NONPARAMETRIC STATISTICAL METHODS

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Chapter One

1.1 Introduction: [Parametric and nonparametric methods]

-**parametric model** or **parametric family** or **finite-dimensional model** is a particular class of statistical models. Specifically, a parametric model is a collection of probability distributions or a family of probability distributions that has a finite number of parameters.

-What is parametric statistical test?

Parametric tests are used when the information about the population parameters is completely known in simple words, parametric test assumes that the data is normally distributed. or a "bell-shaped curve. Involve Population Parameters (Mean)

-What is Nonparametric method(sometimes called a distribution free test) ?

Nonparametric statistics is the branch of statistics that is not based solely on parameterized families of probability distributions (common examples of parameters are the mean and variance).

Nonparametric statistics is based on either being distribution-free or having a specified distribution but with the distribution's parameters unspecified.

(Hint)Generally

-data transformation can sometimes make the data suitable for parametric analyses.

- if both parametric and nonparametric methods are applicable to a particular problem, we should use the more efficient parametric method.

Advantages of nonparametric statistics

The following are some of the advantages of the available nonparametric statistical procedures.

1. Make fewer assumptions.

Nonparametric Statistical Procedures are procedures that generally do not need rigid parametric assumptions with regards to the populations from which the data are taken.

2. Wider scope.

Since there are fewer assumptions that are made about the sample being studied, nonparametric statistics are usually wider in scope as compared to parametric statistics that actually assume a distribution.

3. Need not involve population parameters.

Parametric tests involve specific probability distributions (e.g., the normal distribution) and the tests involve estimation of the key parameters of that distribution (e.g., the mean or difference in means) from the sample data. However, nonparametric tests need not involve population parameters.

4. The chance of their being improperly used is small.

Since most nonparametric procedures depend on a minimum set of assumptions, the chance of their being improperly used is small.

5. Applicable even when data is measured on a weak measurement scale.

For interval or ratio data, you may use a parametric test *depending* on the shape of the distribution. Non-parametric test can be performed even when you are working with data that is nominal or ordinal.

6. Easy to understand.

Researchers with minimum preparation in Mathematics and Statistics usually find nonparametric procedures easy to understand.

7. Computations can quickly and easily be performed.

Nonparametric tests usually can be performed quickly and easily without automated instruments (calculators and computers). They are designed for small numbers of data,

Some Hypothesis Testing Procedures
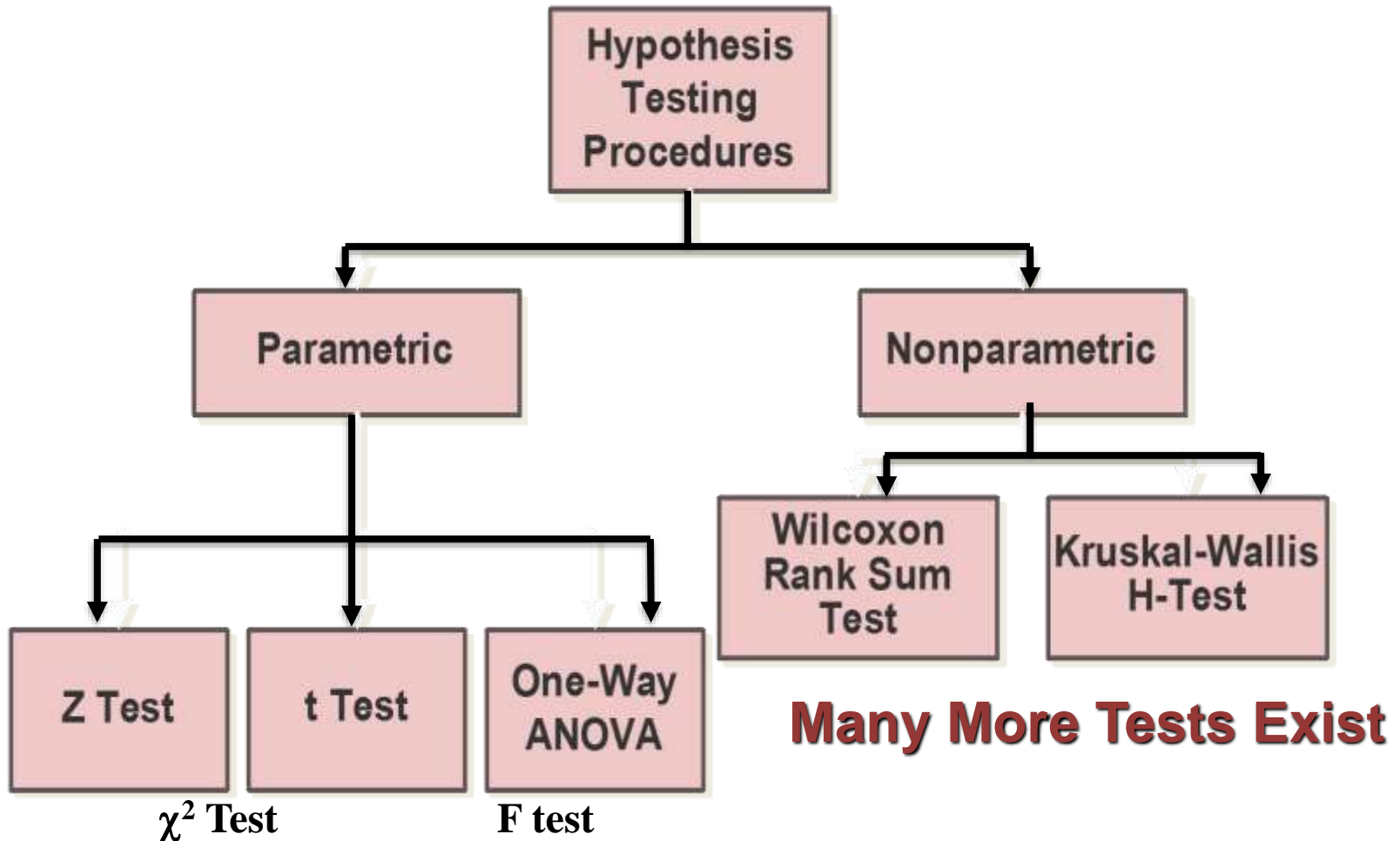
Chapter Two

