

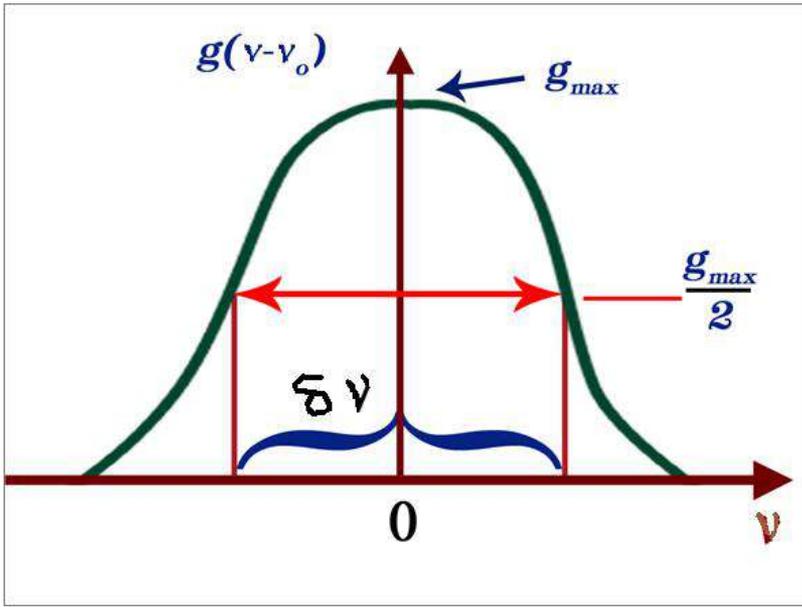
Then according to the **Doppler phenomenon:-**

$$v = v_o \left(1 \pm \frac{v}{c} \right) \text{ ----- (1)}$$

- v = frequency of radiation after interaction
- v_o = the original frequency of radiation
- v = velocity of the atomic medium

The distribution function of velocity is the exponential type

$$f(v) = \left[\frac{M}{2\pi KT} \right]^{1/2} \exp\{(-M v^2)/ 2KT\} dv$$



These arise to a function as a change in v_o, which is called **Gaussian distribution function** as:-

$$g(v - v_o) = \frac{C}{v_o} \left\{ \frac{M}{2\pi KT} \right\}^{1/2} \exp \left[\frac{-MC^2}{2KT} \left(\frac{v - v_o}{v_o} \right)^2 \right] \text{ ----- (2)} \quad \text{at } v = v_o$$

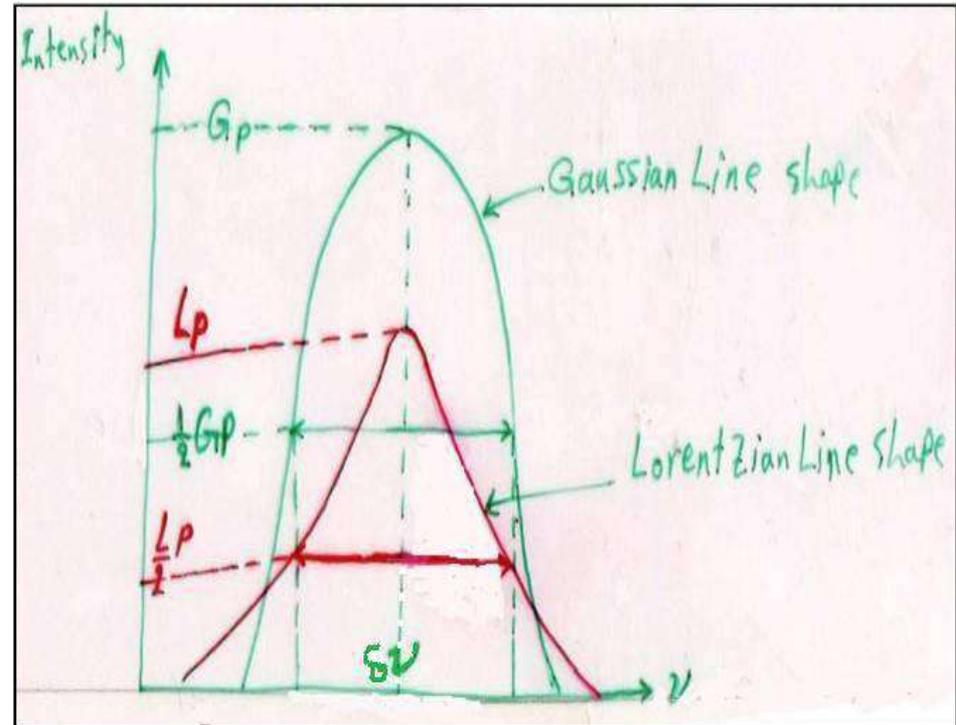
$$g(0) = \frac{C}{v_o} \left\{ \frac{M}{2\pi KT} \right\}^{1/2} \exp^0 = \frac{2}{\delta v} \left(\ln 2 / \pi \right)^{1/2} = \frac{0.939}{\delta v} = \frac{1}{\delta v}$$

$$\delta v = \frac{2v_o}{c} \sqrt{\frac{2kT \ln 2}{M}}$$

Comparison between the two line shapes are shown that the Max. Intensity of Gaussian line is 50% > from max. Intensity of Lorentzian line shape

$$g_G(0) = \frac{2}{\delta\nu} \{(\ln 2)/\pi\}^{1/2} = \frac{0.939}{\delta\nu}$$

$$g_L(0) = \frac{2}{\pi \delta\nu} = \frac{0.637}{\delta\nu}$$



Linewidth broadening

Figure 5.5 show the result of broadening of the fluorescence linewidth.

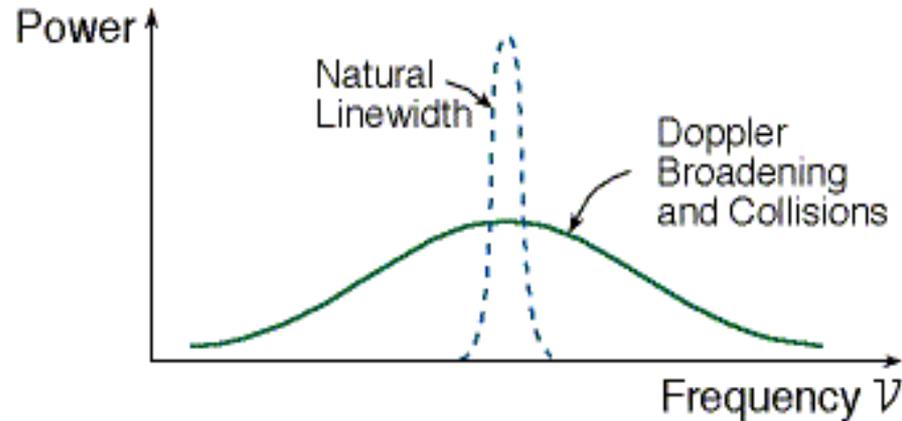


Figure 5.5: Fluorescence Linewidth broadening

Numerical Example : Typical He-Ne Laser Center frequency of the emitted radiation is: $4.74 \cdot 10^{14}$ [Hz].

Linewidth of single longitudinal mode: 1 [KHz] = 10^3 [Hz].

Optical cavity linewidth: 1 [MHz] = 10^6 [Hz].

Natural Linewidth: 100 [MHz] = 10^8 [Hz].

Doppler Linewidth: 1,500 [MHz] = $1.5 \cdot 10^9$ [Hz].

5.3 Loop Gain

Each time the laser radiation pass through the active medium, it is amplified, as was explained on “**population Inversion**” ([section 2.6](#)).

Contrary to amplifying the radiation, there are many **losses**:

1. Scattering and absorption losses at the end mirrors.
2. Output radiation through the output coupler.
3. Scattering and absorption losses in the active medium, and at the side walls of the laser.
4. Diffraction losses because of the finite size of the laser components.

These losses cause some of the radiation not to take part in the lasing process.

A necessary condition for lasing is that the total gain will be a little higher than all the losses.

Loop Gain is defined as the **net gain** (amplification less losses) that the radiation see in a round trip transmission through the laser. It is measured as the ratio between radiation intensity at a certain plane (perpendicular to the laser axis), and the radiation intensity at the same plane after a round trip through the laser.

Loop Gain (G_L)

Figure 5.6 show the round trip path of the radiation through the laser cavity.

The path is divided to sections numbered by 1-5, while point "5" is the same point as "1".

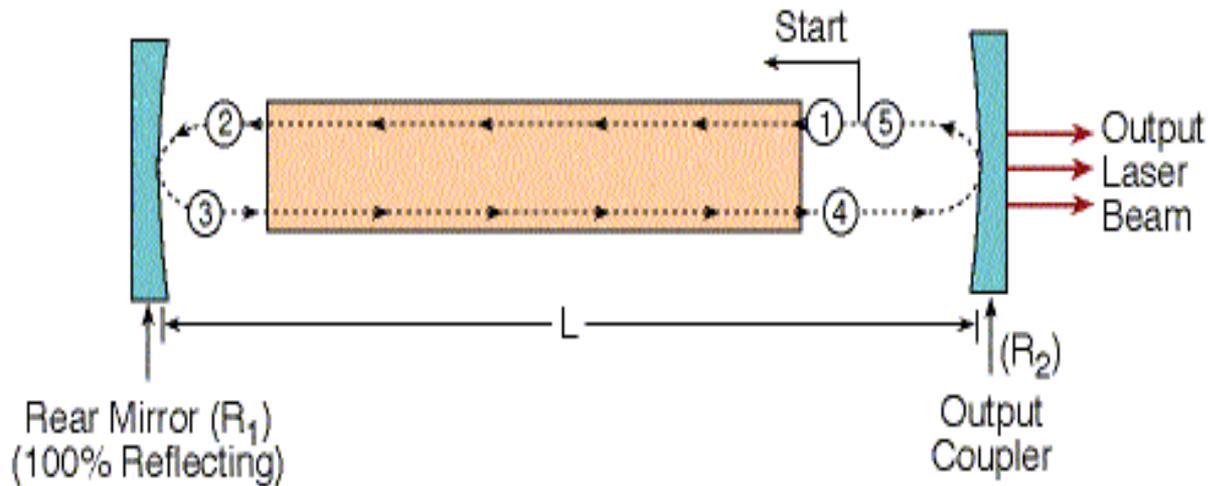


Figure 5.6: Round trip path of the radiation through the laser cavity.

By definition, **Loop Gain** is given by:

$$G_L = E_5/E_1$$

G_L = Loop Gain.

E_1 = Intensity of radiation at the beginning of the loop.

E_5 = Intensity of radiation at the end of the loop.

Calculating Loop Gain (G_L) Without Losses

On the way from point “1” to point “2”, the radiation pass through the active medium, and amplified.

Defining:

G_A = Active medium gain (passing through a length L of the active medium). Thus:

$$E_2 = G_A * E_1$$

For simplicity we assumed that the length of the active medium is equal to the length of the cavity, such that the active medium feel the length of the laser cavity.

On the way from point “2” to point “3”, the radiation is reflected from the mirror with the high reflectivity R_1 (close to 100%). As a result:

$$E_3 = R_1 * G_A * E_1$$

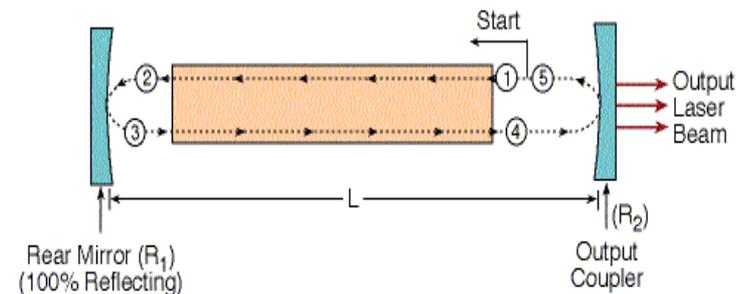
On the way from point “3” to point “4”, the radiation pass again through the active medium, and amplified. Thus:

$$E_4 = R_1 * G_A^2 * E_1$$

On the way from point “4” to point “5”, the radiation is reflected from the output coupler, which have a reflectivity R_2 . Thus:

$$E_5 = R_1 * R_2 * G_A^2 * E_1$$

This completes the loop.



Calculating Loop Gain (G_L) With Losses

We assume that the losses occur uniformly along the length of the cavity (L).

In analogy to the Lambert formula for losses ([which was explained in section 2.3](#)), we define **loss coefficient** (α), and using it we can define **absorption factor (Loss factor) M**:

$$M = \exp(-2\alpha L)$$

M = Loss factor, describe the relative part of the radiation that remain in the cavity after all the losses in a round trip loop inside the cavity.

All the losses in a round trip loop inside the cavity are $1-M$ (always less than 1).

α = **Loss coefficient** (in units cm^{-1}).

$2L$ = **Path Length**, which is twice the length of the cavity.

Adding the **loss factor (M)** to the equation of E_5 :

$$E_5 = R_1 * R_2 * G_A^2 * E_1 * M$$

From this we can calculate the **Loop gain**:

$$G_L = E_5/E_1 = R_1 * R_2 * G_A^2 * M$$

As we assumed uniform distribution of the loss coefficient (α), we now define **gain coefficient (β)**, and assume **active medium gain (G_A)** as distributed uniformly along the length of the cavity.

$$G_A = \exp(+\beta L)$$

Substituting the last equation in the Loop Gain:

$$G_L = R_1 * R_2 * \exp[2(\beta - \alpha)L]$$

Calculating Gain Threshold $(G_L)_{th}$

$$G_L = R_1 * R_2 * \exp[2(\beta - \alpha)L]$$

When the **loop gain (G_L) is greater than 1 $(G_L > 1)$** , the beam intensity will increase after one return pass through the laser.

When the **loop gain (G_L) is less than 1 $(G_L < 1)$** , the beam intensity will decrease after one return pass through the laser. laser oscillation decay, and no beam will be emitted.

Conclusion:

There is a threshold condition for amplification, in order to create oscillation inside the laser.

This Threshold Gain is marked with index “**th**”.

For continuous laser , the threshold condition is:

$$(G_L)_{th} = 1 = R_1 R_2 G_A^2 M = R_1 * R_2 * \exp[2(\beta - \alpha)L]$$

Or

$$\beta = \alpha + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

Example 5.3:

Active medium gain in a laser is 1.05.

Reflection coefficients of the mirrors are: 0.999, and 0.95.

Length of the laser is 30 [cm].

Loss coefficient is: $\alpha = 1.34 \cdot 10^{-4}$ [cm⁻¹].

Calculate:

1. The loss factor M.
2. The Loop gain (G_L).
3. The gain coefficient (β).

Solution to example 5.3:

1. The loss factor M:

$$M = \exp(-2\alpha L) = \exp[-2(1.34 \cdot 10^{-4}) \cdot 30] = 0.992$$

2. The Loop gain (G_L):

$$G_L = R_1 R_2 G_A^2 M = 0.999 \cdot 0.95 \cdot (1.05)^2 \cdot 0.992 = 1.038$$

Since $G_L > 1$, this laser operates above threshold.

3. The gain coefficient (β):

$$G_A = \exp(\beta L)$$

$$\ln(G_A) = \beta L$$

$$\beta = \ln(G_A)/L = \ln(1.05)/30 = 1.63 \cdot 10^{-3} \text{ [cm}^{-1}\text{]}$$

The gain coefficient (β) is greater than the loss coefficient (α), as expected

Example 5.4: Calculating Cavity Losses

Helium Neon laser operates in threshold condition.

Reflection coefficients of the mirrors are: 0.999, and 0.97.

Length of the laser is 50 [cm].

Active medium gain is 1.02.

Calculate:

1. The loss factor M.
2. The loss coefficient (α).

Solution to example 5.4:

Since the laser operates in threshold condition, $G_L = 1$.

Using this value in the loop gain:

$$G_L = 1 = R_1 R_2 G_A^2 M$$

1. The **loss factor M**:

$$M = 1 / (R_1 R_2 G_A^2) = 1 / (0.999 * 0.97 * 1.02^2) = 0.9919$$

As expected, $M < 1$.

Since $G_L > 1$, this laser operates above threshold.

2. The **loss coefficient (α)** is calculated from the loss factor:

$$M = \exp(-2\alpha L)$$

$$\ln M = -2\alpha L$$

$$\alpha = \ln M / (-2L) = \ln(0.9919) / (-100) = 8.13 * 10^{-5} \text{ [cm}^{-1}\text{]}$$

Attention:

If the loss factor was less than 0.9919, then $G_L < 1$, and the oscillation condition was not fulfilled.

Example 5.5: Active Medium Gain in cw Argon Ion Laser

Reflection coefficients of the mirrors are: 0.999, and 0.95.

All the losses in round trip are 0.6%.

Calculate:

1. The active medium gain.

Solution to example 5.5:

For finding the active medium gain G_A , the loss factor (M) must be found.

All the losses are $1-M$.

$$1-M = 0.006$$

$$M = 0.994$$

Using this value in the threshold loop gain:

$$(G_L)_{th} = 1 = R_1 R_2 G_A^2 M$$

$$(G_A)_{th} = 1/\sqrt{R_1 R_2 M} = 1/\sqrt{0.999 \cdot 0.95 \cdot 0.994} = 1.03$$

The active medium gain must be at least 1.03 for creating continuous output from this laser

Summary

$1.G_L = \text{Loop Gain}$, determines if the output power of the laser will increase, decrease, or remain constant. It include all the losses and amplifications that the beam have in a complete round trip through the laser.

$$G_L = R_1 R_2 G_A^2 M$$

$R_1, R_2 = \text{Reflection coefficients}$ of the laser mirrors.

$G_A = \text{Active medium gain}$ as a result stimulated emission.

$$G_A = \exp(+\beta L)$$

$\beta = \text{Gain coefficient}$.

$L = \text{Active Medium length}$.

$M = \text{Optical Loss Factor}$ in a round trip path in the laser cavity.

$$M = \exp(-2\alpha L)$$

$\alpha = \text{Loss coefficient}$.

1. When $G_L = 1$, The laser operate in a **steady state mode**, meaning the output is at a constant power. This is the **threshold condition for lasing**, and the active medium gain is:

$$(G_A)_{th} = 1/\text{sqrt}(R_1 R_2 M)$$

The **Loop Gain** is:

$$G_L = R_1 * R_2 * \exp(2(\beta - \alpha)L)$$

5.4 “Hole Burning” in the Laser Gain Curve

The **active medium gain depends on population inversion, and the fluorescence line shape.**

This gain is influenced by the lasing process itself, since lasing change the population inversion conditions.

Stimulated emission causes depletion of the upper laser level, and reduces the population inversion.

