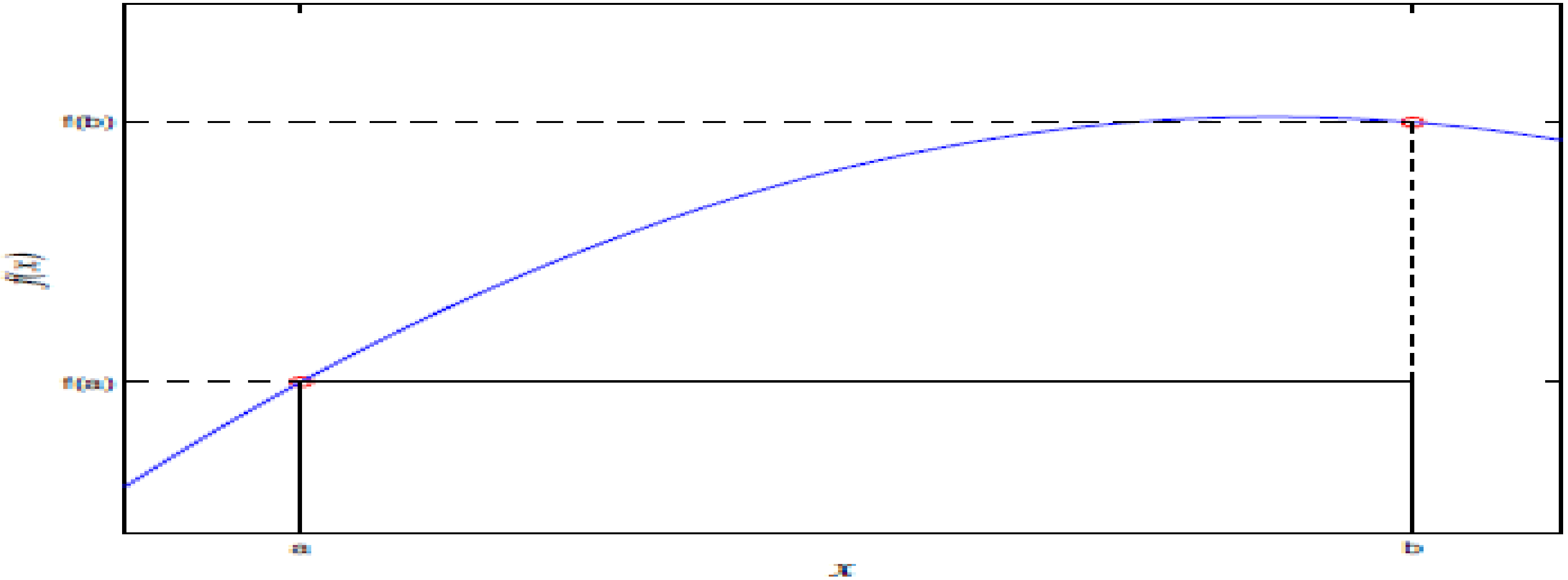
Chapter Seven

Numerical Integration

7-1 Basic Concepts : In this chapter we are going to explore various ways for approximating the integral of a function over a given domain. Since we can not analytically integrate every function , the need for approximate integration formulas is obvious. In addition, there might be situations where the given function can be integrated analytically, and still, an approximation formula may end up being a more efficient alternative to evaluating the exact expression of the integral. is called the rectangular method