2. One-Sample Nonparametric Methods

2.1 Introduction

In classical parametric tests (which assume that the population from which the sample data have been drawn is normally distributed), the parameter of interest is the population mean. In this chapter, we shall be concerned with the nonparametric analog of the one-sample z and t tests. These are nonparametric procedures *(which utilize data consisting of a single set of observations that are appropriate when the location parameter is the median, rather than the mean).*

Several nonparametric procedures are available for making inferences about the median. Two of the nonparametric tests which are useful in situations where the conditions for the parametric *z and t tests are not met, are the one-sample sign test and the Wilcoxon signed-ranks test.*

Recall that the median of a set of data is defined as the middle value when data are arranged in order of magnitude. For continuous distributions, we define the median as the point μ for which the probability that a value selected at random from the distribution is less than μ and the probability that a value selected at random from the distribution is greater than are both equal to(1/2) When the population from which the sample has been drawn is symmetric, any conclusions about the median are applicable to the mean, since in symmetrical distributions the mean and the median coincide.

In this chapter, we shall also discuss procedures for making inferences concerning the population proportion and testing for randomness and the presence of trend .

Steps for presenting the hypothesis-testing procedures:

1. Assumptions

We list the assumptions necessary for the validity of the test, and describe the data on which the calculations are based.

2. Parameter of interest

From the problem context, we identify the parameter of interest.

3. Hypotheses

We state the null hypothesis H_0 *and the alternative hypothesis* H_1

4. Significance level

We choose a significance level $\propto$.

5. Test statistic

We write down a formula or direction for computing the relevant test statistic. When we give a formula, we describe the methodology for evaluating it.

6. Value of the test statistic

We compute the value of the test statistic from the sample data.

7. Decision rule

We determine the critical region. The Appendix gives appropriate tables for the distribution of the test statistic. From these tables, we can determine the critical values of the test statistic corresponding to the chosen $\propto$.

8. Decision

If the computed value of the test statistic is as extreme as or more extreme than a critical value, we reject H_0 *and conclude that* H_1 *is true. If we cannot reject* H_0 *we conclude that* **there is not enough information to warrant its falsity.**

$$H_1$$

2.2 The one-sample sign test

The sign test is perhaps the oldest of all nonparametric procedures. Let $X_1, X_2, X_3, ..., X_n$ be an observed random sample of size (n) from a population with median ($\tilde{\mu}$) The sign test utilizes only the signs of the differences between the observed values (X_i) and the hypothesized median ($\tilde{\mu}_0$)

Thus, the data is converted into a series of plus (+) and minus (−) signs.

2.2.1 Assumptions

1. The sample available for analysis is a random sample of independent measurements from a population with an unknown median $\tilde{\mu}$

2. The variable of interest is measured on at least an ordinal scale.

3. The variable of interest is continuous.

2.2.2 Hypotheses

The hypothesis to be tested concerns the value of the population median. To test the hypothesis
$$H_0 : \tilde{\mu} = \tilde{\mu}_0$$
Where $\tilde{\mu}_0$ is a specified median value, against a corresponding one-sided or two-sided alternative, we use the ***Sign Test.***

The test statistic S *depends on the alternative hypothesis,* H_1 .

(a) One-sided test

For a one sided test, the alternative hypothesis is either $H_1: \widetilde{\mu} < \tilde{\mu}_0 \ or \ \widetilde{\mu} > \tilde{\mu}_0$

(1) If we wish to test
$$H_0 : \tilde{\mu} = \tilde{\mu}_0 \qquad against \qquad H_1: \widetilde{\mu} < \tilde{\mu}_0$$
then the ***test statistic*** is defined *by* $s_0 = N^+$

where $N^+ = $ *Number of observations* X_i *greater than* $(\tilde{\mu}_0)$

$N^+ = $ Number of + signs when the differences(X_i - $\tilde{\mu}_0$) *are computed, $i = 1,2, \dots, n$*

If the alternative hypothesis is true, then we should expect ($X_i - \tilde{\mu}_0$) *to* yield significantly fewer positive (+) signs than negative (−) signs. Thus, a smaller number of (+) signs leads to the rejection of H_0

When H_0 *is true, we expect the* number of (−) signs to be equal to that of the (+) signs and hence
$$P(S < \tilde{\mu}_0) = P(S > \tilde{\mu}_0) = \frac{1}{2}$$

Thus, when H_0 is true, S has the binomial distribution with parameters n and $\frac{1}{2}$

That is, $S \sim b(n, \frac{1}{2})$

Decision rule

The *p*-value of the test is defined by $(P = P(S \leq s_0 | H_0 \, is \, true))$

Where s_0 is the observed value of the test statistic S

Decision We reject H_0 at significance level $\propto$ if $P \leq \propto$.