Thus, gain is reduced, until pumping increase the upper level population again.

Energy level diagram in a 4 level laser

In figure 5.7, an energy level diagram of a 4 level laser is shown (similar to figure 2.7 in [section 2.12](#)).

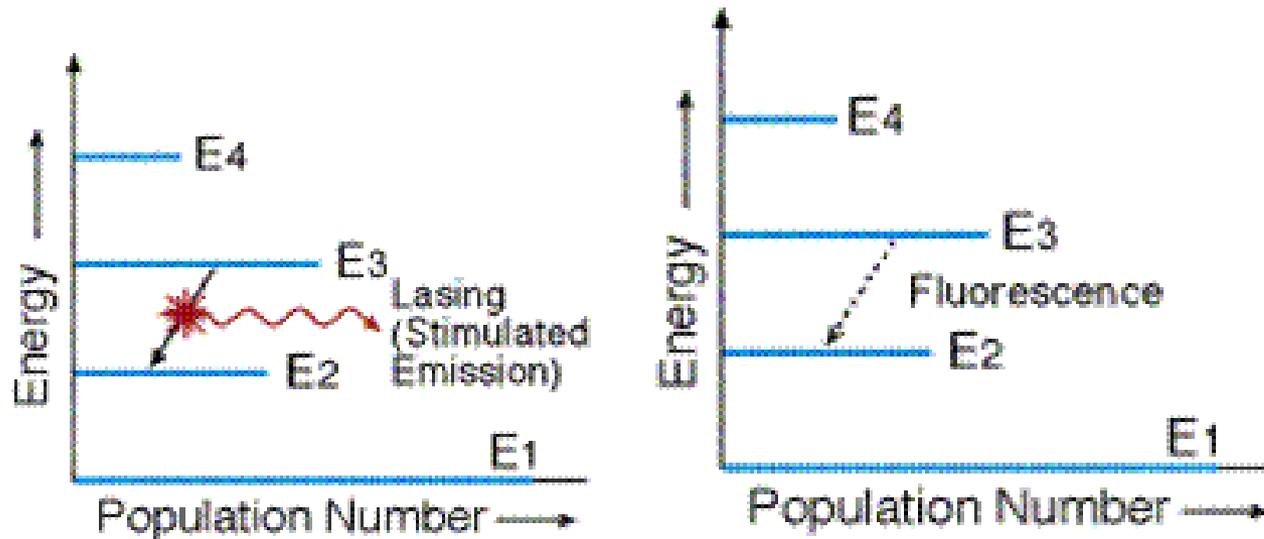


Figure 5.7: Energy level diagram in a 4 level laser

When the cavity mirrors are taken away from the laser, since there is no lasing, the [population inversion](#) will remain almost constant. Only the [spontaneous emission](#) from energy level E_3 to E_2 continue. Thus, active medium gain (G_A) is almost constant.

This gain is called “**Small Signal Gain**” (when there is no lasing process), and it is the **maximum gain of the active medium**.

When the cavity mirrors are back inside the laser, lasing occurs, and population inversion decreases, thus reducing the gain. In this case, the gain is “Saturation Gain”, and is always less than the small signal gain.

Active Medium Gain Curve without and with lasing- Hole Burning

In figure 5.8, both small signal gain and saturation gain are plotted as a function of frequency.

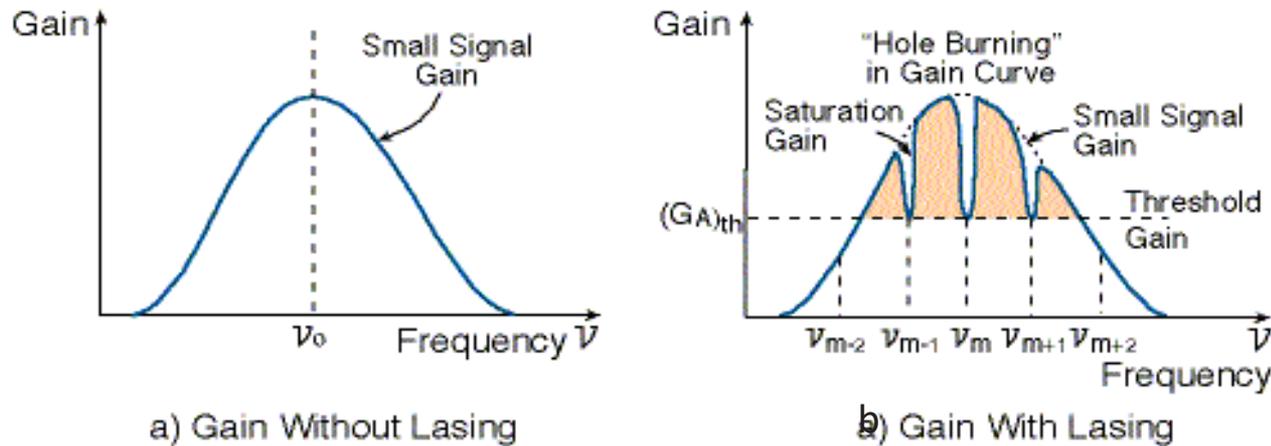


Figure 5.8: Active Medium Gain Curve without and with lasing - Hole Burning

Small signal gain Curve appears identical to the fluorescence line shape ([figure 5.3](#)), with one maximum at the frequency of the basic mode (ν_0).

The value of the saturation gain drops for each lasing mode, from the small signal gain to threshold gain $(G_A)_{th}$.

This process is called "**hole burning**" in the gain curve.

Conclusion:

Each moment, most of the energy stored inside the active medium is not used to create the radiation out of the laser.

Saturation Gain in a Continuous Wave Laser

In a **continuous laser**, energy is supplied continuously to the active medium.

Thus constant gain and constant output power are created.

We saw in [section 5.3](#) that the **threshold gain** was defined as the active medium gain, for which the loop gain is equal to 1.

It was explained that the threshold gain depends on the mirror's reflectivity's, and the losses inside the cavity.

At the moment of lasing, holes are generated in the gain curve, at frequencies of the [laser longitudinal modes](#).

These holes reduce the value of the gain from the small signal gain to the saturation gain.

Conclusion:

While operating in continuous mode, the saturation gain is equal to the threshold gain:

$$(G_A)_{th} = 1 / \text{sqrt}(R_1 * R_2 * M)$$

and Output Power of CW Laser

For the same laser, **increasing pumping cause increase in small signal gain**, but the saturation gain is unaffected, and remain equal to threshold gain $(G_A)_{th}$.

The **output power of the laser will increase** since both the small signal gain and the population inversion increases.

Increasing pumping cause the holes inside the gain curve to be filled more quickly, since the number of excited atoms is larger.

Saturation Gain in a Continuous Wave Laser

Figure 5.9 shows the influence of the input power in CW laser on the following factors:

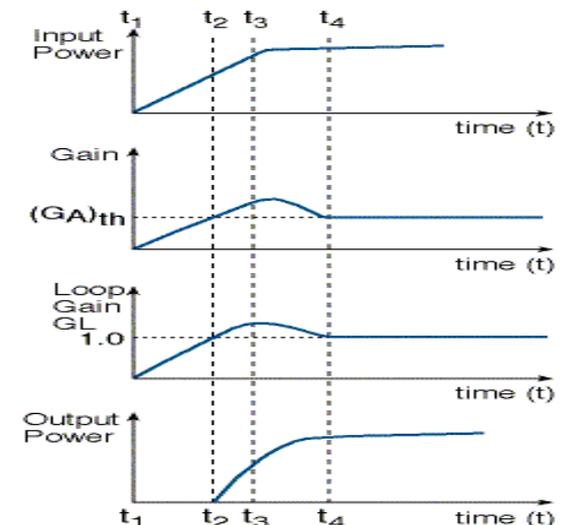
- 1.Active medium gain.
2. Loop gain.
3. Output power of the laser.

At time t_1 the excitation mechanism is activated. As a result, the active medium gain and loop gain increase.

At time t_2 the active medium gain is equal to the threshold gain, and the loop gain is equal to 1. Lasing starts, and output power of the laser start to increase.

At time t_3 the input power reaches its steady state (constant input power). The active medium gain is a little above threshold, and the loop gain is a little above “1”.

Output power from the laser continues to rise, **until t_4** , when it reaches its steady state value. Then the active medium gain is equal to the threshold gain, and the loop gain is equal to “1”.



Continuous Wave Laser

In a continuous wave laser at steady state lasing, the loop gain (G_L) is always “1”.

At this state, the gain value for each longitudinal laser mode is dropping from the value of the small signal gain to the threshold gain $(G_A)_{th}$, which is equal to the saturation gain.

Increasing pumping cause an increase in the output power of the laser.

The system will stabilize on higher power when the loop gain will be equal to the threshold gain.

Conclusions for continuous wave laser:

1. The saturation gain of the active medium is equal to the threshold gain $(G_A)_{th}$.
2. The loop gain in steady state operation is always equal to (1)

Pulsed Laser

Pulsed laser is pumped at high intensity for a short period of time.

As a result, the active medium gain, and the loop gain are much higher than for continuous wave laser, so the output power is higher.

We shall explain the **principle of operation of a pulsed solid state laser**, with the example of the [Ruby laser](#). [Section 7.3](#) examples on laser pulses.

Pulse Shape Out of a Pulsed Ruby Laser

Figure 5.10 describes the shape of a single pulse out of a Ruby laser, compared to the pumping pulse from the flash lamp.

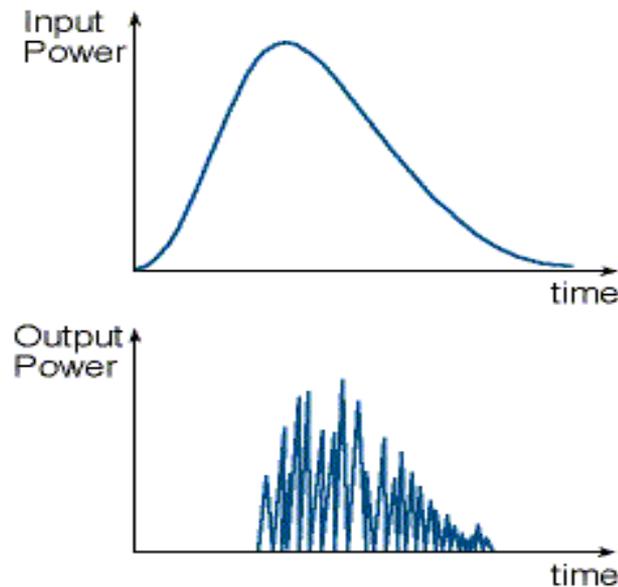


Figure 5.10: single pulse out of a Ruby laser, compared to the pumping pulse from the flash lamp

The output laser pulse is about 1 millisecond, and it is **composed of hundreds or thousands of small pulses**. Each of the small pulses is called a **spike**, and last about a microsecond.

The spikes appear randomly in time, and differ from each other in its length and peak power. Usually only the entire pulse is measured, without consideration of each spike.

The **average power per pulse** is calculated by timing the entire pulse, and measuring its energy.

In figure 5.10 it can be seen that **the laser pulse starts after a short time from the pumping pulse**. This is the time it takes the active medium to arrive at the threshold value for lasing.

Analysis of a single pulse from a solid state laser

The linewidth of a laser beam from a solid state laser is more than 30 [GHz] (3×10^{10} [Hz]). Each pulse has hundreds of longitudinal modes in it. For each of these modes, the process described in figure 5.11 applies.

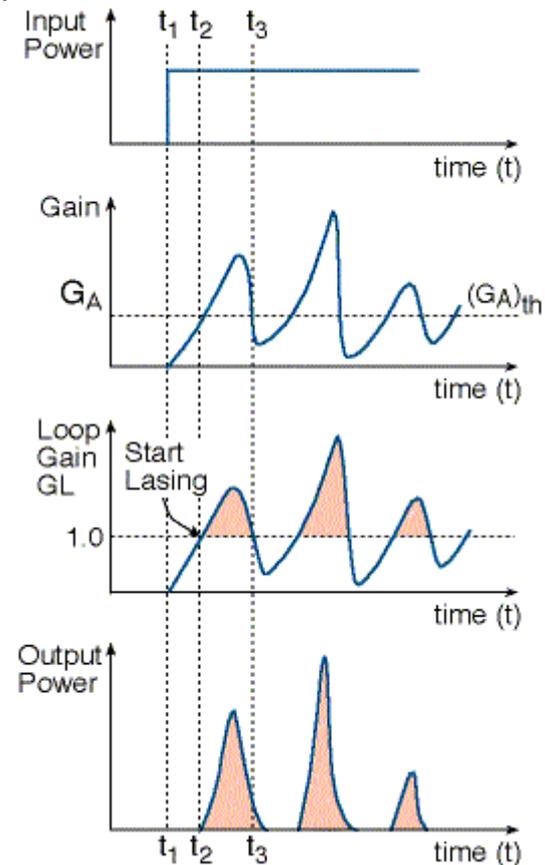


Figure 5.11: Gain and output power from a pulsed solid state laser.

Figure 5.11 describes a simple case of constant pumping of the active medium that starts at time t_1 .

- 1. Starting from t_1 ,** the active medium gain and the loop gain increase rapidly as a result of continuous strong pumping.
 - 2. At time t_2 ,** the active medium gain arrive to the threshold value, and the loop gain arrive to "1" - lasing starts. The active medium gain and loop gain continue to rise since the output power has not reach the saturation value that cause "hole burning" in the gain curve.
 - 3. Until time t_3 ,** the high value of the loop gain causes intense pulse of laser radiation. Thus, the active medium gain drops below the threshold value. When the loop gain is below "1", lasing stops, and the whole process starts again as long as the pumping continue.
- Each longitudinal laser mode starts at a different time, with a different photon. There is a competition between the longitudinal modes on the energy inside the active medium. Thus, the **random nature of the spikes: Each spike has its own peak power and duration.**

5.5 Summary of Chapter 5

1. **Lasing action** is possible only in those wavelengths for which the active medium has spontaneous emission.
2. The **Fluorescence line** describes the intensity of the fluorescence as a function of the frequency.
3. The **fluorescence linewidth** is measured the width of the fluorescence line at half its maximum height.
4. The **gain curve of the active medium** depends on the linewidth of the spontaneous emission of the specific laser transition.
5. Laser Linewidth can contain many longitudinal laser modes, and is determined by the upper part of laser gain curve above the threshold value: $(G_L) = 1$.
6. A **condition for lasing** is that the total gain will be a little more than the total loss.
7. **Loop gain (G_L)** is the **net gain** (Gain minus losses) of the radiation in a round trip through the laser cavity.

$$G_L = R_1 * R_2 * G_A^2 * M$$

M = Absorption Loss factor, describe the relative part of the radiation that remain in the cavity after all the losses in a round trip loop inside the cavity.

All the losses in a round trip loop inside the cavity are **1-M** (always less than 1).

$$M = \exp(-2\alpha L)$$

α = **Loss coefficient** (in units of cm^{-1}).

2L = Path Length, which is twice the length of the cavity.

For **continuous laser**, the **Threshold Loop Gain condition** is:

$$(G_L)_{\text{th}} = 1 = R_1 R_2 G_A^2 M = R_1 * R_2 * \exp(2(\beta - \alpha)L)$$

Chapter 6

Different Laser Types and their Characteristics

Chapters 1-5 laid the theoretical background which explains the **Lasing Process**, and the main parts of the laser.

This chapter explains the different types of lasers.

Lasers can be divided into groups according to different criteria:

1. The **state of matter of the active medium**: solid, liquid, gas, or plasma.
2. The **spectral range of the laser wavelength**: visible spectrum, Infra-Red (IR) spectrum, etc.
3. The **excitation (pumping) method of the active medium**: Optic pumping, Electric pumping, etc.
4. The **characteristics of the radiation emitted from the laser**.
5. The **number of energy levels which participate in the lasing process**

6.1 Gas Lasers

Most **elements** can be made to lase when they are in the gas state. Also many **molecules** (composed of a few atoms each) have been demonstrated to lase.

In a **gas laser**, **the laser active medium is a gas at a low pressure** (A few milli-torr).

The main reasons for using low pressure are:

- To enable an **electric discharge in a long path**, while the electrodes are at both ends of a long tube.
- To obtain narrow spectral width not expanded by collisions between atoms.

(A few types of special lasers use gas at high pressure).

The **first pulsing gas laser** was operated by **T. H. Maiman in 1961**, one year after the first laser (**Ruby**) was demonstrated.

The first gas laser was a **Helium-Neon laser**, operating at a wavelength of 1152.27 [nm] (Near Infra-Red).

Excitation of a gas laser

Two main excitation techniques are used for gas lasers:

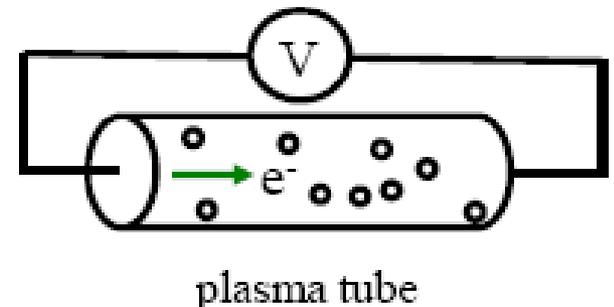
- Electrical Discharge
- Optical Pumping

• Excitation of Gas Laser by Electrical Discharge

- Applying high voltage to electrodes at both sides of the tube containing the gas causes **electrical breakdown through the gas**.

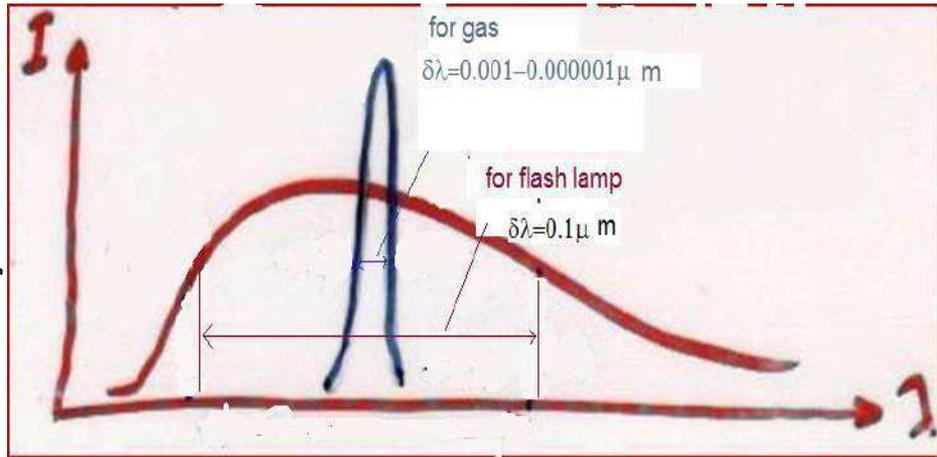
Electrons are ejected from the cathode, accelerated toward the anode, and collide with the gas molecules along the way.

