6-5Lagrange Interpolation for unequal intervals

$$y_n = \sum_{i=0}^{n} L_k(x_n) y_i$$

If k=2 only we find

$L_0(x_n)$ $L_1(x_n)$ *and*

$$L_0(x_n) = \frac{(x_n - x_1)(x_n - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$$L_1(x_n) = \frac{(x_n - x_0)(x_n - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$

$$L_2(x_n) = \frac{(x_n - x_0)(x_n - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

Example

i	y_i	x_i
0	x_0=1	y_0=6
1	x_1=3	y_1=2
2	x_2=4	y_2=3

$$L_0(x_n) = \frac{(x_n - x_1)(x_n - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x_n - 3)(x_n - 4)}{(1 - 3)(1 - 4)} = \frac{(x_n - 3)(x_n - 4)}{(-2)(-3)} = \frac{(x_n - 3)(x_n - 4)}{6}$$

$$L_1(x_n) = \frac{(x_n - x_0)(x_n - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x_n - 1)(x_n - 4)}{(3 - 1)(3 - 4)} = \frac{(x_n - 1)(x_n - 4)}{(2)(-1)} = \frac{(x_n - 1)(x_n - 4)}{(-2)}$$

$$L_2(x_n) = \frac{(x_n - x_0)(x_n - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{(x_n - 1)(x_n - 3)}{(4 - 1)(4 - 3)} = \frac{(x_n - 1)(x_n - 3)}{(3)(1)} = \frac{(x_n - 1)(x_n - 3)}{3}$$

$$y_n = (x_n - 3)(x_n - 4) - (x_n - 1)(x_n - 4) + (x_n - 1)(x_n - 3)$$

$$y_n = x_n^2 - 7x_n + 12 - x_n^2 + 5x_n - 4 + x_n^2 - 4x_n + 3$$

$$y_n = x_n^2 - 6x_n + 11$$