(2) For a ***one-sided test***, we test

$H_0: \tilde{\mu} = \tilde{\mu}_0$ 　　　　 against 　　　　 $H_1: \widetilde{\mu} > \tilde{\mu}_0$

The ***test statistic*** is

$S = N^-$

where N^- = Number of observations ***less than*** $\tilde{\mu}_0$

= Number of – signs when the differences ($X_i - \widetilde{\mu}_0$) are computed, $i = 1, 2, \ldots n$.

If the alternative hypothesis H_1 is true, then we should expect ($X_i - \widetilde{\mu}_0$) to yield less

negative ($-$) signs than would be expected if the null hypothesis were true. Likewise, when H_0 is true, S has the binomial distribution with parameters n and $\frac{1}{2}$. That is, $S \sim b(n, \frac{1}{2})$

Decision rule

The p-value of the test is defined by 　　　　 $(p = P(S \leq s_0 | H_0 \, is \, true)$

where s_0 is the observed value of $.s_0 = N^-$

Decision We reject H_0 at significance level α if $p \leq \propto$.

b) **Two-sided test**

If we wish to test

$H_0: \tilde{\mu} = \tilde{\mu}_0$ against

$H_0: \tilde{\mu} \neq \tilde{\mu}_0$,

then the *test statistic* is defined by

$s_0 = \min\{N^-, N^+\}$s the number of $-$ signs and N^+ is the number of $+$signs when the differences $X_i - \tilde{\mu}_0$ are computed.

We should reject the null hypothesis if we have too few negative ($-$) signs or too few positive ($+$) signs. When H_0 is **true**, S has the binomial distribution with parameters n and $\frac{1}{2}$

Decision Rule

The *p*-value of the test is defined by$(P = 2P(S \leq s_0 | H_0 is\ true)$where s_0 is the observed value of the test statistic S.

Decision We reject H_0 at significance level $\propto$ if $P \leq \propto$.

Problem with zero differences

- We assume that the variable of interest is continuous. Therefore, in theory, no zero differences should occur when we compute $X_i - \tilde{\mu}_0$.

- In practice, however, zero differences do occur. The usual procedure is to discard observations leading to zero differences and reduce n accordingly. In that case the hypothesis may be re-stated in probability terms. For example, a two-sided case will have its null hypothesis as. $P(S < \tilde{\mu}_0) = P(S > \tilde{\mu}_0) = \frac{1}{2}$

Example 2.1

Appearance transit times for 11 patients with significantly occluded right coronary arteries are given below:

Subject	1	2	3	4	5	6	7	8	9	10	11
Transit time(insec)	1.80	3.30	5.65	2.25	2.50	3.50	2.25	3.10	2.70	2.70	3.00

Can we conclude, at the 0.05 level of significance, that the median appearance transit time in the population from which the data were drawn, is different from 3.50 seconds?

Solution

The parameter of interest is $\tilde{\mu}$ the median appearance transit time in the population. We wish to

The parameter of interest is $\tilde{\mu}$ the median

Test the hypothesis $H_0: \tilde{\mu} = 3.50$ against $H_1: \tilde{\mu} \neq 3.50$

Level of significance at the $\propto = 0.05$,

Test statistic Since this is a two-sided test statistic is $s_0 = min\{N^-, N^+\}$

Where N^- is the number of observations less than 3.50 and N^+ is the number of observations greater than 3.50.

	1	2	3	4	5	6	7	8	9	10	11
X_i	1.80	3.30	5.65	2.25	2.50	3.50	2.25	3.10	2.70	2.70	3.00
Sign of X_i- 3.50	−	−	+	−	−	0	−	−	−	−	−

From the above table, $N^- = 9$ and $N^+ = 1$

The observed value of the test statistic is therefore given by $s_0 = \min\{9,1\} = 1$

Decision rule

Since this is a two-sided test, the *p-value of the test is given by*

$p = 2P(S \leq s_0|10, 1) = 2 \text{ X } 0.0107$

Since the *p-value of the test, 0.0214, is less than 0.05, we reject* H_0 *at the 0.05 level of* significance and conclude that the population median is not 3.50.

Note: We discard one observation which has the same value as the hypothesized median, leaving us with a usable sample size of 10.

Example 2.2

The following data are IQs of arrested drug abusers who are aged 16 years or older. Is there any evidence that the median IQ of drug abusers in the population is greater than 107?
Use $\alpha = 0.05$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
99	100	90	94	135	108	107	111	119	104	127	109	117	105	125

Solution

The parameter of interest is $\tilde{\mu}$ the median IQ of drug abusers in the population. We wish to test the hypothesis,

The parameter of interest is $\tilde{\mu}$ the median

Test the hypothesis $H_0: \tilde{\mu} = 107$ against $H_1: \tilde{\mu} > 107$

Level of significance   at the $\alpha = 0.05$

Test statistic $s_0 = N^-$

Where N^- is the number of observations less than 107. When H_0 is true $S \sim b(14, 0.5)$

The following table gives the signs of $X_i - 107$

1	2	3	4	5	6	7	8	9	10	11	12	13	14
99	100	90	94	135	108	111	119	104	127	109	117	105	125
−	−	−	−	+	+	+	+	−	+	+	+	−	+

Let s_0 be the observed value of the test statistic. We reject H_0 at the 0.05 level of significance when $p \leq \alpha = 0.05$, where the p-value of the test is given by $p = P(S \leq s_0 | 14, 0.5)$
Here, $N^- = 6$
The observed value of the test statistic is $s_0 = N^- = 6$ *Thus* $s_0 = 6$
Decision rule
Since this is a one-sided test, the *p-value of the test is given by*
$p = P(S \leq s_0 | 14, 6) = 0.3953$
Since the *p-value of the test, 0.3953, is greater than 0.05, we fail to reject* H_0
at the 0.05 level of significance. Hence, there is not enough evidence to conclude that the median IQ of the subjects in the population is greater than 107.