During the collision, the mechanical kinetic energy of the electrons is transferred to the gas molecules, and excites them. (This same method of energy transfer is used in common fluorescent lights).



•Excitation of Gas Laser by Optical Pumping

- Exciting a laser medium by optical pumping, requires that the absorption spectrum of the medium will be similar to the **emission spectrum** of the pumping source, so that a big amount of the radiation will be absorbed.
- Conventional light sources used for optical pumping have broad emission spectrum, so only a small part of the light is used in the excitation process. Because gas atoms absorb only a small portion of the spectrum, optical pumping is not generally an efficient method for gas lasers.
- The absorption spectrum of solids are wider than the absorption spectra of gases, so the pumping efficiency of solid state lasers by conventional light sources are higher than that for gas lasers. Thus **gas lasers are usually excited by an electric discharge**. When we want to excite a gas laser by optical pumping, we need to find an **optical source with very narrow bandwidth**, which fits the narrow absorption spectral lines of the gas. A good source for optical pumping of a gas laser is another laser. This method is used for pumping **Far-Infra-Red (FIR) gas lasers** by a **CO₂ laser**



Groups of Gas Lasers

For convenience, **gas lasers are divided into 3 groups:**

- I. Atoms** - The laser active medium is composed of **neutral gas atoms** such as Helium-Neon and Copper Vapor.
- II. Ions** - The laser active medium is composed of **ionized gas** such as Argon ion gas or Helium-Cadmium gas.
- III. Molecules** - The laser active medium is composed of **gas molecules**, like Carbon Dioxide (CO₂), Nitrogen (N₂), Excimer laser, Chemical lasers (HF, DF), Far Infra-Red (FIR) laser.

I. Atoms

6.1.1 Helium-Neon (He-Ne) Laser

The Helium-Neon laser was the most common laser until the spread of [diode lasers](#) in the last few years. It was first built in 1961 by Ali Javan.

The [active medium](#) is a noble gas Neon (Ne), and it is a 4 level laser.

The energy level diagram of a Helium-Neon laser is described in figure 6.1.

Two **meta-stable energy levels** act as **upper laser levels**. The He-Ne laser have two lower laser levels, so quite a few wavelengths can come out of the transitions between these levels.

The important wavelengths are:

$\lambda_1=632.8$ [nm], $\lambda_2=1.152$ [μm], $\lambda_3=3.3913$ [μm],

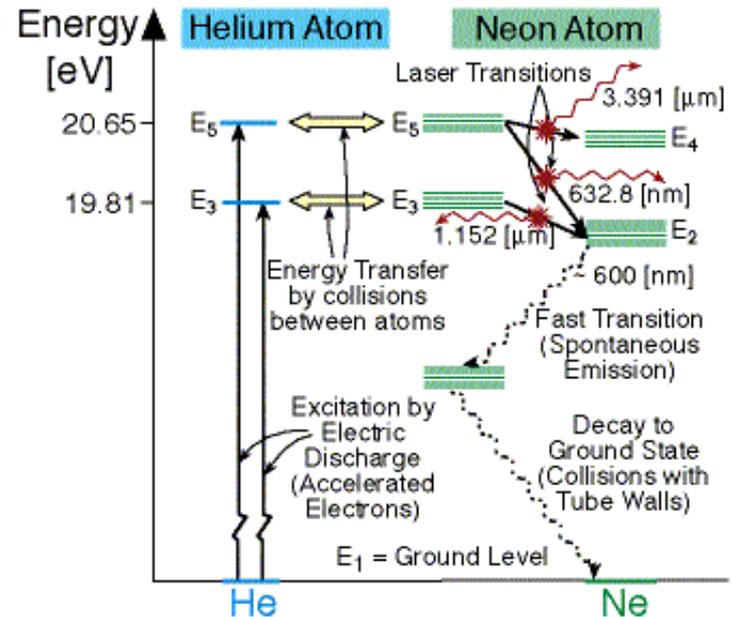
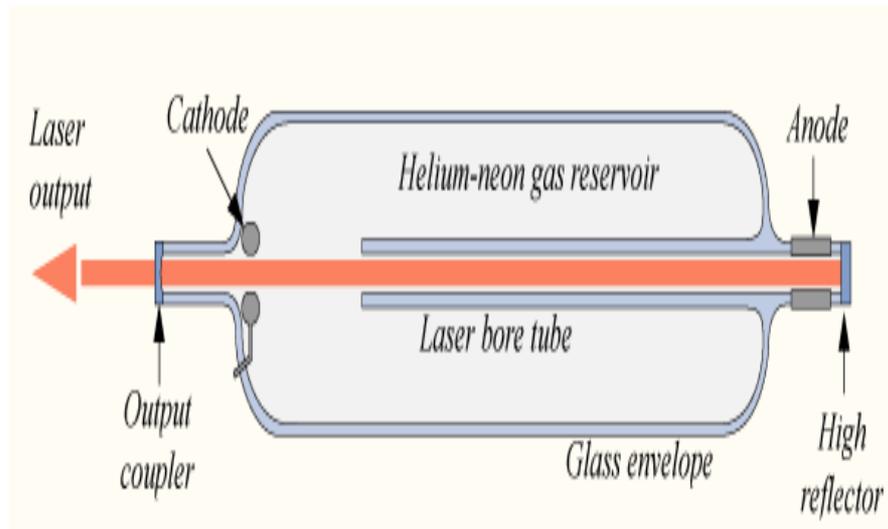


Figure 6.1: Energy Level Diagram of He-Ne Laser

The role of the Helium gas in He-Ne laser

The role of the Helium gas in He-Ne laser is to increase the efficiency of the lasing process. Two effects make Helium particularly important :

1. The direct excitation of **Neon** gas is inefficient, but the direct excitation of **He** gas atoms is very efficient.
2. An excited state of the **He** atom (labeled E_5) has an energy level which is very similar to the energy of an excited state of the **Neon** atom (also labeled E_5).

The excitation process of the **Neon** atoms is a **two stages process**:

- The high voltage causes electrons to accelerate from the cathode toward the anode. These electrons collide with the **He** atoms and transfer kinetic energy to them.
- The excited **Helium** atoms collide with the **Neon** atoms, and transfer to them the energy for excitation.

Thus **Helium gas does not participate in the lasing process**, but **increases the excitation efficiency** so that the lasing efficiency with it increase by a factor of about 200 (!).

Red Wavelength out of He-Ne Laser

Most of the applications of He-Ne Laser use the **red wavelength**, because it is the strongest line and it is in the visible region of the spectrum.

As shown in figure 6.1, this red light is emitted when the Neon atom goes from the energy level labeled E_5 to the energy level labeled E_2 , a much bigger energy difference than for the other transitions.

A problem with creating this red light is that a Neon atom in state E_5 may also emit **3.3913 $[\mu\text{m}]$** radiation. This emission decreases the population of the E_5 level, without producing visible radiation.

The solution to this problem is to use a **special coating** on the laser mirrors which **selectively reflect only the red light**. This coating causes reflection back into the optical cavity of only the desired (red) wavelength, while all other wavelengths are transmitted out, and not forced to move back and force through the active medium.

In a similar way, other selective reflecting coating can be used on the mirrors to select other transitions. This procedure allows commercial production of He-Ne lasers at other wavelengths in the visible spectrum. For example, **orange, yellow and green** He-Ne lasers can be produced, but the laser efficiency is much lower than for the red.

Absorption and Amplification in He-Ne Laser

As light moves through the active medium, two different processes act on the radiation:

absorption and **amplification**. In a standard He-Ne laser, the **amplification by the active medium is about 2%**. During one pass through the active medium (from one mirror to the other) the amount of radiation inside increases by 1.02.

Thus, to get amplification of light, all the losses, including collisions of the excited atoms with the walls of the gas tube, absorption by other molecules, etc. should be less than 2%.

He-Ne laser is a 4 level laser, so the lifetime of the **lower laser energy level** needs to be very short. In a Neon gas, which is the active lasing gas, the transition (decay) from the lower laser level is not fast enough, but it is accelerated by collisions with the tube walls.

Because the number of collisions with the tube walls increase as the tube becomes narrow, the laser gain is inversely proportional to the tube radius. So, **the tube diameter of a He-Ne laser must be as small as possible**.

The **low gain of the active medium in a He-Ne laser limits the output power to low power**. In laboratory prototypes an output power of the order of 100 [mW] was achieved, but commercial lasers are available only in the output range of 0.5-50 milliwatts [mW].

The **output coupler** of He-Ne laser is a mirror with coating that transmits about 1% of the radiation to the output. This means that the power inside the optical cavity is a 99 times more than the emitted power.

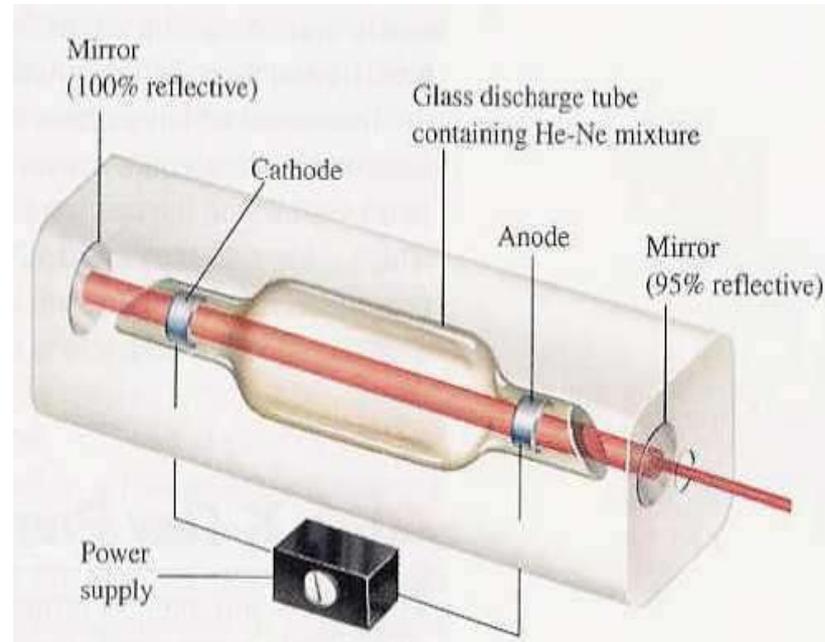
He-Ne laser structure

Helium Neon laser has three main elements :

› Plasma Tube.

› Optical Cavity.

› Power Supply.



› Plasma Tube of He-Ne Laser:

The **thin inner tube** has a diameter of about 2 [mm] and length of tens of centimeters. The inner tube is surrounded by **thick outer tube** with diameter of about 2.5 [cm] and is sealed from the outside.

The purposes of the outer tubes are:

- ▶ To make a **stable structure** which protects the inner tube and the laser mirrors from movements.
- ▶ To act as a **large gas reservoir** which refreshes the Neon gas that has been absorbed by the cathode.

The lasing process, which creates the electromagnetic radiation, is confined to the inside of the inner tube which is filled with the gas mixture.

The **gas mixture** is 85-90% Helium gas, and 10-15% Neon gas, a ratio of 1:6 to 1:10.

The **gas pressure** is 0.01 Atmosphere (≈ 10 [torr]).

At the end of the tube, the electrodes are attached to a high voltage power supply (AC or DC).

› Optical Cavity of He-Ne Laser:

The cavity in a common He-Ne laser uses a **semi confocal optical cavity**. It is composed of **one planar mirror**, which reflects about 98-99 % of the light striking it, and a **second concave mirror reflecting 100%**. This concave mirror has a focal length equal to the length of the cavity (see figure 6.2).

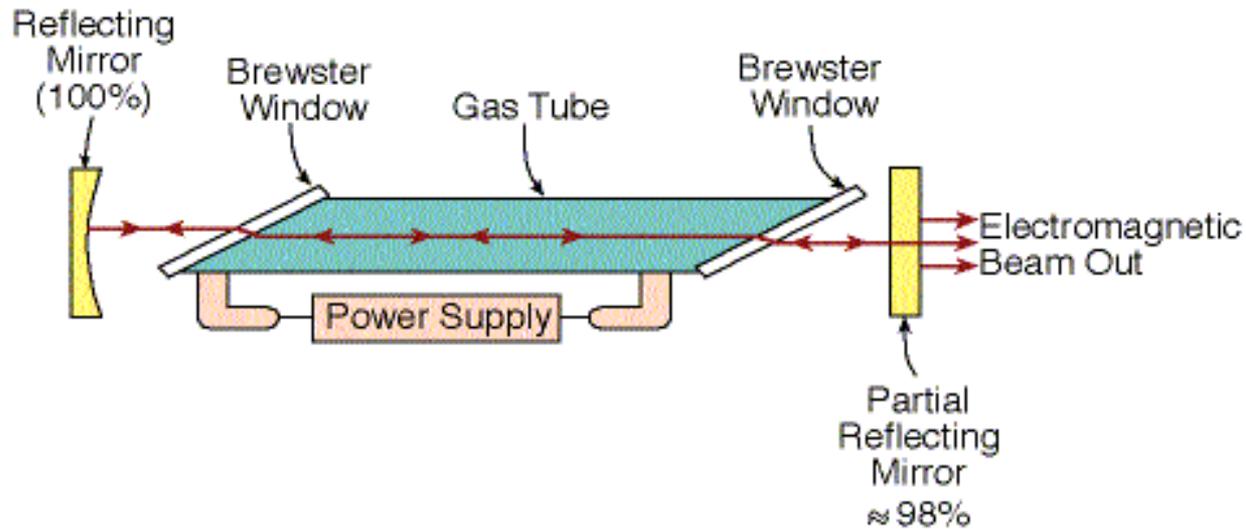


Figure 6.2: The Cavity Structure of He-Ne Laser

This arrangement of the mirrors causes the radiation to be an almost parallel beam. The importance of **Brewster windows** is for create polarization of the out put of the laser beam.

› Power Supply of He-Ne Laser:

He-Ne lasers which gives up to 1 [mW] , usually use a **DC Current**, High Voltage power supply of 2,000 [Volts].

The laser needs a constant current (constant supply of electrons), so a **stable current supply** is used.

To initialize the lasing action, The high voltage pulses applied to the electrodes at the ends of the tube cause the accelerated electrons to collide with the gas in the tube. This voltage is called the **Ignition Voltage** of the laser. At the moment of beginning breakdown, the electrical resistance of the tube suddenly falls to a low value. This means that the voltage suddenly drops, while the current rises. Thus, by Ohm's law it is a **negative electrical resistance** (Decrease in voltage with increase in current).

To overcome this problem, a **Ballast Resistor** is connected very close to the anode, in series with the power supply. The role of the ballast resistor is to limit the current through the tube when the tube resistance falls.

For example, for a laser with operating current of 5 [mA], the Ballast Resistor is 60-90 $K\Omega$, and the voltage on it is 300-450 [Volts].

After lasing action begins, the supply voltage is dropped to about 1,100 [Volts], needed for the continuous operation of the laser.

Commercial He-Ne Lasers:

Wavelength:	632.8 [nm]
Output Power:	0.5-50 [mW]
Beam Diameter:	0.5-2.0 [mm]
Beam Divergence:	0.5-3 [mRad]
Coherence Length:	0.1-2 [m]
Power Stability:	5 [%/Hr]
Lifetime:	>20,000 [Hours]

6-1-2 Metal Vapor Laser

As the name implies, **the active medium in this laser is a vapor consisting of metal atoms**. There is a distinction between two types:

a. Neutral metal vapor lasers, which include:

1. Copper vapor laser (CVL).
2. Gold Vapor Laser (GVL).

b. Ionized metal vapor laser, which includes:

Helium-Cadmium (He-Cd) Laser.

All metal vapor lasers emit visible electromagnetic radiation in a form of rapid pulses and with high efficiency.

We shall concentrate on Copper Vapor Laser as an example for neutral vapor lasers.

Copper vapor laser (CVL)

Lasing action in copper vapor was first demonstrated in 1966.

The first commercial copper vapor lasers appeared around 1980.

This laser was attractive because of its **relative high efficiency (up to 1%)** for lasers in the visible spectrum range, and the high pulse power achieved.

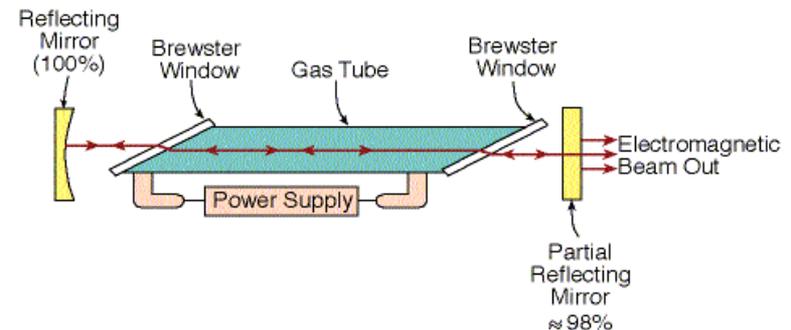
Copper Vapor Laser Structure

Copper vapor laser is a gas laser, build as a tube with windows at both ends (like He-Ne laser).

The tube is filled with an **inert gas** and a small quantity of pure copper.

In order to have **copper vapor**, the metal needs to be at very high temperatures, so the tube is build from **Aluminum or Zirconium**, which are high temperature resistant materials.

The tube diameter is 10-80 [mm], and it contain **Neon gas** at a pressure of 25-50 [Torr].



Copper Vapor Laser operation

The melting temperature of Copper is 1083°C . At temperatures higher than the melting point, **Copper vapors** are created at high enough concentration so they can serve as an **active medium** of the laser.

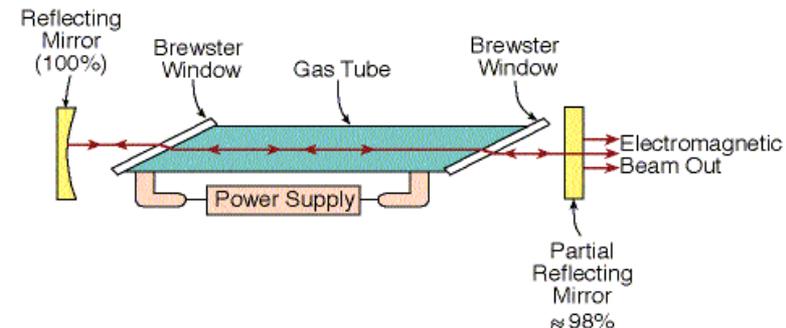
A solid bulk of **pure Copper metal** is inserted in the middle of the tube before it is filled by the **neon gas**.