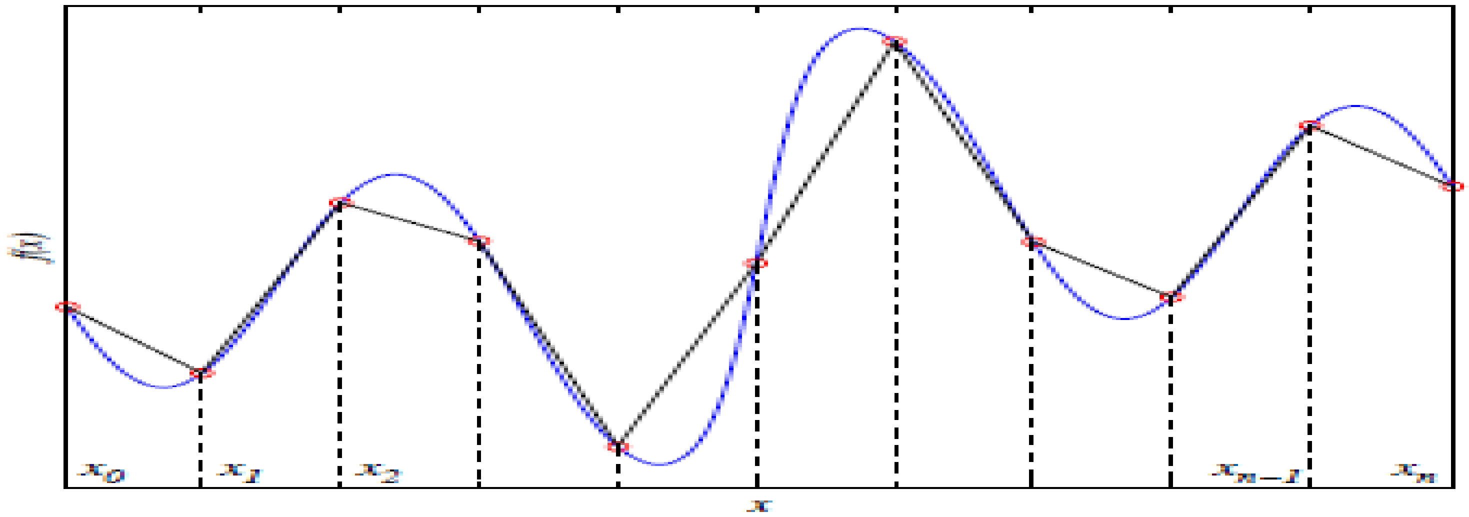
$$\int_a^b f(x)dx = f(b) - f(a)$$



while the lower (Darboux) sum of f(x) with respect to the partition P is defined The upper integral of f(x) on [a, b] is defined as $U(f) = \inf(U(f, P))$, the lower integral of f(x) is fined as $L(f) = \sup(L(f, P))$, where both the infimum $U(f, P) = \sum_{i=1}^{n} M_i \Delta x_i$ and the supremum $L(f, P) = \sum_{i=1}^{n} m_i \Delta x_i$. are taken over all possible partitions, P, of the interval [a, b]. If the upper and lower integral of f(x) are equal to each other, their common value is denoted by and is referred to as the Riemann integral of f(x) $\int_a^b f(x)dx$.

7-2Composite Integration Rules

In a composite quadrature, we divide the interval into subintervals and apply an integration rule to each subinterval. We demonstrate this idea with a couple of examples.



If f (x) is a function, then the integral of f from a to b is Written by $\int_a^b f(x)dx$
This integral gives the area under the graph of f , with the area under the positive part counting as positive area, and the area under the negative part of f counting as negative area, $1 = \int_a^b f(x)dx$

7-3 Integration via Interpolation Methods

In this section we will study how to derive quadratures by integrating an interpolant .As always, our goal is to evaluate. We select nodes $x_0, \ldots, x_n$ [a, b],and write the Lagrange interpolate (of degree $\leqslant n$) through these points Univariate quadrature methods are designed to approximate the integral of areal-valued function f depened on a bounded interval [a; b] of the real line.

7-4 -1 **Newton-Cotes method are widely used,**

7-4 -2 Simpson's Rule

Both rules are easy to implement and are typically adequate for computing the area under a continuous function. Both rules are easy to implement and are typically adequate for computing the area under a continuous function

7-4-1Newton-Cotes Method

If $P_n(x)$ is the Lagrange interpolation Polynomial for the function y=f(x) for which the following(n+1) points are given:

(x_0, y_0) , (x_1, y_1), (x_2, y_2) , . . . , (x_n, y_n)

then

$$\int_{x_0}^{x_n} f(x)\,dx \approx \int_{x_0}^{x_n} P_n(x)\,dx$$

and if

$$P_n(x_1) = \sum_{r=0}^{n} L_r(x) y_r$$

$$\int_{x_0}^{x_n} P_n(x_1)\,dx = \int_{x_0}^{x_n} \sum_{r=0}^{n} L_r(x) y_r\,dx$$

$$= \sum_{r=0}^{n} y_r \int_{x_0}^{x_n} L_r(x)\,dx$$

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x_k	y_k
x_0	y_0
x_1	y_1

$$P_1(x)\text{dx} = L_0(x)y_0 + L_1(x)y_1$$

$$\int_{x_0}^{x_1} P_1(x)\text{dx} = \int_{x_0}^{x_1} [L_0(x)y_0 + L_1(x)y_1]\,\text{dx}$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$\int_{x_0}^{x_1} P_1(x)\text{dx} = \int_{x_0}^{x_1} \left[\frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1\right]\text{dx}$$

7-4 -2 Simpsons Method

If $P_n(x)$ is the Lagrange interpolation Polynomial for the function y=f(x) for which the following(n+1) points are given:

$$(x_0, y_0), (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

then

$$\int_{x_0}^{x_n} f(x)\,dx \approx \int_{x_0}^{x_n} P_n(x)\,dx$$

x_k	y_k
x_0	y_0
x_1	y_1
x_2	y_2

$$x_1 - x_0 = x_2 - x_1 = h \qquad x_0 - x_2 = -2h$$

$$\int_{x_0}^{x_2} P_2(x)\,dx = \int_{x_0}^{x_2} \sum_{r=0}^{2} L_r(x) y_r\,dx$$

$$\int_{x_0}^{x_2} P_2(x)\,dx = \int_{x_0}^{x_2} [L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2]\,dx$$

$$L_0(X) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-x_1)(x-x_2)}{2h^2}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-x_0)(x-x_2)}{-h^2}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-x_0)(x-x_1)}{2h^2}$$

$$\int_{x_0}^{x_2} P_2(x)\,\mathrm{dx} = \int_{x_0}^{x_2} [L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2]\,\mathrm{dx}$$

$$\int_{x_0}^{x_2} L_0(x)dx = \int_{x_0}^{x_2} \frac{(x-x_1)(x-x_2)}{(x-x_1)(x-x_2)} y_0 dx$$

$$\int_{x_0}^{x_2} L_1(x)\,dx = \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1\,dx$$

$$\int_{x_0}^{x_2} L_2(x)\,dx = \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_1)}{(x-x_0)(x-x_1)} y_2 dx$$

$$\int_{x_0}^{x_2} P_2(x)\,dx = \int_{x_0}^{x_2} \frac{(x-x_1)(x-x_2)}{(x-x_1)(x-x_2)} y_0 dx + \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1\,dx + \int_{x_0}^{x_2} \frac{(x-x_0)(x-x_1)}{(x-x_0)(x-x_1)} y_2 dx$$

We must solve the final integral by part the result is :

$$\int_{x_0}^{x_2} P_2(x)\,\mathrm{dx} = \frac{\mathrm{h}}{3}[\mathrm{y}_0 + 4\mathrm{y}_1 + \mathrm{y}_2]$$