2.2.3 Large sample approximation

If the sample size is larger than 15, we can use the normal approximation to the binomial distribution with a continuity correction. Thus, if *n is large and* $S \sim b\left(n, \frac{1}{2}\right)$

then it can be shown that S is approximately normally distribution with mean (np)and variance $np(1-p)$ *That is,* $S \sim N\big(np, np(1-p)\big)$

Thus, for the sign test, when $p = \frac{1}{2}$ *and* $n > 15$, *we can use the*

test statistic $. Z = \dfrac{S - \frac{1}{2}n}{\sqrt{n \times \frac{1}{2} \times \frac{1}{2}}} = \dfrac{S - \frac{1}{2}n}{\frac{1}{2}\sqrt{n}}$ ----------(1)

When H_0 *is true and* $n > 15$, Z *is approximately* $N(0,1)$
For the large sample approximation, *it is common to use a continuity correction, by replacing* S *by* $S + \frac{1}{2}$ *in the* definition of Z. *Equation (1) then becomes*

$$Z_0 = \frac{\left(s_0 + \frac{1}{2}\right) - \frac{1}{2}n}{\frac{1}{2}\sqrt{n}}$$ ----------(2)

Example 2.3

The following data give the ages, in years, of a random sample of 20 students from a training course. It is believed that the median age of students in this school is smaller than 22 years. Based on these data, is there sufficient evidence to conclude that the median age of students from a training course is smaller than 22 years?

9	13	16	16	16	17	18	19	19	19
19	20	20	21	21	23	24	25	25	27

Solution

The parameter of interest is the median age of students from a training course.

The parameter of interest is $\tilde{\mu}$ the median

Testing the hypothesis

$H_0 : \tilde{\mu} = 22$ *against* $H_1 : \tilde{\mu} < 22$

Level of significance at the $\propto = 0.05$

The test statistic is $s_0 = N^+$

When H_0 is true, $S \sim b(20, \frac{1}{2})$.

Since n > 15, we use the normal approximation to the binomial distribution with a continuity correction.

The test statistic then becomes

$$Z_0 = \frac{\left(s_0+\frac{1}{2}\right)-\frac{1}{2}n}{\frac{1}{2}\sqrt{n}} = \frac{\left(s_0+\frac{1}{2}\right)-\frac{1}{2}X\,20}{\frac{1}{2}\sqrt{20}}$$

9	13	16	16	16	17	18	19	19	19	19	20	20	21	21	23	24	25	25	27
−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	+	+	+	+	+

From this table $N^+=5$, Thus, the observed value of the statistic S is $s_0 = 5$.

This gives, When H_0 is true, $Z \sim N(0, 1)$.

Decision rule

Let z_0 denote the observed value of the test statistic Z.

$$Z_0 = \frac{\left(s_0 + \frac{1}{2}\right) - \frac{1}{2}X\,20}{\frac{1}{2}\sqrt{20}}$$

$$Z_0 = \frac{\left(5+\frac{1}{2}\right)-\frac{1}{2}X\,20}{\frac{1}{2}\sqrt{20}} = -2.0125$$

Since -2.0125 is less than -1.645, we reject H_0 at the 0.05 level of significance and we conclude that the median age of students of training course is less than 22 years.

2.3 The Wilcoxon signed-ranks test

As we have seen, the sign test utilizers only the signs of the differences between observed values and the hypothesized median. For testing $H_0 : \tilde{\mu} = \tilde{\mu}_0$ there is another procedure that uses the magnitude of the differences when these are available.

The Wilcoxon signed-ranks procedure makes use of additional information to rank the differences between the sample measurements and the hypothesized median. The Wilcoxon signed-ranks test uses more information than the sign test, making it a more powerful test when the sampled population is symmetric. However, the sign test is preferred when the sampled population is not symmetric.

2.3.1 Assumptions

1. The sample available for analysis is a random sample of size n from a population with an unknown median $\tilde{\mu}$
2. The variable of interest is measured on a continuous scale.
3. The sampled population is symmetric.
4. The scale of measurement is at least interval.
5. The observations are independent.

2.3.2 Hypotheses

The parameter of interest $\tilde{\mu}$ is the population median. To test the hypothesis ,
$H_0 : \tilde{\mu} = \tilde{\mu}_0$
Where $\tilde{\mu}_0$ is the hypothesized median, against a corresponding one-sided or two-sided alternative H_1, we can also use the Wilcoxon signed-ranks test.

2.3.3 Test statistic

To obtain the test statistic, we use the following procedure.
1-Subtract the hypothesized median $\tilde{\mu}_0$ from each observation X_i that is, for each observation X_i , find $D_i = X_i - \tilde{\mu}_0$ $\qquad \forall\ i = 1,2,3, \cdots, n$
2- If any observation X_i is equal to the hypothesized median, $\tilde{\mu}_0$ eliminate it from the calculations and reduce the sample size accordingly.