Electric breakdown is created by high voltage to the electrodes at the ends of the tube. As a result, the temperature rises inside the tube cavity, until the Copper evaporates, and the vapor pressure of the Copper is about 1 [Torr].

The measured temperature on the outside of the tube can reach $1400\text{-}1500^{\circ}\text{C}$.

During the laser operation only a small fraction of the Copper atoms are ionized, and they are moving (electrical attraction) toward the ends of the tube. There, the vapor cool down, and transform to solid metal.

As a result, the amount of Copper vapor in the tube is reduced. After a few hundred hours of operation, new Copper must be inserted into the tube.



The high voltage pulses applied to the electrodes at the ends of the tube cause the accelerated electrons to collide with the Copper vapor molecules, exciting them into one of the two available high laser energy levels, as seen in figure 6.3.

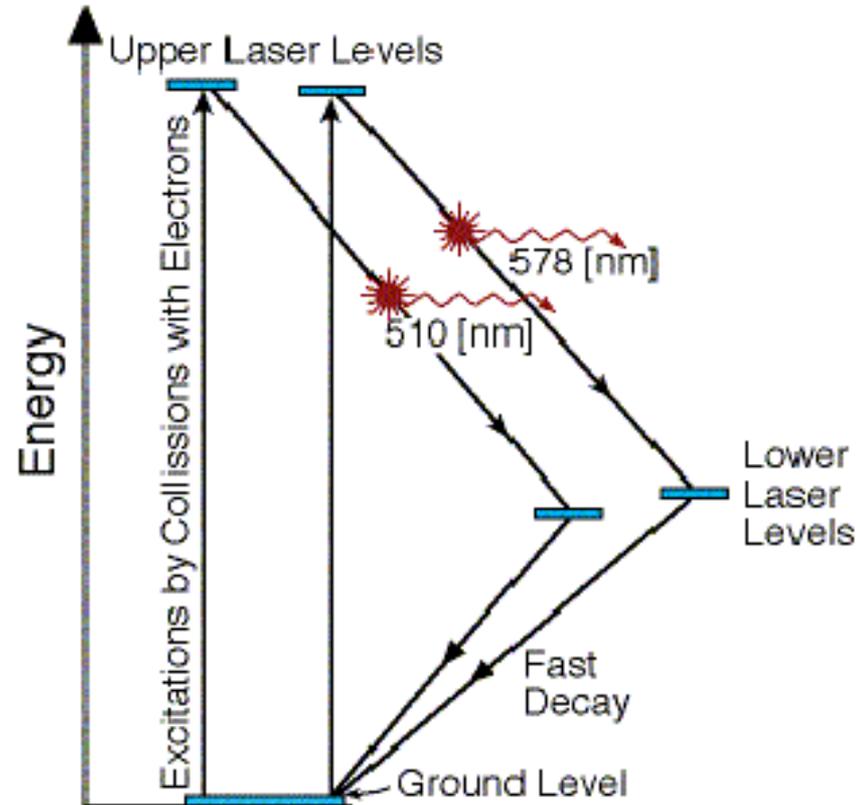


Figure 6.3: Energy Level Diagram of Copper Vapor Laser

The wavelengths emitted by Copper vapor lasers are:

$\lambda_1 = 510.6$ [nm] (Green),

$\lambda_2 = 578.2$ [nm] (Yellow)

Why copper vapor laser is restricted to pulsed operation?

Unfortunately, both of these laser transitions end at lower laser energy levels which are meta-stable (with long lifetimes of hundreds of micro-seconds).

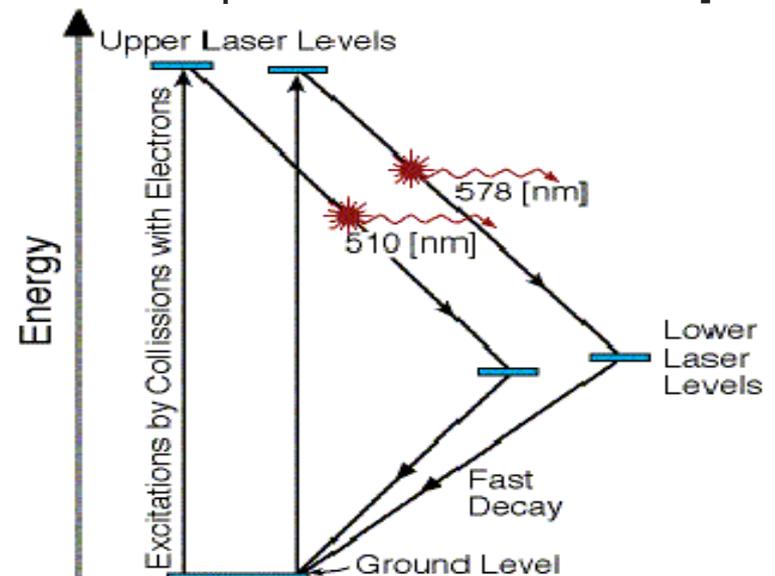
Since the population of these energy levels increases rapidly, the condition of population inversion is destroyed, and laser action stops.

After the lasing stops, the lower energy levels decay to the ground state by collisions of the excited molecules with the tube walls.

Then, another laser pulse can form. The time of each laser pulse is less than 100 [nsec]

The copper vapor laser is a three level laser:

1. Ground state of the Copper atom.
2. Upper laser energy level.
3. Lower laser energy level.



Summary of Copper laser properties:

- **Copper vapor pressure** is about 1 [Torr].
- **Optimal operating temperatures:** $1650^{\circ}\text{C} \pm 50^{\circ}\text{C}$.
- The laser is very sensitive to the purity of the active gas.
- The laser **operates simultaneously on two spectral lines** (Green and Yellow) with no competition between them (separate levels).
- The energy per pulse of the green line (510.6 [nm]) depends on the frequency of the applied electric pulse .
- Copper vapor lasers have very high gain, and can operate even without an optical cavity.
- In practice, one mirror reflects 100%, and the other about 10% (Fresnel reflection without coating can be used).
- The **high temperature** required for the lasing process, is achieved by heating as a result of the electric breakdown in the gas.
- It is possible to achieve lasing at lower temperatures (400°C), by using Copper salts like CuCl , but there are still problems with these lasers, and they are in experimental stages.

II. Ionic Gas Lasers:

- The electronic configuration of inert gases is $(1s^2 2s^2 \dots np^6..)$ so the electronic configuration of the ionized gases is the same as the neutral case **losing one electron.**
- For exp. The electron configuration of **Ar⁺:** $1s^2 2s^2 2p^6 3s^2 3p^5$

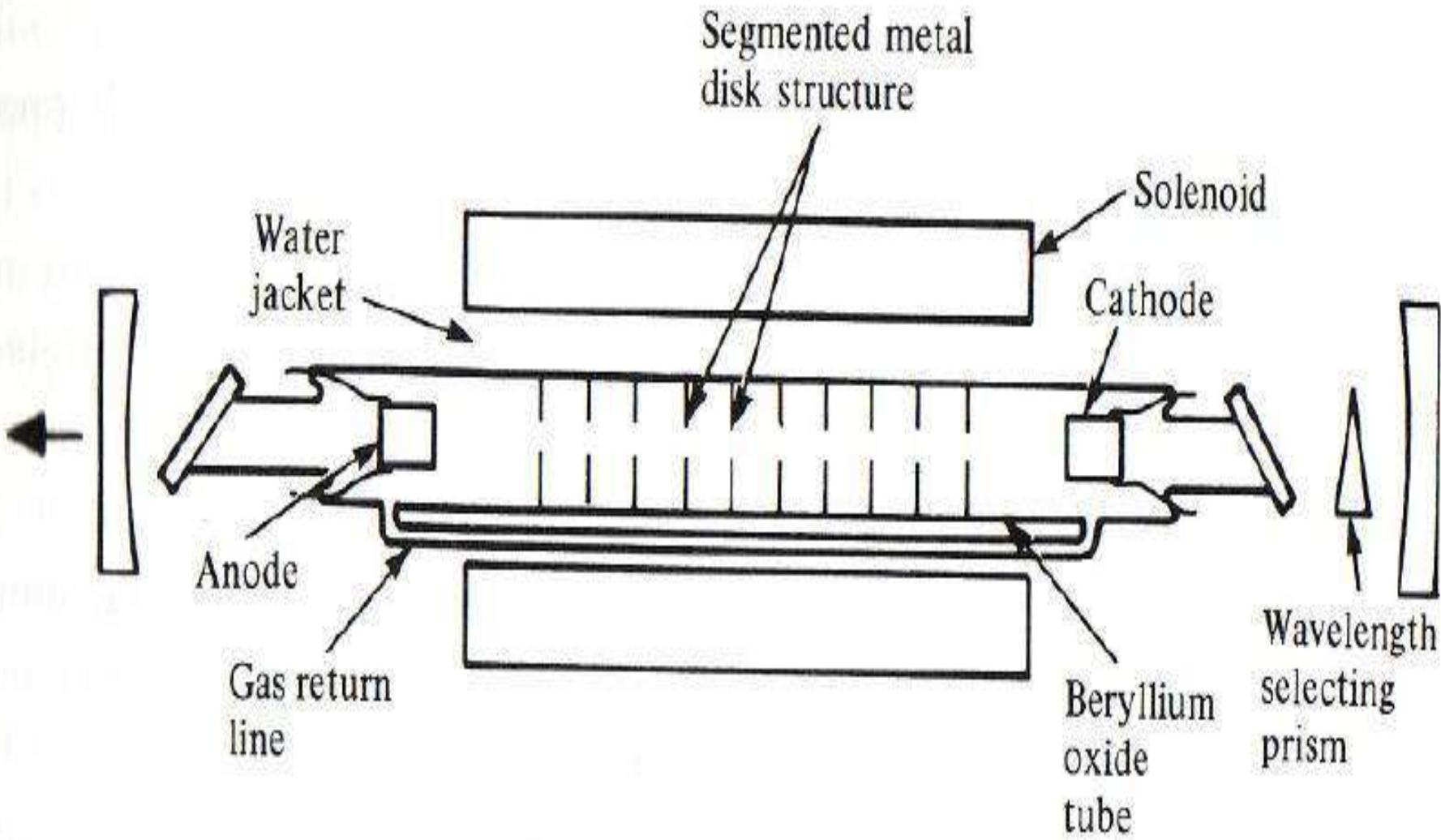
6.1.3 Argon Ion (Ar^+) Gas Laser

- The Argon laser was invented in 1964 by **William Bridges**.
 - Argon ion laser contains a tube filled with Argon gas which transforms into **plasma** in an excited state.
- The principle of operation is similar to atomic gas lasers but high discharge currents are used to strip the atoms of electrons to form ions.

Powerful Continuous Wave operation (CW) can be achieved by complicated discharge geometry to maximize the pumping power.

Several watts CW output or up to 1 kilowatt in microsecond pulses can be generated.

The schematic diagram of the Ar⁺ ion laser device is shown in the following figure;



•A schematic diagram of the energy levels of the Argon Ion laser is shown in figure 6.4.

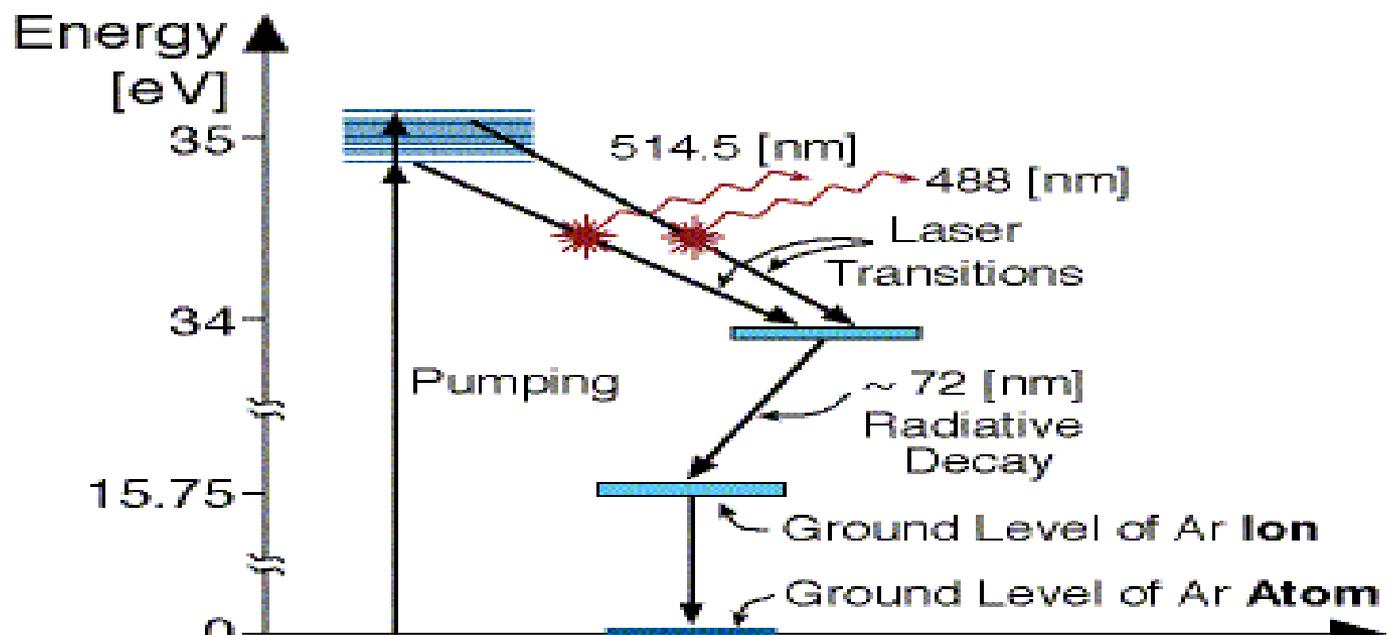


Figure 6.4: Energy Level Diagram of Ion Argon Ion Laser.

- The two main laser transitions are at visible wavelengths:
- Blue 0.488 [μm]
- green 0.5145 [μm],
- but the Argon ion laser emits also in the UV spectrum:
- 0.3511 [μm]
- 0.3638 [μm].

- Argon ion laser pumped electronically; they operate in continuous wave, the kind of pumping is the collision of first kind:



High discharge current 15-50 A. Pump to 4P states, 35eV above ground state by multiple collisions.

Transitions correspond to 4P-4S, 514.5nm and 488nm.

Use Brewster window at ends of gas tube to isolate a single polarization with minimum reflection losses.

• Power Output from Argon Laser

When considering the power output of the Argon Ion laser, it is **important to state if the power output is at all the laser lines together, or at a specific wavelength.**

Some applications require specific wavelength which can be chosen by a grating or prism at the end of the optical cavity.

The ionized gas lasers are the only visible lasers that produce many colors with comparatively high power (up to a few Watts).

• Argon (Ar^+) laser efficiency

We see from the diagram in figure 6.4 that the lasing energy levels belong to the **Argon ion**, so **the atoms of the gas inside the tube need to be ionized first.**

• As seen in the diagram, **the ground state of the laser is at about 16 [eV] above the ground state of the neutral Argon atom.** This is a large amount of energy that must be supplied to the laser, but is not used for creating laser radiation. This "wasted" energy is one of the reasons for the **very low efficiency of the Argon laser (0.1%).**

Output Power from Argon Laser

The **gain of the active medium in Argon ion lasers is very high**, so **high power** can be achieved from Argon ion lasers (tens of Watts), although as we saw, with low efficiency. The **output power** increases as a nonlinear function of the **current density** in the tube. Thus it is common to **use narrow tubes (small cross section) and very high currents (100-500 [A/cm²])**. Argon Ion lasers require a separate three phase electrical power lines.

The ignition of the Argon Ion laser is done by a **pulse of high voltage** (about 10 KV DC) ionizes the argon gas.

After ionization, a few hundreds volts DC are maintained across the laser tube. A high DC current (more than 50 Amperes) maintains lasing, Such high current densities create **large amounts of heat** which must be taken away from the laser. **Argon Ion lasers require water cooling.**

In order to withstand the high temperatures, the laser tube is made from special high melting materials such as Beryllium Oxide. This material has very high thermal conductivity, and is not destroyed by the electrical discharge.

The radiation of Argon Ion laser is hazardous to view, and working with it **requires special protecting goggles** for everyone in the room.

- Characteristic of Ar ion laser:-

(1) Out put wave length is in $\lambda_{\text{out}}=488$ or 514.5 nm

(2) Max out put power $p_{\text{max}} \approx 200$ watt

(3) Full angle of divergence $=2.3$ m.rad

(4) Beam diameter ≈ 0.6 m.m

6.1.4 Helium-Cadmium Laser

Helium-Cadmium lasers can be categorized among ion gas lasers - The properties of Helium-Cadmium laser are similar to those of Helium-Neon laser which is a neutral atom gas laser.

The He-Cd laser is a gas laser, and the metal Cadmium can be transform into the gas phase by heat.

The excitation to the upper laser level of the Cadmium atoms in the gas is similar to the excitation process of the Neon gas in a Helium-Neon laser:

Helium atoms are excited by collisions with accelerated electrons, and than they pass their energies to Cadmium atoms by collisions.

The transitions in Helium-Cadmium laser are between energy levels of **singly ionized Cadmium atoms**, and about **twelve lines are available**. These wavelengths are in the **shorter wavelength region, violet and Ultra-Violet (UV)**.

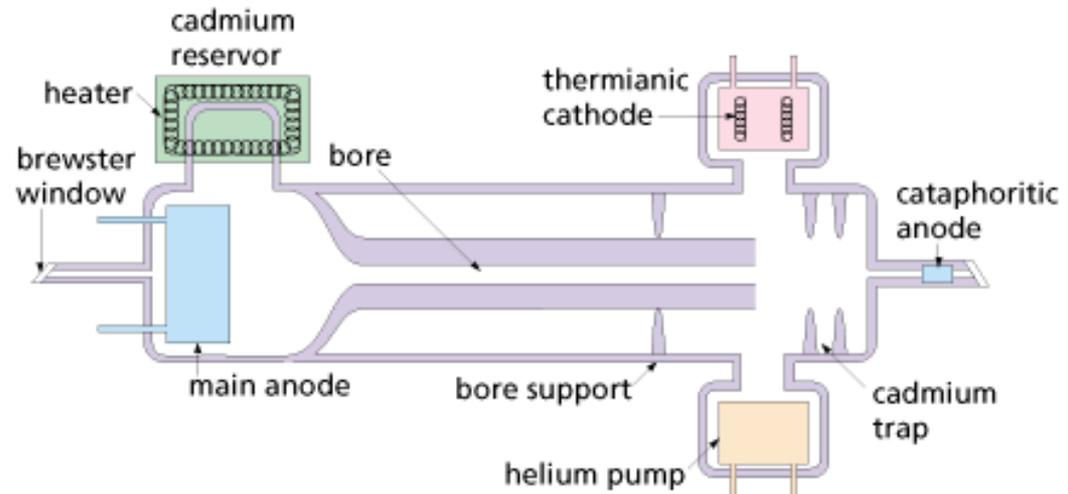
Lasing action in a Helium-Cadmium Laser:

The Cadmium metal is heated to a temperature of 250°C , to create the appropriate vapor pressure.

