Data Distribution (Normal Distributions and the Standard Distribution)

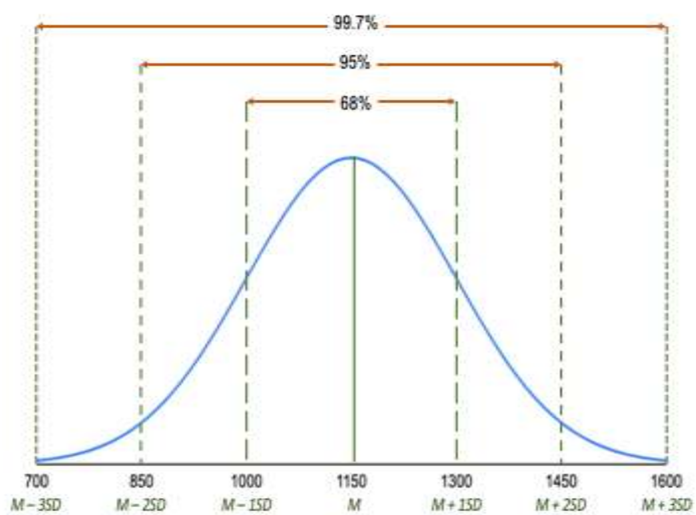
1. Normal Distribution

- Example:1 Using the empirical rule in a normal distribution You collect SAT scores from students in a new test preparation course. The data follows a normal distribution with a mean score (*M*) of 1150 and a standard deviation (*SD*) of 150.

Solution:

Following the empirical rule:

- Around 68% of scores are between 1000 and 1300, 1 standard deviation above and below the mean.
- Around 95% of scores are between 850 and 1450, 2 standard deviations above and below the mean.
- Around 99.7% of scores are between 700 and 1600, 3 standard deviations above and below the mean.

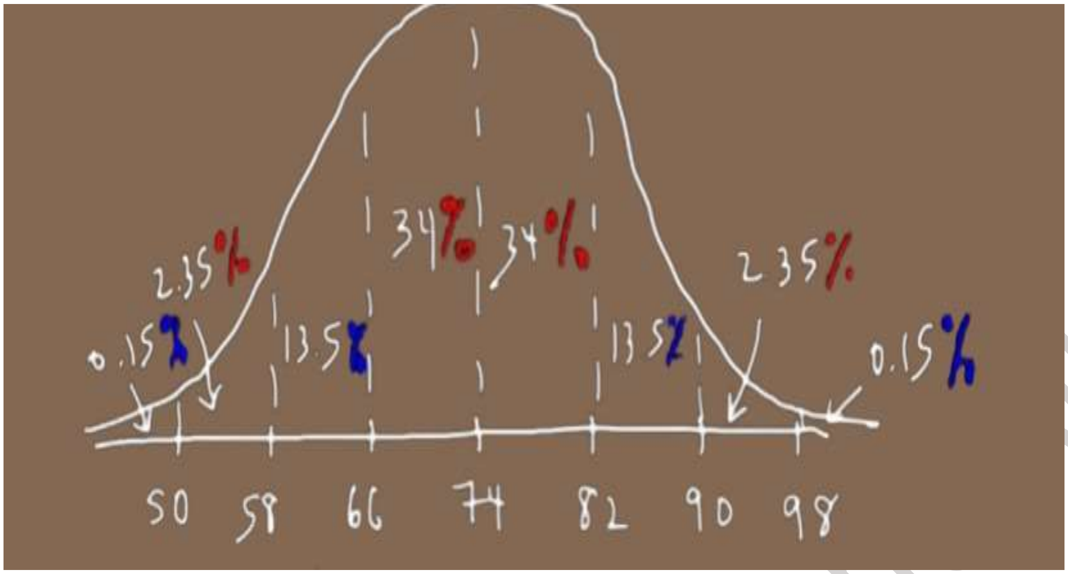


- Example 2: the average test score in a certain class as 74 with a standard deviation of 8. There are 2000 students in this class. Use the empirical rule to answer the following questions:

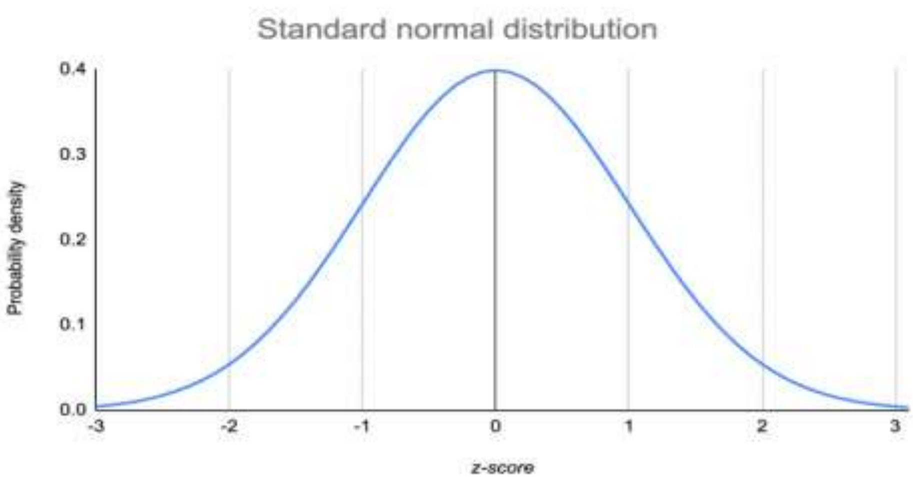
1. What percentage of students scored less than 58?
2. What is the probability that a student scored between 66 and 82 on the exam?
3. How many students scored at most 90?
4. What percentage of students scored at least 66
5. How many students scored more than 98 on the test?

Solution:

- 1. $P(x < 58)$ = 2.35%+ 0.15 %= 2.50 %
- 2. $P(66 < x < 82)$ = 34 % + 34 % = % 68
- 3. $p(x < 90)$ =13.5 +34+34+13.5+ 2.35+0.15= 97.5 %
- 97.5/100*2000=0.975*2000=1950
- 4. $p(x > 66)$ = 34%+34%+ 13.5%+ 2.35%+0.15%=84%
- 5. $p(x > 98)$ = 0.15% = 2000* (0.15/100) =3



2. The Standard Normal Distribution (Z-distribution)



Standard deviation below the mean *Standard deviation above the mean*

- To standardize a value from a normal distribution, convert the individual value into a *z*-score:

Z-score formula

$$z = \frac{x - \mu}{\sigma}$$

- x = individual value
- μ = mean
- σ = standard deviation

Transforming z Scores into Raw Scores (x)

$$X = z\sigma + \mu$$

- Example 1: Given X~N (*50, 10*),

1. what are the values of the mean and standard deviation?
2. What value of x has a z-score of 1.4?
3. What is the z-score that corresponds to x=30

1/ X ~ N(μ, σ) μ=50 σ=10

2/ Z=1.4 , x=?

X= μ+zσ

X=50+1.4 (10)= 64

3/ x=30 z=?

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{30-50}{10} = \frac{-20}{10} = -2$$

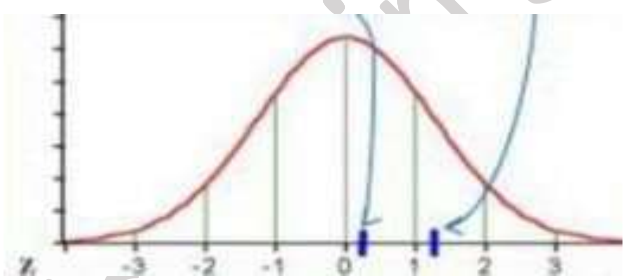
Example 2: Mark scored a 43 on chemistry test. The mean score in the class was a 38 and the standard deviation was 4. Marshall scored a 67 on his AP Calculus test and the mean in that class was a 65, with a standard deviation of 7. Whose score was better?

<u>Mark</u>	<u>Marshall</u>
X= 43	x=67
Mean=38	mean=65
SD=4	SD=7

$$z = \frac{x - \mu}{\sigma}$$

Mark: z = (43 – 38)/4 = 1.25

Marshall: z = (67 – 65)/7 **0.286** **1.25**



✓ Mark did better because he was further above the mean than Marshall.