3- Rank the differences $|D_i|$, from the smallest to largest without regard to their signs. If two or more $|D_i|$ are tied, assign each tied value the mean of the rank positions of the tied differences.

4- Assign to each rank the sign of the difference of which it is ranked.

5-Obtain the sum of the ranks with positive signs; call it W^+.

 Obtain the sum of the ranks with negative signs; call it W^- .

6- Note that: $W^+ = \frac{n(n+1)}{2} - W^-$

7-For a given sample, we do not expect W^+ to be equal to W^- .

2.3.4 Carrying out the Wilcoxon signed ranks test

When the null hypothesis, $H_0 : \tilde{\mu} = \tilde{\mu}_0$

is true, we do not expect a great difference between W^+ and . W^- Consequently, a sufficiently small value of W^+ or a sufficiently small value of W^- causes us to rejectH_0

 a) *One-sided test*: To test

$H_0 : \tilde{\mu} = \tilde{\mu}_0$, against

$H_1 : \tilde{\mu} < \tilde{\mu}_0$ at the α level of significance.

Test statistic

A sufficiently small value of W^+ leads to the rejection of the null hypothesisH_0

 The test statistic therefore is $W_0 = W^+$.

Decision rule

We reject H_0 at significance level $\propto$ if the observed of W value $= W_0$is less than or equal to the tabulated W value for n and a preselected $\propto$

(b) *One-sided* test: To test
$H_0 : \tilde{\mu} = \tilde{\mu}_0$, against $H_1 : \tilde{\mu} > \tilde{\mu}_0$
at the $\propto$ level of significance.

Test statistic

For a sufficiently small W^- value, we reject H_0
The test statistic therefore is, $W_0 = W^-$
since a small value causes us to reject the null hypothesis.

Decision rule

We reject H_0 at significance level $\propto$ if the observed of W value $= W_0$ is less than or equal to the tabulated W value for n and a preselected value of $\propto$.

(c) For a *two-sided* test, we test
$H_0 : \tilde{\mu} = \tilde{\mu}_0$, against $H_1 : \tilde{\mu} \neq \tilde{\mu}_0$
at the $\propto$ level of significance.

Test statistic

The test statistic is
$W_0 = \min(W^-, W^+)$ since a small value of either $W^- \ or \ W^+$ causes us to reject the null hypothesis H_0.

Decision rule

We reject H_0 at significance level $\propto$ if the observed W value, w_0 is less than or equal to the tabulated W value for n and a preselected value of $\dfrac{\propto}{2}$

The distribution of W

1. The smallest value W can take is zero (0) and the largest value that W can take is the sum of the integers from 1 to n: that is, $\frac{n(n+1)}{2}$.

W is therefore a discrete random variable whose support ranges between 0 and $\frac{n(n+1)}{2}$

2. It can be shown that the probability mass function of the discrete random variable W is given by

$$P(W = w) = f(w) = \frac{c(w)}{2^n} \quad , \quad 0 < w < \frac{n(n+1)}{2}$$

where $c(w)$ = the number of possible ways to assign a +sign or a −sign to the first n integers the sum of the ranks with +signs (or −signs) is equal to w.

Example 2.4

The following are the systolic blood pressures (mmHg) of 13 patients undergoing a drug therapy for hypertension

183	178	152	157	194	163	144	114	179	150	118	158	165

Can we conclude on the basis of these data that the median systolic blood pressure is less than 165 mmHg? Take $\alpha = 0.05$.

Solution

The parameter of interest is $\tilde{\mu}$ the median systolic blood pressure of the population. We wish to Test the hypothesis

H_0: $\tilde{\mu} = 165$ against H_1: $\tilde{\mu} < 165$

Level of significance $\alpha = 0.05$.

Test statistic

Wilcoxon signed rank test, the test statistic is $W_0 = W^+$

Where W^+ is the sum of the ranks with positive signs

No	1	2	3	4	5	6	7	8	9	10	11	12		
X_i	114	118	144	150	152	157	158	163	178	179	183	194		
$D_i = X_i - 165$	-51	-47	-21	-15	-13	-8	-7	-2	13	14	18	29		
$\|D_i\|$ Ranks$(-)$	12	11	9	7	4.5	3	2	1					49.5	
$\|D_i\|$ Ranks$(+)$									4.5	6	8	10		28.5

From this table,

the sum of the ranks with negative signs $= W^- = 49.5$ and

the sum of the ranks with positive signs $= W^+ = 28.5$

the test statistic is $W_0 = W^+$

The value of the test statistic is therefore $W_0 = 28.5$

Decision rule

We reject H_0 if at 0.05 level of significance the value of $W_0 \leq W_{n,0.05}$

In **this case for** $\propto = \mathbf{0.05} \ and \ n = \mathbf{12}$

the tabulated value $\quad W_{n,\propto} = W_{12,0.05} = 13$

Decision

$W_0 > W_{12,\mathbf{0.05}}$ $\qquad \qquad 28.5 > 13$

We fail to reject H_0

We conclude that the median systolic blood pressure of the subjects in the population

is not less than 165 mm Hg.