The Cadmium vapor pressure of a few millitorr is added to Helium gas at a pressure of 3-7 millitorr.

Since Helium is a noble gas, its excitation energy is very high (24.46 [eV]) compared to the Cadmium which is a metal with low excitation energy 8.96 eV.

Thus in He-Ne laser the Helium remains electrically neutral, and fills the cavity of the tube, while the **positive Cadmium atoms are moving toward the negative cathode.**



The **practical problem** in Helium-Cadmium laser is to maintain homogeneous distribution of the metal vapor inside the electrical discharge tube. The ions are attracted to the cold windows at the ends of the cavity. In order to prevent coating of the windows with Cadmium, cold traps are put before the laser windows.

In the design of the tube of Helium-Cadmium laser most of the effort is to reduce to a minimum the amount of Cadmium ions on the cathode. The best He-Cd lasers lose about 1 [g] Cadmium metal for 1,000 hours of operation of the laser.

For comparison, the gain and power output of the main two lines of He-Cd laser are higher than for the He-Ne laser, but less than for the Ar⁺ laser.

Characteristics of He-Cd lasers:

- **Output wavelengths:** Blue light $0.4416 \text{ } [\mu\text{m}]$, and Ultra-Violet (UV) light $0.3250 \text{ } [\mu\text{m}]$.
- **Maximum output power:** 150 [mW] in the blue line, and 50 [mW] at UV.
- **Maximum total efficiency:** in the blue line 0.02% , and in the UV 0.01% .
- **Spectral width:** 0.003 [nm] (about 5 [GHz]), and **coherence length:** about 10 [cm] .
- **Distance between two longitudinal modes:** about 200 [MHz] .

III. Molecular Gas Lasers

In a molecule, the main energy levels are subdivided into **vibrational energy levels**. Each vibrational energy level can be subdivided into rotational energy levels:

- 1. Vibrational energy levels** - energy levels associated with the **oscillation of the atoms** in the molecule.
- 2. Rotational energy levels** - energy levels associated with the **rotation** of the molecule.

Since these energy levels are subdivisions of the main energy levels, the difference between the two vibrational energy levels in which lasing occurs, is much smaller than the difference between main electronic energy levels. Thus, the wavelengths associated with the energy transitions among these levels is longer, and is usually in the Infra-Red (IR) spectrum.

Among the molecular lasers, the most common laser is the Carbon-Dioxide (CO₂) laser.

6.1.5 Carbon-Dioxide (CO₂) Laser

These are lasers having very high efficiencies (20%) compared with other gas lasers {like He-Ne and Ar⁺ lasers}.

CO₂ is the gas in which the lasing process occurs, but other gas additives to the laser tube improve the total efficiency of the laser.

The standard CO₂ laser includes in the active medium a mixture of CO₂ with N₂ and He. The optimal proportion of these 3 gases in the mixture depends on the laser system and the excitation mechanism. In general, for a continuous wave laser the proportions are:

CO₂:N₂:He - 1:1:8

CO₂ is a linear molecule, and the three atoms are situated on a straight line with the Carbon atom in the middle.

In figure 6.5 the **three vibrational modes of CO₂ molecule** are illustrated:

1. Symmetric stretch mode (ν_1).
2. Bending mode (ν_2).
3. Asymmetric stretch mode (ν_3).

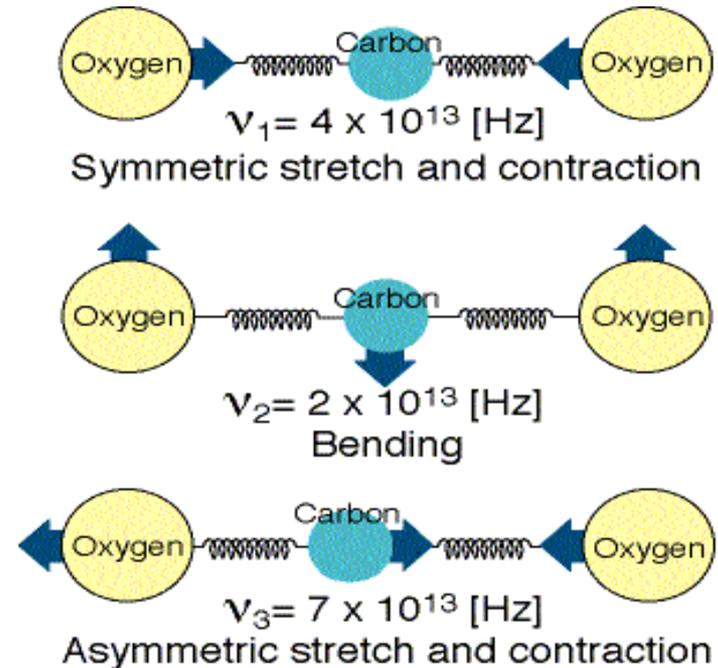


Figure 6.5: Oscillation Modes of CO₂ Molecule

Lasing transitions in CO₂ laser

Lasing transitions in CO₂ laser occur when the molecule is going from higher energy level of the asymmetric mode into one of the other two, as can be seen in figure 6.6.

1. The transition to the symmetric stretching mode correspond to the wavelength of 10.6 [μm].
2. The transition to the bending mode correspond to the wavelength of 9.6 [μm].

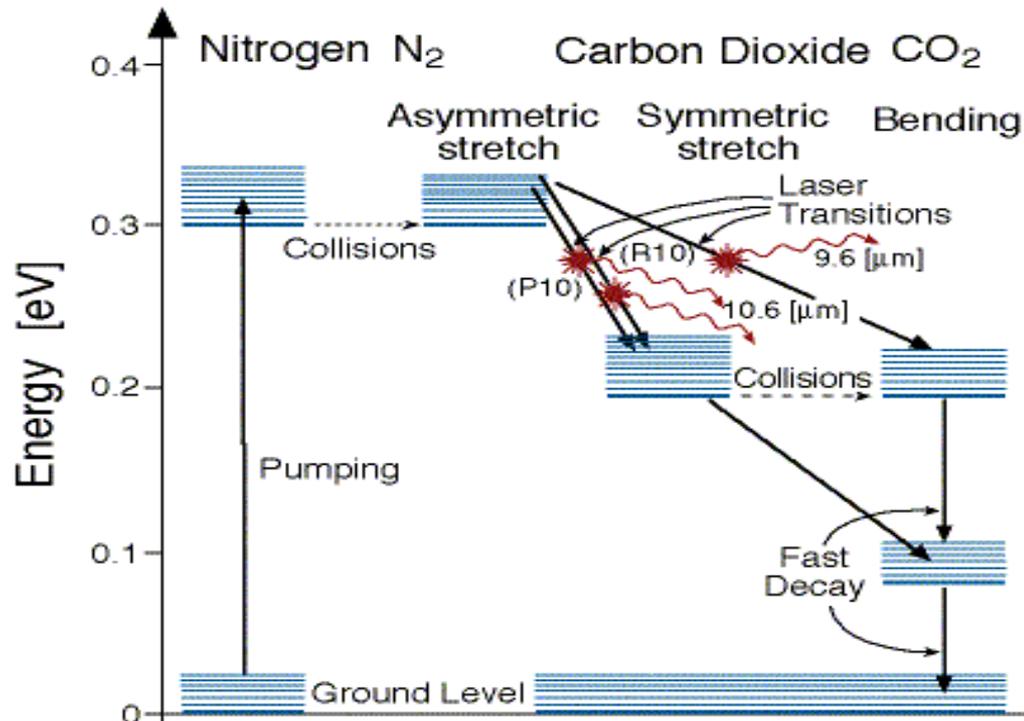


Figure 6.6: Energy Level Diagram of CO₂ Laser

Each of the vibrational energy level is subdivided into **many rotational levels**. Transitions can occur between vibrational energy levels with different rotational levels, so there are many lasing lines around the main vibrational transitions.

CO₂ laser operation

Electric discharge is created in the laser tube. The energy of the accelerated electrons is transferred by collisions to the **Nitrogen molecules** and to the **CO₂ molecules**.

Excitation of CO₂ laser is given as;



Nitrogen molecules help in the process of the excitation of the CO₂ molecules. The first vibrational energy level of the Nitrogen molecule is very similar to the asymmetric stretching mode of the CO₂ molecule (see figure 6.6), so energy can be easily transferred from the excited Nitrogen molecules to the CO₂ molecules.

Helium molecules are added to the gas mixture in order to:

1. Empty the lower laser energy level so that population inversion is maintained.
2. Stabilize the electrical discharge by **taking heat away** from the lasing area.

Gas pressure inside the CO₂ laser tube is 5-30 [Torr], of which 10% CO₂ gas, 10% N₂ and the rest is He.

Types of CO₂ Lasers:

There are many types of CO₂ lasers, **all based on the same physical principles**. The difference between them is in their **structure**, **excitation mechanism**, and **the output radiation**.

A Few CO₂ lasers are described below.

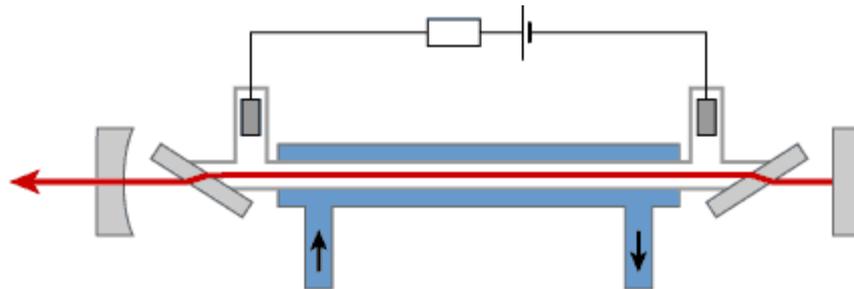
Sealed off CO₂ laser

The gas laser is filled with the appropriate mixture of gasses and sealed (as we saw in [He-Ne laser](#)).

High electric voltage is applied to electrodes at both ends of the gas tube. The accelerated electrons excite the gas molecules.

The problem with sealed off lasers is the dissociation of the CO₂ molecules into CO and Oxygen with time. To reduce this effect, a **catalyzing agent** is added to the gas mixture. This catalyzing agent reverses the dissociation reaction and restores the CO₂ molecule which is required for lasing.

Sealed off CO₂ lasers are usually limited to output power of less than 200 watts. For higher output power it is necessary to take away the heat generated inside the laser, and a flowing gas is needed.

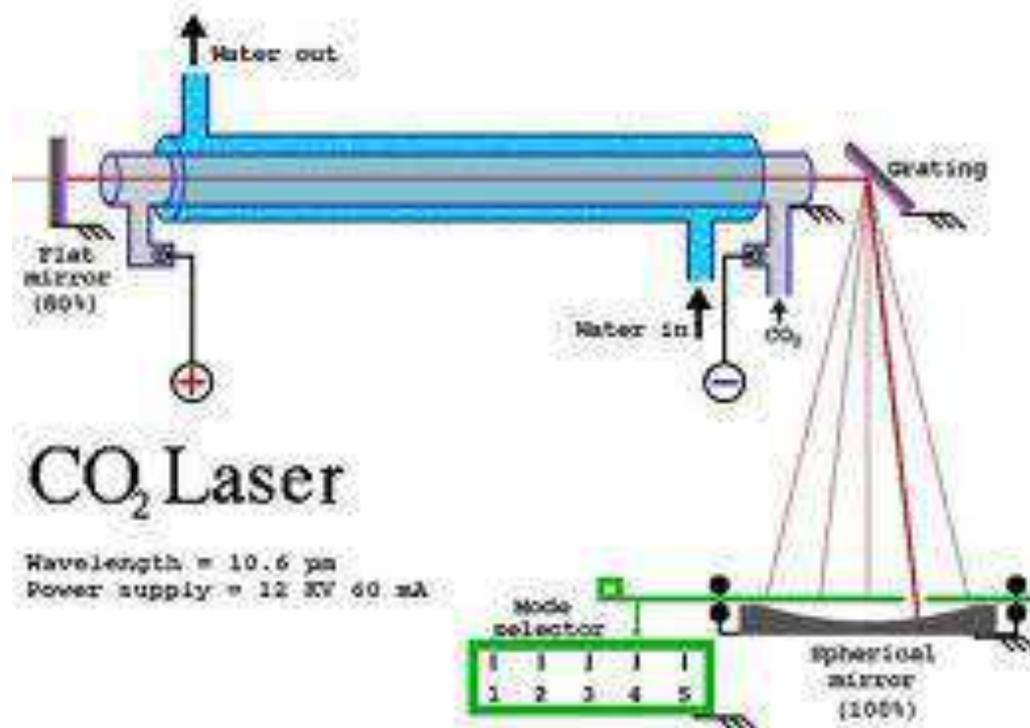


Flowing CO₂ gas lasers

In these lasers a **fresh gas mixture is flowing continuously through the laser tube while lasing lasts.**

Flowing gas is used when the **maximum power is needed out of the CO₂ laser.**

The gas flows along the tube and is released out into the atmosphere (since it is non poisonous). These lasers are very simple, and the requirements from the gas purity are small. Hundreds of watts can be achieved at the output of these lasers.

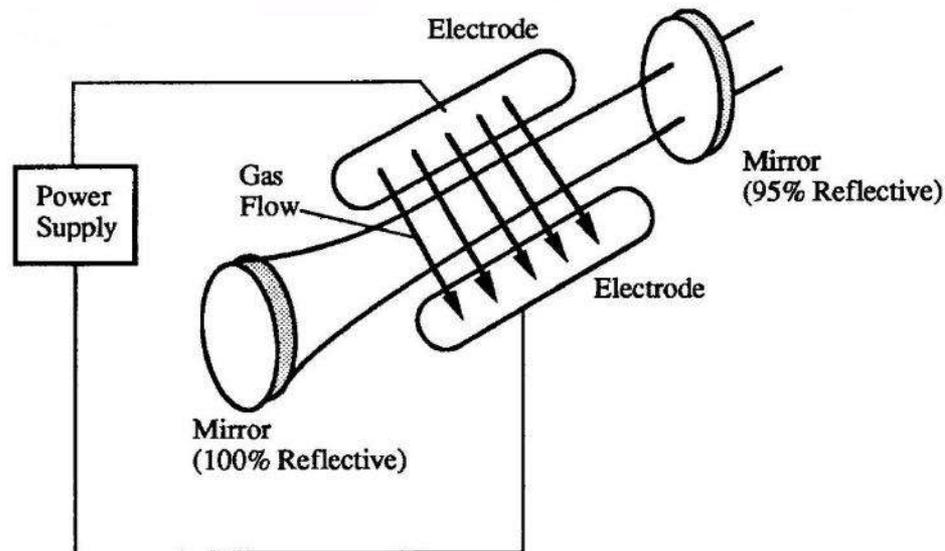


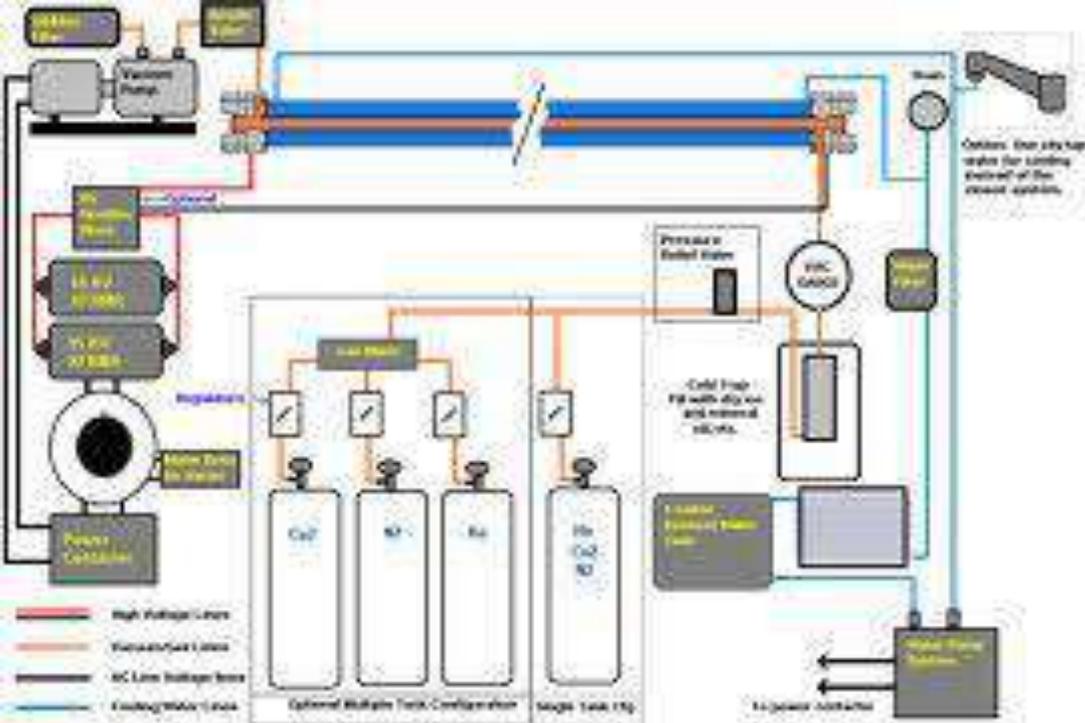
Transverse Flow CO₂ gas lasers

When the **gas flow inside the laser is perpendicular to the laser axis**, it is possible to have a **much larger flow** since the distance is very short.

Since **cooling by gas flow is very efficient**, it is possible to **get very high power output out of these lasers**.