Example 2.5

The following data are IQs of arrested drug abusers who are aged 16 years or older. Use the ***Wilcoxon signed-ranks test*** to determine if there is any evidence that the median IQ of drug abusers in the population is different from 107? Use $\alpha = 0.05$

Solution

The parameter of interest is $\tilde{\mu}$ denote the median IQ of drug abusers who are aged 16 years or older We wish to

Test the hypothesis

$H_0: \tilde{\mu} = 107$ against $H_1: \tilde{\mu} \neq 107$

Level of significance. $\alpha = 0.05$

The test statistic is

$W_0 = \min (W^-, W^+)$.

We arranged the data ascending

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
X_i	90	94	99	100	104	105	107	108	109	111	117	119	125	127	135	
$D_i = X_i - 107$	-17	-13	-8	-7	-3	-2	0	1	2	6	10	12	18	20	28	
$\lvert D_i\rvert$ Ranks$(-)$	11	10	7	6	4	2.5										40.5
$\lvert D_i\rvert$ Ranks$(+)$								1	2.5	5	8	9	12	13	14	64.5

ascending

The value of the test statistic is therefore $= min(W^-, W^+) = min(40.5, 64.5) = 40.5$

$$W_0 = 40.5$$

Decision rule: We reject H_0 if the 0.05 level of significance if $w_0 \leq w_{14,\mathbf{0.05}}$

Decision

the tabulated value $W_{14,\mathbf{0.05}} = 21$

$40.5 > 21$ $W_0 > W_{14,\mathbf{0.05}}$

We fail to reject H_0

We conclude that the median IQ of the subjects in the population may be 107

2.3.5 Large sample approximation

Theorem 2.1

When the null hypothesis is true,

$$E(W) = \frac{n(n+1)}{4} \qquad \text{and} \qquad V(W) = \frac{n(n+1)(2n+1)}{24}$$

Proof

When H_0 true, W can be defined as

$$W = \sum_{i=1}^{n} w_i \qquad \text{where}$$

$w_i = 0$ with probability $\frac{1}{2}$

$w_i = i$ with probability $\frac{1}{2}_i$

Thus, $\quad E(W) = E(\sum_{i=1}^{n} w_i) = \sum_{i=1}^{n} E(w_i) = \sum_{i=1}^{n}[w_i \cdot p(w_i)$

$$V(W) = ?$$

Since $(w_1, w_2, w_3, \cdots, w_n$ are independent

$$V(W) = V\left(\sum_{i=1}^{n} w_i\right) = \sum_{i=1}^{n} V(w_i)$$

$$V(w_i) = E(w_i^2) - [E(w_i)]^2 = \left[0^2\left(\frac{1}{2}\right) + i^2\left(\frac{1}{2}\right)\right] - \left(\frac{i}{2}\right)^2$$

$$= \left(\frac{1}{2}\right)i^2 - \left(\frac{1}{4}\right)i^2$$

$$V(w_i) = \left(\frac{1}{4}\right)i^2$$

$$V(W) = \sum_{i=1}^{n} V(w_i) = \sum_{i=1}^{n}\left(\frac{1}{4}\right)i^2 = \frac{1}{4}\sum_{i=1}^{n}i^2$$

$$V(W) = \frac{1}{4}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{n(n+1)(2n+1)}{24}$$

Theorem 2.2

When the null hypothesis is true, for large *n:*

$$Z = \frac{W - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$ follows an approximate standard normal

distribution $N(0, 1)$.

Proof

If *W* is a random variable with mean $\frac{n(n+1)}{4}$

and variance $\frac{n(n+1)(2n+1)}{24}$ then by the central limit theorem

$$Z = \frac{W - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$ is approximately $N(0, 1)$.

Adjustment for Ties

- We can incorporate an adjustment for ties among nonzero differences in the large sample approximation in the following way.

- Let t be the number of absolute differences tied for a particular nonzero rank. Then the correction factor is

The correction factor =
$$\frac{\sum t^3 - \sum t}{48}$$

We can subtract this quantity from the expression in the denominator under the square root sign.

☐ Thus the adjusted statistic for a large sample approximation is

$$Z = \frac{W - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24} - \frac{\sum t^3 - \sum t}{48}}}$$

We illustrate the calculation of an adjustment for ties in the following data:

Computation of correction factor

Observation	Rank	t	t^3
3	1.5		
3	1.5	2	8
4	3		
6	5		
6	5	3	27
6	5		
8	7.5		
8	7.5	2	8
9	10.5		
9	10.5		
9	10.5	4	64
9	10.5		
		11	107

$$\frac{\sum t^3 - \sum t}{48} = \frac{107 - 11}{48} = 2$$

Example 2.6

The following data show the life span, in years, of a random sample of 21 recorded deaths in a certain country. It has been known in the past years that the median life span in the country is 50 years. Can we conclude from these data that the median life span in the country has improved? Use α = 0.05

| 39 42 42 47 47 53 59 59 59 60 62 65 66 68 69 70 72 |
| 75 75 85 90 |

Solution:

Let $\tilde{\mu}$ denote the median life span in the population. We wish to test the hypothesis

Test the hypothesis $H_0: \tilde{\mu} = 50$ aginst $H_1: \tilde{\mu} > 50$

Level of significance: $\propto = 0.05$.

Test statistic: $W_0 = W^-$

But we have Ties we must use Adjustment for Ties

The test statistic is :

computation of test statistic

	X_i	D_i	$|D_i|$	Rank −	Rank +	t	t^3
1	39	−11	11	10			
2	42	−8	8	4.5		2	8
3	42	−8	8	4.5			
4	47	−3	3	2			27
5	47	−3	3	2		3	
6	53	3	3		2		
7	59	9	9		7		27
8	59	9	9		7	3	
9	59	9	9		7		
10	60	10	10		9		
11	62	12	12		11		
12	65	15	15		12		
13	66	16	16		13		
14	68	18	18		14		
15	69	19	19		15		
16	70	20	20		16		
17	72	22	22		17		
18	75	25	25		18.5	2	8
19	75	25	25		18.5		
20	85	35	35		20		
21	90	40	40		21		
				23	210	10	70

The value of the test statistic is

$$Z_0 = \frac{W^- - \dfrac{n(n + 1)}{4}}{\sqrt{\dfrac{n(n + 1)(2n + 1)}{24} - \dfrac{\sum t^3 - \sum t}{48}}}$$

$$Z_0 = \frac{23 - \dfrac{21(21 + 1)}{4}}{\sqrt{\dfrac{21(21 + 1)(2 \times 21 + 1)}{24} - \dfrac{70 - 10}{48}}} = -3.2175$$

Decision rule

Reject H_0 at the 0.05 level of significance if $Z_{,\propto} = -1.645$
where Z_0 is the computed value of $Z = -3.2175$

Decision

Since calculated $Z_0 \leq$ tabulated $Z_{\propto,}$ $(-3.2175 < -1.645)$, we reject H_0 at the 0.05 level of significance. We therefore conclude that, the median life span in the country has improved significantly.

Chapter Three
Two Independent Samples
3.1The Mann-Whitney U test (Wilcoxon rank-sum) test

3.1The Mann-Whitney U test (Wilcoxon rank-sum) test

The key contributions of **Mann Whitney U Test or(Wilcoxon Rank Sum Test) is** by Henry Mann and Donald Whitney in their original paper on the test in 1947

this is a popular nonparametric test to compare outcomes between two independent groups is used to test whether two samples are likely to derive from the same population (i.e., that the two populations have the same shape)or (both have the same median).

This test in the nonparametric equivalent of the two sample t-test. While the t-test makes an assumption about the distribution of a population (i.e. that the sample came from a t-distributed population), the Mann Whitney U Test makes no such assumption.

Some investigators interpret this test as comparing the medians between the two populations.

Assumptions

1- The sample is independent   random sample and

 2-The variable should be measured on an ordinal scale or a continuous scale.

3-The variable should be two independent, categorical groups. In other words, there should be no relationship between the two groups or within each group.

4-The-sample does not require normally distributed. However, they should follow the same shape (i.e. both are bell-shaped and skewed

Test the hypotheses
H_0: The two populations are equal versus
H_1: The two populations are not equal.
Level of significance α =0.05
The test statistic
The test statistic is U= min$\{U_1 , U_2 \}$
1-The test is often performed as a two-sided test and, thus, the research hypothesis indicates that the populations are not equal as opposed to specifying directionality.
 2-A one-sided research hypothesis is used if interest lies in detecting a positive or negative shift in one population as compared to the other. The procedure for the test involves pooling the observations from the two samples into one combined sample, keeping track of which sample each observation comes from, and then ranking lowest to highest from 1 to n_1+n_2, respectively.
 to Compute the test statistic assign ranks the smallest through largest values in the total sample, and find U_1 and U_2 as follows
$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \text{ and } U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

where R_1 and R_2 are the sums of the ranks in groups 1 and 2, respectively.
Decision rule.
The appropriate critical value can be found in the table above. To determine the appropriate critical value we need sample sizes and level of significance α
 The decision rule is as follows: Reject H_0 if U calculated $\leq$ U tabulated

Mann-Whitney Test Works

What does Mann Whitney U test show?

The **Mann-Whitney U test is** used to compare whether there **is** a difference in the dependent variable for two independent groups. It compares whether the distribution of the dependent variable **is** the same for the two groups and therefore from the same population.

Why we use Mann Whitney test?

The **Mann-Whitney U test** is a non-parametric **test** that can be **used** in place of an unpaired t-**test**. **It** is **used** to **test** the null hypothesis that two samples come from the same population (i.e. have the same median) or, alternatively, whether observations in one sample tend to be larger than observations in the other.

The Mann-Whitney test can be completed in four steps:

1-Combine the data from the two samples into one

2-Rank all the values, with the smallest observation given rank 1, the second smallest rank 2, etc.

3-Calculate and assign the average rank for the observations that are tied (the ones with the same value)

4-Calculate the sum of the ranks of the first sample (the *U*-value)

Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. A total of n=10 participants are randomized to receive either the new drug or a placebo. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of the assigned treatment. The data are shown below.

Placebo	7	5	6	4	12
New Drug	3	6	4	2	1

Is there a difference in the number of episodes of shortness of breath over a 1 week period in participants receiving the new drug as compared to those receiving the placebo and is this statistically significant at the 5% level by the Mann-Whitney U test ?

Solution

In addition, the sample size is small ($n_1=n_2=5$), so a nonparametric test is appropriate.

The hypothesis test

H_0: The two populations are equal   versus

H_1: The two populations are not equal.

level of significance ($\alpha=0.05$)

The test statistic

$U= \min\{U_1 , U_2 \}$

The first step is to assign ranks and to do so we order the data from smallest to largest. This is done on the combined or total sample (i.e., pooling the data from the two treatment groups ($n=10$)), and assigning ranks from 1 to 10, as follows. We also need to keep track of the group assignments in the total sample.