Both the gas flow and the electric discharge in these lasers are along the width of the laser. Thus, the distance between the electrodes is short, so electric discharge can be achieved even for gas at very high pressure (up to a few atmospheres). Transverse flow is used for very high power CO₂ lasers.





properties of CO₂ Laser

- **High output power.** Commercial CO₂ Lasers produce more than 10,000 watts continuously.
- Output spectrum is in the **Infra-Red (IR) spectrum: 9-11 [μm]**.
- **Very high efficiency (up to 30%).**
- **Can operate both continuously or pulsed.**
- **Average output power is 75 [W/m] for slow flow of gas, and up to few hundreds [W/m] for fast gas flow.**
- **Very simple to operate, and the gasses are non-toxic.**

Summary of CO₂ lasers according to groups:

- Gas laser.
- Emit in the Infra-Red (IR) spectrum ($\lambda = 9-11$ [μm]).
- Electrical excitation.
- Continuous wave, although pulsed operation is possible.
- Four level laser.

6.2 Solid State Lasers

As we mention at the beginning of chapter 6.1, it is convenient to excite a laser whose active medium is solid with "**Optical Pumping**".

The atoms in a solid are close to each other, and the interaction between neighbors is strong. Thus, **the absorption and emission spectrum ranges in solids are much wider than those of gasses**. Wide absorption spectrum allows pumping of the active medium with a "conventional" light source, which has a wide emission spectrum.

In **Optical Pumping** the active medium is excited by illuminating it with external electromagnetic source. The photons from the external source are absorbed by the material of the active medium, thus transferring energy to its molecules.

Two types of electromagnetic sources are used in optical pumping:

- Source of **wide band electromagnetic spectrum**- such as Flash lamps.
- Source of **narrow band electromagnetic spectrum** - **another laser**.

Structure of the active medium in Solid State Laser

The **active medium** in solid state lasers is a medium of one solid material, in which **impurity ions of another material** are scattered. These impurity ions are replacing atoms of the solid background, and **the energy levels which participate in the lasing process are those of the ions of impurity.**

The solid background influence on the energy level structure is minor. Thus, the same impurity ion embedded in different host material will emit at very close wavelengths. **The optical properties of the laser are dictated mostly by the impurity ion.**

On the other hand, the **physical properties of the active medium** such as thermal conductivity, thermal expansion, are determined by the solid host. Thus, **the solid host determine the maximum power levels which can be emitted from the laser.**

Optically Pumped Solid State Lasers

The active medium in these lasers is a crystal or glass.

The shape of the active medium is usually a **rod with circular or square cross section.**

The pumped beam usually enter the active medium via its surface area along the rod, while the laser radiation is emitted through the ends of the rod. The ends of the rod are usually at right angles to the rod axis, and are optically polished.

Solid state lasers emit radiation in either **pulsed mode** or in **continuous mode**.

The pump lamps for pulsed lasers are usually Xenon (or Krypton) flash lamps, in which a low pressure gas is contained within quartz tube.

The pump lamps for continuous lasers are usually Halogen lamps, or high pressure Mercury discharge lamps .

Arrangement of Pump and Laser Rod

There are many ways to transfer as much pump light as possible from the lamp to the active medium.

The most common method is to use an **elliptic optical cavity**. The lamp is at one focus of the **ellipsoid**, and the rod of the active medium at another focus, as described in Figure 6.12.

The another method is the **Spiral lamp** around the laser rod.

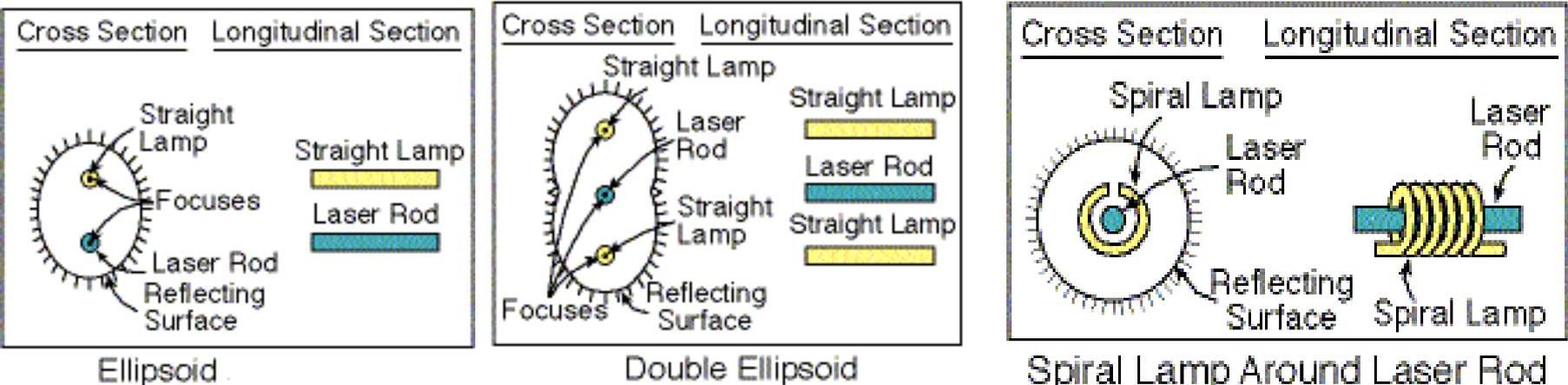


Figure 6.12: Methods of Optical Pumping of Solid State Lasers.

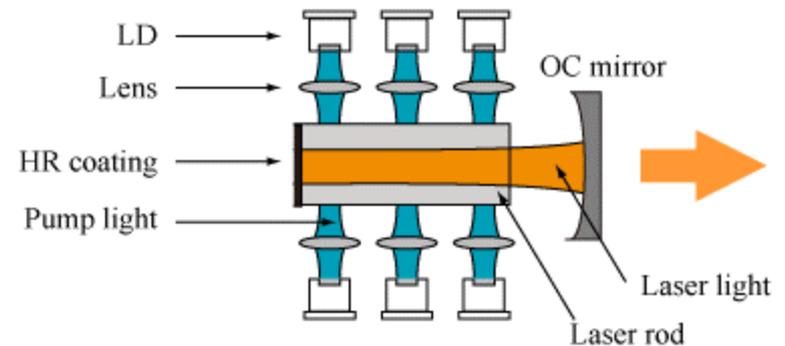
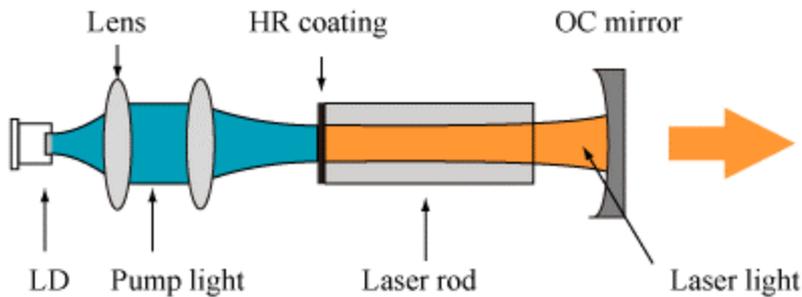
The inner surface of the cavity are coated with a reflective coating (usually Gold), such that all the radiation emitted from the lamps ended at the active medium.

Diode Pumped Solid State Lasers (DPSSL).

During the last few years, with the new developments of **diode lasers** at high powers, a new pumping method is being developed for solid state lasers. Instead of broad spectrum pumping source, Diode Lasers are used as pumping sources.

The wavelength in these diode lasers can be adjusted to fit the absorption spectrum of the active medium.

These diode lasers are very efficient sources, and almost all their light is absorbed by the active medium. Thus, very little energy is lost (converted into unwanted heat). These solid state lasers which are pumped by diode lasers are called: **Diode Pumped Solid State Lasers (DPSSL).**



6.2.1 Ruby Laser

Ruby laser was the **first man made laser**, which was build by **Theodore Maiman in 1960**.

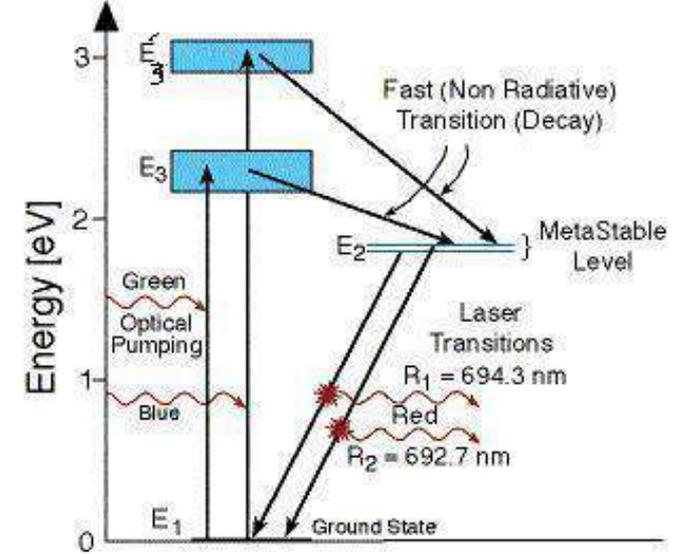
Ruby is a synthetic crystal of Aluminum Oxide (Al_2O_3), and is more familiar in daily life as a **precious stone** for jewel.

The chemical structure of Ruby is of **Al_2O_3** (which is called **Sapphire**), with impurity of about 0.05% (by weight) of **Chromium Ions (Cr^{+3})**.

The active ion is Cr^{+3} , which replace **Al** atom in the crystal. This ion causes the red color of the crystal. The impurity ion of **Cr^{+3}** is responsible for the energy levels which participate in the process of lasing.

Energy Levels of Ruby Laser

The energy level diagram of a Ruby laser (Cr^{+++}) is described in figure 6.13.



This system is a **three level laser** with lasing transitions between E_2 and E_1 . The excitation of the Chromium ions is done by **light pulses** from flash lamps (usually Xenon).

The **Chromium ions** absorb light at wavelengths around 545 [nm] (500-600 [nm]). As a result the ions are transferred to the excited energy level E_3 . From this level the ions are going down to the **metastable energy level E_2** in a **non-radiative transition**. The energy released in this non-radiative transition is transferred to the **crystal vibrations** and changed into **heat** that must be removed away from the system.

The lifetime of the meta stable level (E_2) is about 5 [msec].

Ruby laser has another absorption band (E_3) which can be used for pumping, in the spectrum range: 350-450 [nm].

It is difficult to achieve **continuous operation of a Ruby laser** since it is a **three level laser**

Example 6.1: Ruby laser wavelength

The energy gap between the **upper lasing level** and the **ground level** in a Ruby laser is 1.789 [eV].

What is the wavelength of the emitted light out of a Ruby laser?

Solution to Example 6.1:

By substitute of the energy values into the wavelength equation we get:

$$\lambda = hc/(E_2 - E_1)$$

$$\lambda = (6.626 \cdot 10^{-34} \text{ [J-s]}) \cdot (3 \cdot 10^{10} \text{ [m/s]}) / (1.789 \text{ [eV]}) \cdot (1.6 \cdot 10^{-19} \text{ [eV}^{-1}\text{]}) =$$

$$= 6.943 \cdot 10^{-7} \text{ [m]} = 694.3 \text{ [nm]}$$

The emitted wavelength from a Ruby laser is at the edge of the visible spectrum.
Since the eye is transparent to this wavelength, it is dangerous to the eye !

Operation of the Ruby Laser

The flash lamp is getting its energy from a discharge of capacitor.

The duration of the discharge is measured in microseconds, so the duration of excitation light pulse out of the flash lamp is of the same order of magnitude. Thus, the duration of a pulse of radiation out of a Ruby laser is of the order of magnitude of microseconds.

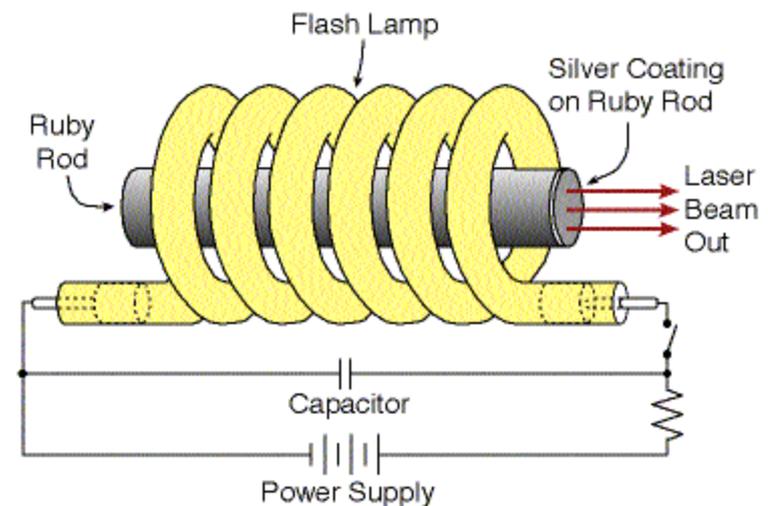
Some of the photons are emitted by **spontaneous emission** in the transition between energy levels E_2 to E_1 .

Because of the **population inversion** between these two energy levels, these spontaneous emitted photons cause other photons to be emitted in **stimulated emission**.

The **control of the direction of emission of radiation**, is determined by the **properties of the optical cavity**, and the **output coupler**. Only photons which are emitted along the laser axis will continue to move back and forth between the mirrors of the optical cavity.

Thus they will stimulate more and more photons to be emitted along the axis.

The **amount of energy** emitted in each pulse, is determined by the **active medium** and the **excitation system**.



Summary of the Ruby Laser

According to Classification into Groups:

- A **solid state laser**.
- Emit radiation in the red range of the **visible spectrum**.
- **Optically pumped**.
- The radiation is emitted as **pulses**.
- A **three level laser**.

6.2.2 Nd Laser

In Nd laser **Nd⁺³ ions** (as impurities of up to a few percent by weight) are replacing the atoms of the solid host in the active medium.

Three **known solid hosts** are used for Nd- laser where Nd⁺³ ions are added as impurities:

- **Glass.**
- **YAG (Yttrium Aluminum Garnet)** Crystal.
- **YLF (YLiF₄)** Crystal.

The choice between the three possible hosts is according to the intended use of the laser:

- **Glass** is used as the host material when a **pulsed laser** is needed, with each pulse at high power, and the pulse repetition rate is slow.

The active medium of Nd-Glass Laser can be manufactured in a shape of **disk or rod**, with diameters of up to 50 cm (!) and length of up to several meters (!). Such dimensions are possible because glass is isotropic material, cheap, and can be easily worked to the right shape.

High percentage (up to about 6%) of Nd ions can be added to glass as impurity.

The **problem** with glass as a host is its **poor thermal conductivity**. Thus cooling the laser when it operates continuously or at high repetition rate is difficult.

• **YAG crystal** is used for **high repetition rate pulses** (more than one pulse per second). In this case a large amount of heat need to be transferred away from the laser, and **the thermal conductivity of the YAG crystal is much higher than that of glass.**

YAG crystal with the high quality needed for lasers can be made with diameters of 0.2-1.5 [cm] and at lengths of 2-30 [cm].

The price of a YAG laser rod is high, since growing crystals is a slow and complicated process.

The percentage of Nd ions in the YAG host is 1-4% by weight.

• **YLF Crystal:** is the same as before except that the host material **Yttrium Lithium Fluoride** .

Energy Level Diagram of Nd- laser

The energy level diagram of a Nd- laser can be seen in figure 6.15.

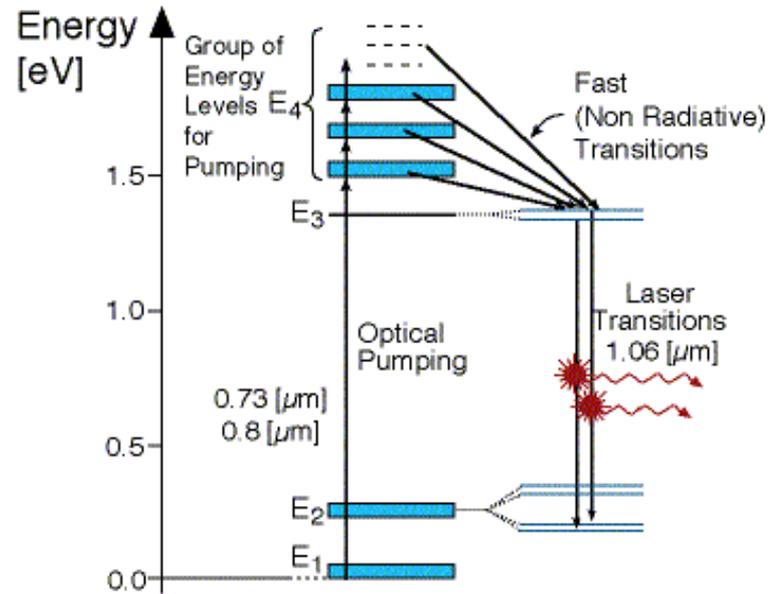


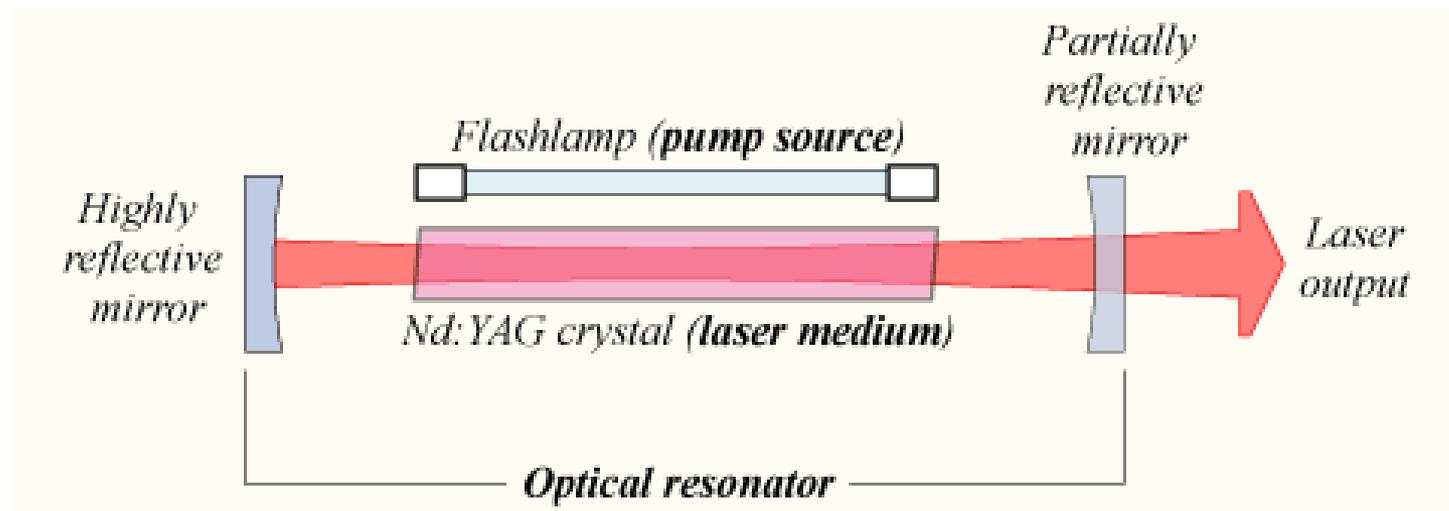
Figure 6.15: Energy Level Diagram of a Nd- Laser

As can be seen from the energy level diagram, Nd- lasers are **four level lasers**. Nd ions have **many absorption band**, and excitation is done by **optical pumping**, either by flash lamps for pulsed lasers, or by arc lamps for continuous wave lasers. From these excited energy levels, the Nd ions are transferring into the upper laser level by a non radiative transition.

The **stimulated emission** is from the upper laser level to the lower laser level, and the wavelengths of the emitted photons are around 1.06 [μm].

From the lower laser level, a non-radiative transition to the ground level.

- Nd- lasers are optically pumped lasers.
- The **population inversion** is created when the active medium absorbs photons from an intense **light source**.
- That **light source** can be anything: a **lamp** , another **laser**.
If the **light source** is a **laser**, it might be a **semiconductor laser**, or it might be a **gas laser** or even another **solid-state laser**.
- Because the energy that creates the population inversion comes from an optical source, these lasers are called **optically pumped** solid-state lasers.



Pulsed Nd lasers

Nd glass lasers can emit a large amount of energy in a single pulse. Usually in pulsed Nd lasers the energy per pulse is in the range 0.01-100 [J], and the pulse repetition rate is up to 300 [Hz].

The average energy of a pulsed Nd laser can be high.

As an example: pulses of 0.5 [msec] with energy of 10 [J] means an average energy of $2 \cdot 10^4$ watt.

The total efficiency of Nd lasers is low, and is in the range: 0.1-2%.

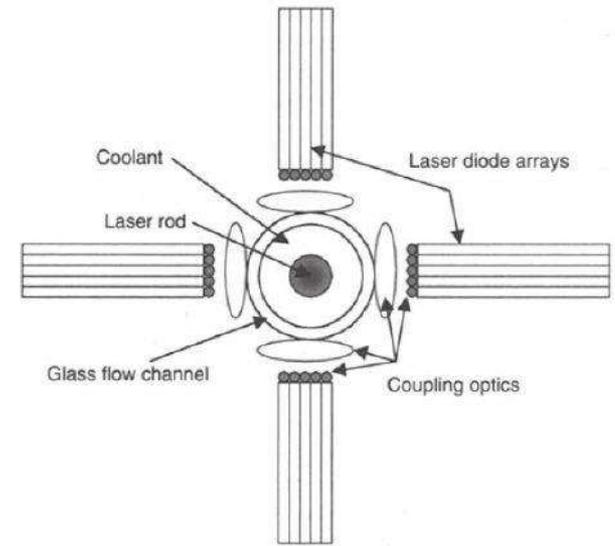
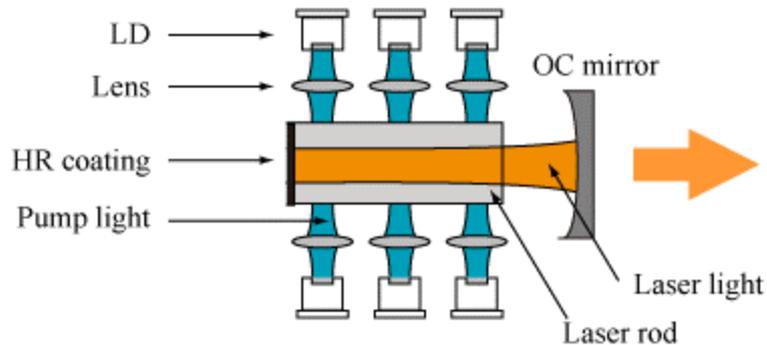
Summary of Nd lasers according to groups:

- **Solid state laser.**
- **Emit in the Near-Infra-Red (NIR) spectrum range.**
- **Optically pumped.**
- **Operate in both pulsed and continuous mode.**
- **Four level laser.**

DIODE-PUMPED SOLID STATE LASERS: DPSSL

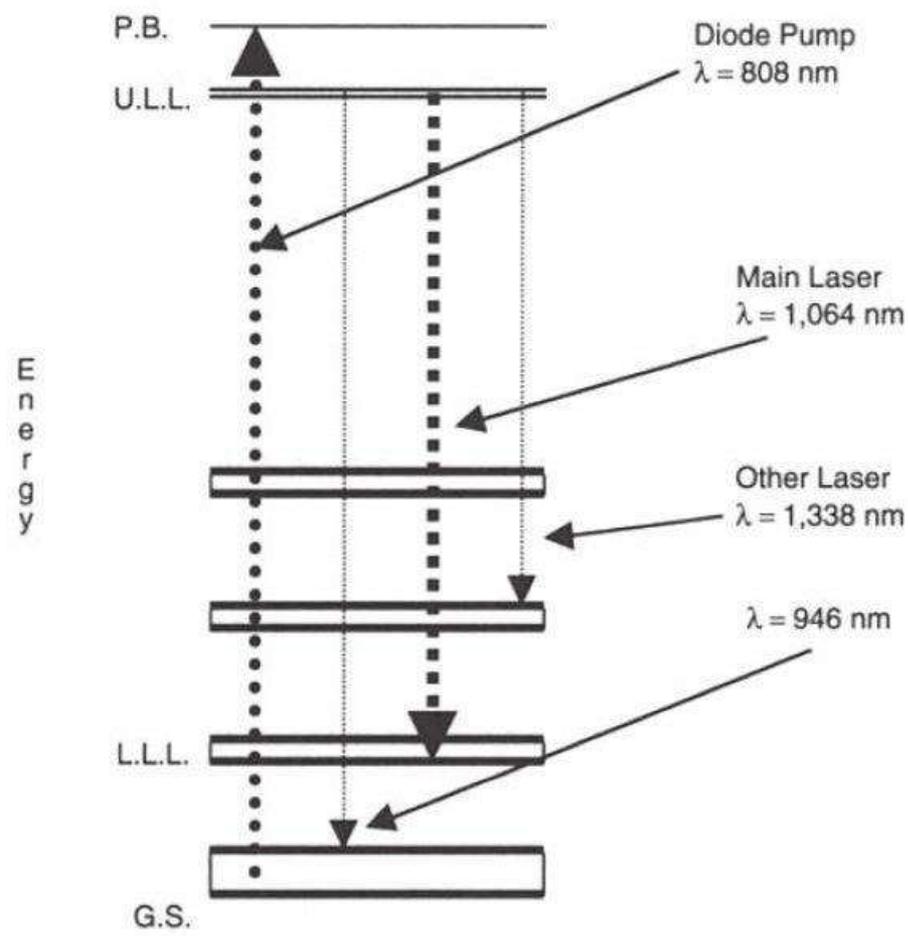
- Diode pumping is intrinsically simpler than lamp pumping, and as the technology evolves, diode pumping is certain to become commercially dominant. diode pumping is often preferable because it produces much less heat in the laser medium and has a significantly greater overall efficiency.
- The higher efficiency pays off in lower cost of power supplies and a simpler thermal design of the laser itself.
- diode can be chosen to match the absorption wavelength of the active ion.

- Because the diode laser is so small, there are several possible geometrical arrangements for diode-pumped solid-state lasers.
- The lasing medium is usually shaped as a cylindrical rod, but slabs and disks are sometimes used.
- The lasing medium can be pumped from the side, or it can be end pumped. It can be directly pumped by the diode laser, or the diode's radiation can be coupled into the gain medium with an optical fiber.
- For an initial diode-pumped solid-state laser, a simple side-pumped rod (see Fig. for a diagram).



- In this case, the laser rod is surrounded by pumping diodes that excite the laser crystal through a cooling jacket.
- Water or antifreeze flows between the jacket and the rod, removing the heat that gets to the surface.
- The principle components of the laser head are the laser rod, a transparent cooling jacket, the laser diodes, the resonator mirrors and coupling optics plus the power supplies.

- Nd^{3+} is a four-level system, Some of these levels can serve as the lower laser level of secondary lasing transitions, at 1338 and 946 nm. (You can force the laser to lase at one of the secondary transitions by maximizing the feedback at that wavelength and minimizing the feedback at competing laser wavelengths.)
- the Figure also illustrates the advantage of diode pumping over lamp pumping. Because the diode laser's output is concentrated at the narrow wavelength at about 808 nm, it can be efficiently absorbed by the narrow energy level identified in Figure as the diode pump band (P.B.).



- It's instructive to examine the energy flow through a diode-pumped Nd:YAG laser, as illustrated in Fig.1. Here, we show typical efficiencies for each major step in the process of converting energy from electricity to light. The overall efficiency is ~8%

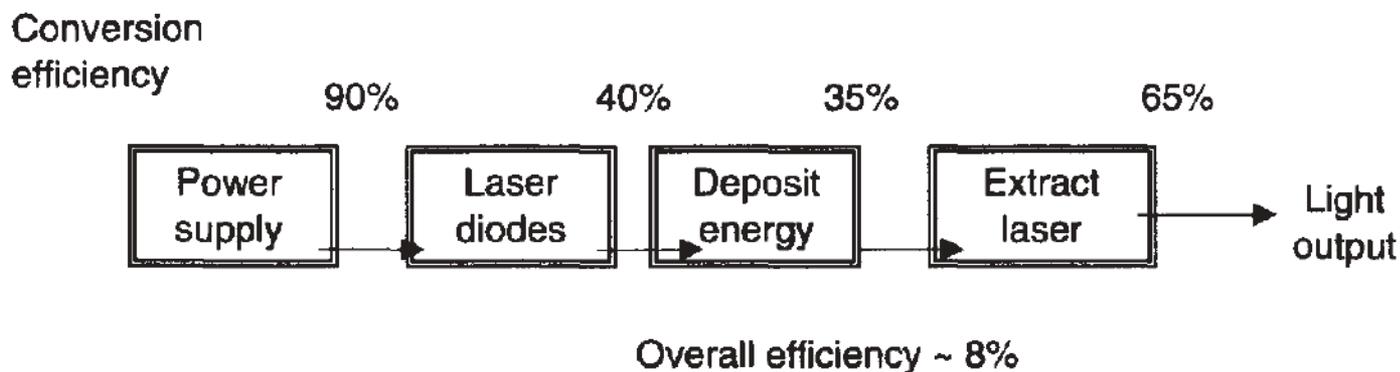


Fig.1

- Figure 2 is a similar diagram for a lamp-pumped Nd:YAG laser. The comparable overall efficiency is on the order of 1 or 2%.

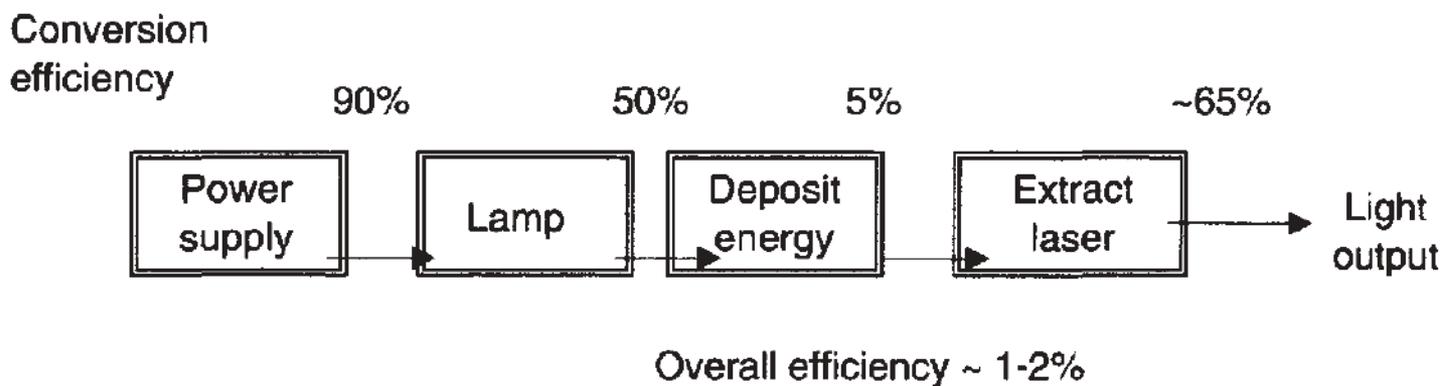
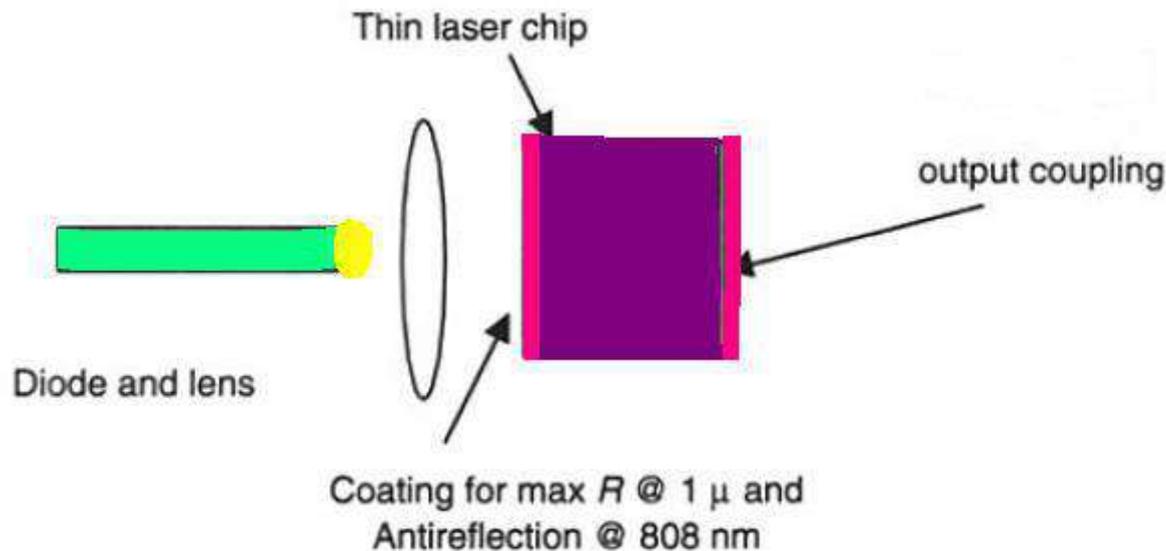


Fig.2

- An important variety of the diode-pumped solid-state laser is the end pumped "micro laser" illustrated in Figure, the simple, compact design allows average powers of 1 W or more in the infrared,
- The laser mirrors are applied directly to the YAG material, so the laser can be quite small, perhaps 1 cm long.



End-pumped microlaser. This simple configuration is used widely in low-power applications of diode-pumped Nd lasers.

6.3 Liquid lasers

- Dye Laser

A **dye laser** can be considered as a special device to convert electromagnetic radiation from one wavelength, to another wavelength which can be tuned.

The output of a dye laser is always a coherent radiation tunable over a specific spectrum region, determined by the Dye material.

History:

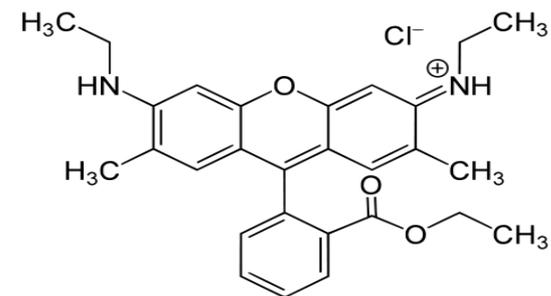
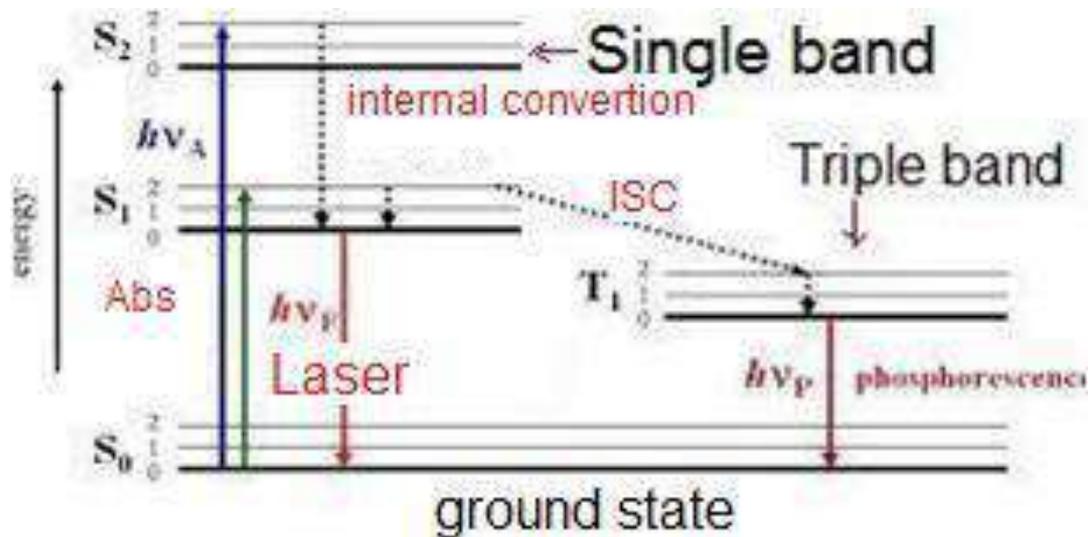
Dye laser was first demonstrated in 1965 at IBM laboratories in the US, by **Peter P. Sorokin and J. R. Lankard.**

They discovered the dye laser action during a fluorescence research of organic dye molecules, which were excited by [Ruby laser](#).

In 1967 scientists discovered the possibility to tune the emitted wavelength, using a grating at the end of the optical cavity.

Active Medium and Energy Levels in Dye Laser:

since the organic dyes have a complex structure, **Color molecule** is made of big organic fluorescent compound which contains large number of cyclic structures. As an example, rhodamine-6G has two excited single states corresponding to two excited triple states.



Molecular structure of the laser dye rhodamine 6 G

Energy level diagram of an organic laser dye.

The **active medium in Dye laser** is made of color molecule dissolved in **liquid** which is usually a type of alcohol.

Because of the interaction of the color molecules and the solvent, there is a **broadening of the vibrational energy levels**.

Solutions of organic color molecules have wide absorption and emission bands.

An example of the spectral bands for the common color:

Rhodamin 6G can be seen in figure 6.33. and the Molecular formula is $\text{C}_{28}\text{H}_{31}\text{N}_2\text{O}_3\text{Cl}$

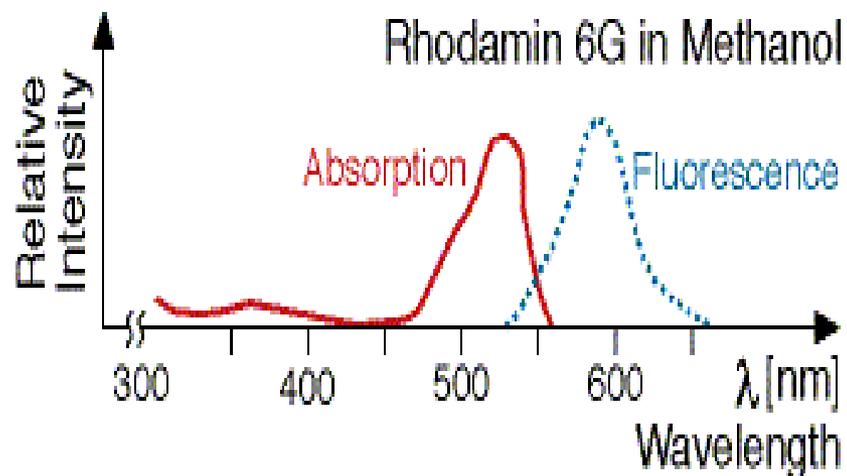


Figure 6.33: Absorption Spectrum (solid line) and Emission Spectrum (dashed line) of Rhodamin 6G in Methanol.

Simplified Energy Level Diagram of Dye Laser

The structure of energy levels of organic dye molecules in a solvent is very complex. The explanation below is based on a **simplified energy level diagram** described in figure 6.34.

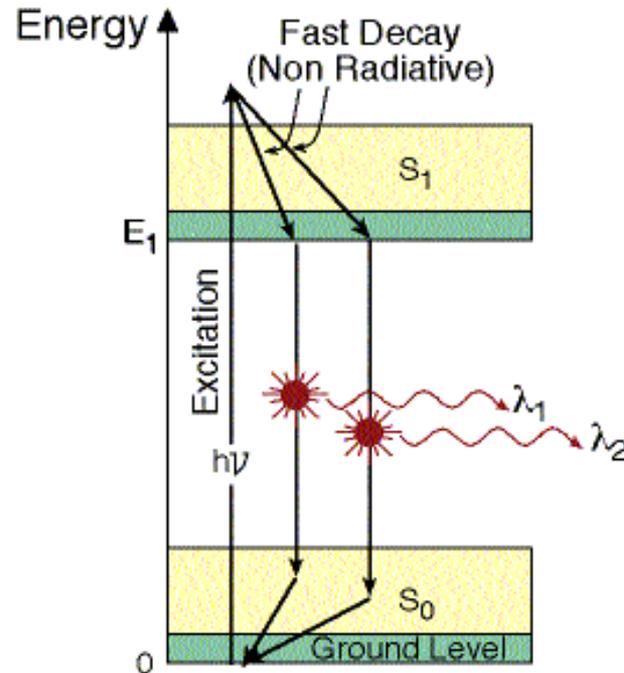


Figure 6.34: Simplified Energy Level Diagram of Dye Laser.

The width of each energy band is of the order of tenths of electron volts [eV]. At room temperature, the thermal energy of vibration is of the order of 1/40 [eV]. Thus only the bottom of the each energy level is filled (marked darker in figure 6.34).

Lifetime of Excited level in Dye Laser

As can be seen in figure 6.33, color molecules absorb and emit in a wide spectrum range.

All the transitions from energy bands create a **continuum of wavelengths**.

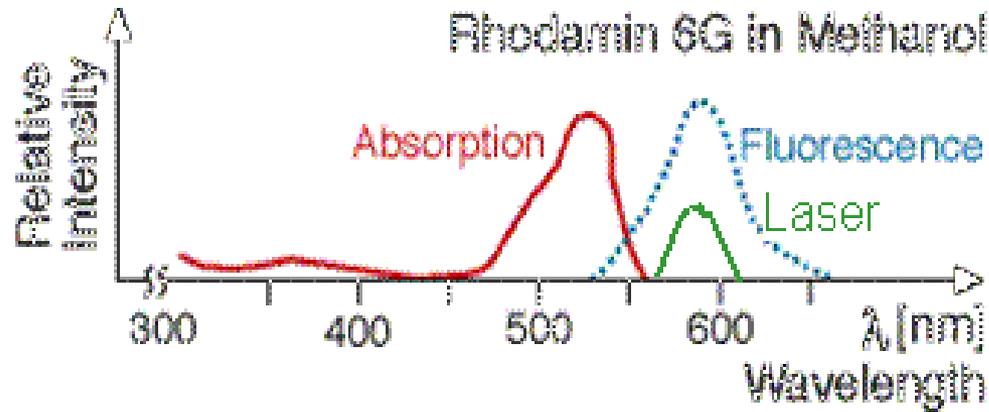


Figure 6.33: Absorption Spectrum ,Emission Spectrum and Laser emission of Rhodamin 6G in Methanol.

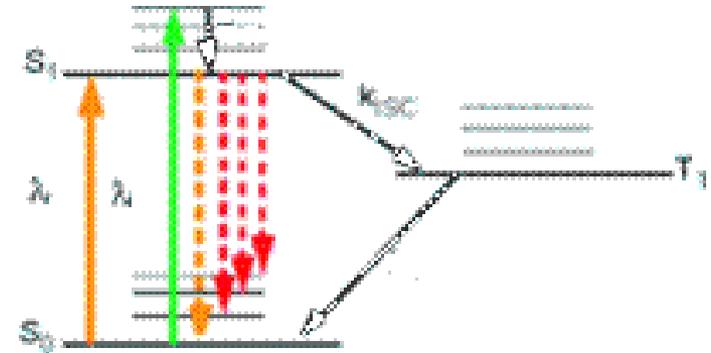
The fact that the energy band is wide, cause the **lifetime of the excited level to be very short** for two reasons:

1. There is a large number of possible transitions for lasing.
2. The number of collisions between molecules in a liquid is very big, and the energy transfer during each collision cause rapid decay of the excited state.

Dye Laser Operation :

Population inversion in a Dye laser is done by **optical pumping** - illumination by electromagnetic radiation at the proper wavelength.

When a color molecule is optically excited by absorbing a photon, it "jump" to an excited state.



In a very short time, of the order of pico-seconds, the excited molecule will decay to the bottom of the energy band by a non-radiative transition.

From this position the molecule can make lasing transition to any place in the lower band, and then, by a non-radiative transition to the bottom of the ground level.

Since lasing transition can occur into any position in the wide lower level, a **wide spectrum range of electromagnetic radiation can be emitted from a Dye laser.**

Since each photon carry a certain amount of energy, and since there is some loss of energy (which turn into heat) in the optical pumping process. Thus, **the wavelength emitted from the laser is of longer wavelength than the wavelength of the pump.**

Conclusions from the simplified energy level diagram:

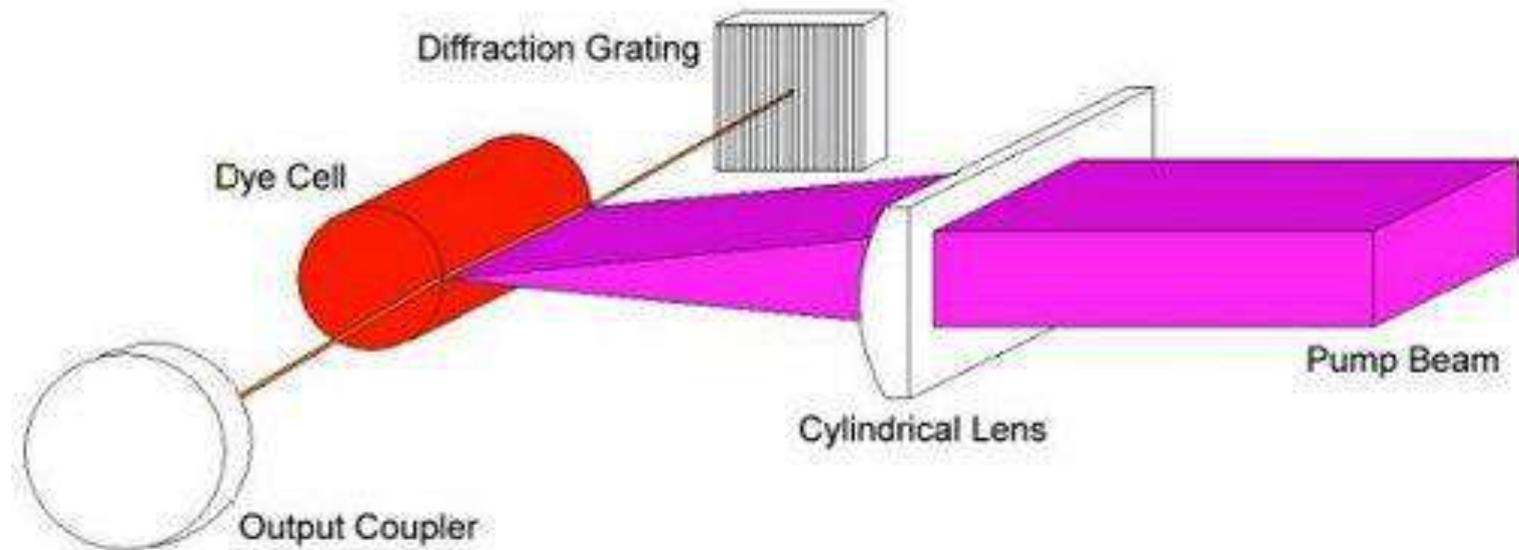
- **Fluorescent spectrum** is independent of excitation spectrum, as long as the excited photon have enough energy to excite the color molecule.
- Since most of the lower laser level (the wide lower band) is empty, it is quite easy to obtain population inversion between the lower part of the upper band and most of the lower band.
- It is possible to get high power of a Dye laser, since there are many color molecules dissolve in the active medium, unlike the low percentage of ions in the active medium in solid state laser.
- The spectral tuning range of a Dye laser is wide since each molecule can make transition to any level within the lower band. Order of magnitude of tuning range is **50-100 [nm]**.

The requirements from a Dye laser color molecule

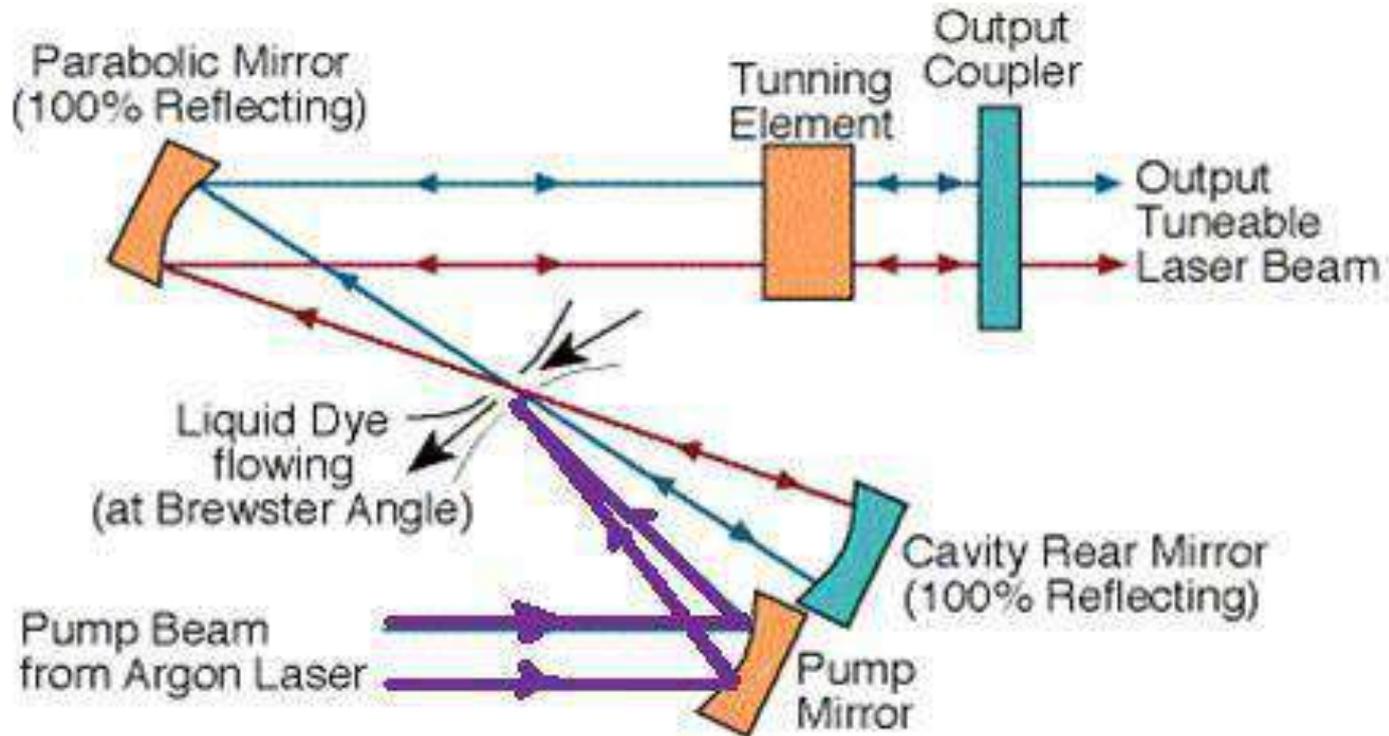
- **High absorption at the excited wavelengths.**
- **Rapid decay to the bottom of the excited band.**
- **Low probability for transitions from the Single band to the Triple band.**
- **Low absorption at the emitted wavelengths.**

There are two common arrangements for the liquid dye in a Dye laser:

1. The liquid dye is inside a transparent container, and the optical pump energy is coming through the walls of the container.



2. The liquid dye is flowing through a special nozzle, and the optical pump energy is shining on it while it flows out of the nozzle.

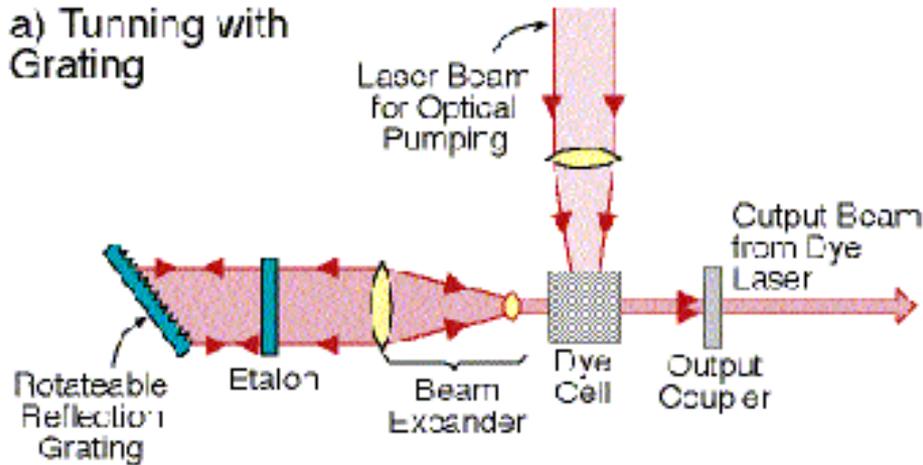


Tunable Dye Laser with Flowing Dye.

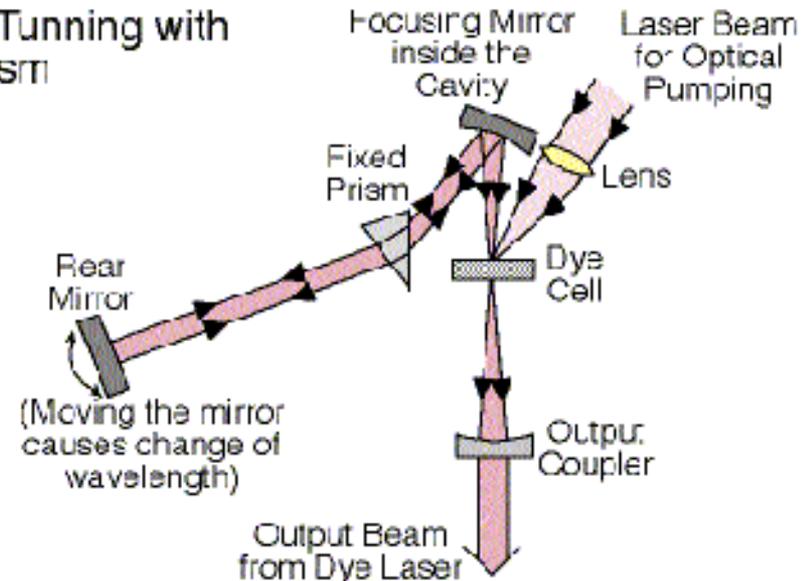
Tuning a Dye laser:

Choosing the right wavelength from a Dye laser is done by a **prism** or **grating at the end of the optical cavity**. An example of tuning with a **Grating** and **Prism** are described in figure (6.36 a&b).

a) Tuning with Grating



b) Tuning with Prism



Advantages of Dye Laser:

- Liquid is homogeneous by nature, and there is no difficulty of manufacturing homogeneous perfect Dye laser with no defects.
- It is relatively easy to change the type of liquid dye used as an active medium. Thus, changing the wavelength range of the emitted radiation.
- The liquid carry with it the heat evolved during the lasing process, so cooling the laser is simple. The active medium is replaced continuously.
- Special properties of the output radiation of a Dye laser are:
 - Very narrow linewidth.
 - Very short pulses.

Disadvantages of Dye Laser:

- Most Dye lasers use **liquid as the active medium**, which complicate maintenance of the laser.
- The **excitation is done by another laser**, which complicate the system.
- **Short dye lifetime**. Dye quality degrade with time, and need to be changed.
- Continuing operating expenses.
- **Potentially toxic chemicals**.
- **Volatile solvents**.
- Hazardous waste disposal.

In recent years, **Solid State Dye Lasers** are being developed. **By embedding the dye molecules in a solid matrix, the disadvantages of the liquid are eliminated.**

Classification of Dye Laser according to Groups:

- ❖ Liquid active medium.
- ❖ Operate mostly in the visible spectrum.
- ❖ Excited optically - usually by another laser.
- ❖ Can emit radiation continuously or in pulses - as determine by the excitation mechanism and the laser structure.
- ❖ Four level laser.
- ❖ Tunable laser.