		Total Sample (Ordered Smallest to Largest)		Ranks	
Placebo	New Drug	Placebo	New Drug	Placebo	New Drug
7	3		1		1
5	6		2		2
6	4		3		3
4	2	4	4	4.5	4.5
12	1	5		6	
		6	6	7.5	7.5
		7		9	
		12		10	

the goal of the test is to determine whether the observed data support a difference in the populations of responses.
we produce a test statistic based on the ranks.
sum the ranks in the placebo group=37=R_1
the sum of the ranks in the new drug group=18= R_2
the sum of the ranks will always equal n(n+1)/2 = 10(11)/2=55 which is equal to 37+18 = 55.

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \qquad U_1 = 5(5) + \frac{5(6)}{2} - 37 = 3$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 \qquad U_2 = 5(5) + \frac{5(6)}{2} - 18 = 22$$

U=min(3 , 22) = 3

Decision rule

The critical value of U can be found in the table below. To determine the appropriate critical value we need sample sizes (for Example: $n_1=n_2=5$) and our two-sided level of significance ($\alpha=0.05$). the critical value is 2, and

The decision

rule is to reject H_0 if $U \leq 2$. We do not reject H_0 because 3 > 2.
We do not have statistically significant evidence at $\alpha = 0.05$, the failure to reach statistical significance may be due to low power.

Example

A new approach to prenatal care is proposed for pregnant women living in a rural community. The new program involves in-home visits during the course of pregnancy in addition to the usual or regularly scheduled visits. A pilot randomized trial with 15 pregnant women is designed to evaluate whether women who participate in the program deliver healthier babies than women receiving usual care.

Usual Care	8	7	6	2	5	8	7	3
New Program	9	9	7	8	10	9	6	

Is there statistical evidence of a difference in women receiving the **New Program** versus usual prenatal care?

Solution

• Test the hypotheses.

H_0: The two populations are equal   versus

H_1: The two populations are not equal.

•Level of significanceα =0.05

•The test statistic.

Because the data are not normally distributed and the samples are small (n_1=8 and n_2=7), we use the Mann Whitney U test. The test statistic is U, the smaller of

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \text{ and } U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

where R_1 and R_2 are the sums of the ranks in groups 1 and 2, respectively.

• Compute the test statistic.

The first step is to assign ranks of 1 through 15 to the smallest through largest values in the total sample, as follows

		Total Sample (Ordered Smallest to Largest)		Ranks	
Usual Care	New Program	Usual Care	New Program	Usual Care	New Program
8	9	2		1	
7	8	3		2	
6	7	5		3	
2	8	6	6	4.5	4.5
5	10	7	7	7	7
8	9	7		7	
7	6	8	8	10.5	10.5
3		8	8	10.5	10.5
			9		13.5
			9		13.5
			10		15
				$R_1 = 45.5$	$R_2 = 74.5$

Next, we sum the ranks in each group. In the usual care group, the sum of the ranks is $R_1=45.5$ and in the new program group, the sum of the ranks is $R_2=74.5$. Recall that the sum of the ranks will always equal $n(n+1)/2$. As a check on our assignment of ranks, we have $n(n+1)/2 = 15(16)/2=120$ which is equal to $45.5+74.5 = 120$.

We now compute U_1 and U_2, as follows:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 8(7) + \frac{8(9)}{2} - 45.5 = 46.5$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 = 8(7) + \frac{7(8)}{2} - 74.5 = 9.5$$

Thus, the test statistic is U=9.5.

•Decision rule.

The appropriate critical value can be found in the table above. To determine the appropriate critical value we need sample sizes ($n_1=8$ and $n_2=7$) and our two-sided level of significance ($\alpha=0.05$). The critical value for this test with $n_1=8$, $n_2=7$ and $\alpha=0.05$ is 10 and the decision rule is as follows: Reject H_0 if $U \le 10$

. Decision
Reject H_0 if $U \le 10$
U=9.5
Reject H_0 $9.5 \le 10$

Normal Approximation

For large samples, U is approximately normally distributed. In that case, the standardized value

$$z = \frac{U - \mu_U}{\sigma_U}$$

where μ_U and σ_U are the mean and standard deviation of U, is approximately a standard normal deviate whose significance can be checked in tables of the normal distribution. μ_U and σ_U are given by

$$\mu_U = \frac{n_1 n_2}{2}$$

$$\sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

Note that since $U_1 + U_2 = n_1 n_2$, the mean $n_1 n_2/2$ used in the normal approximation is the mean of the two values of U. Therefore, the absolute value of the z statistic calculated will be same whichever value of U is used.

Example

Two processing systems were used to clean wafers. The following data represent the (coded) particle counts. is there a difference between two processing systems were used to clean wafers.
The solution shows the typical kind of output software for this procedure would generate, based on the large sample approximation.

Solution:

Test the hypothesis

H_0 :there is no difference between the two processing systems were used to clean wafers.
H_1 :there is difference between the two processing systems were used to clean wafers.

Significance level $\alpha = 0.05$

Test the Statistic:

The test statistic is U= min$\{U_1 , U_2 \}$

Compute test statistic

Group A	Rank	Group B	Rank
0.55	8	0.49	5
0.67	15.5	0.68	17
0.43	1	0.59	9.5
0.51	6	0.72	19
0.48	3.5	0.67	15.5
0.60	11	0.75	20.5
0.71	18	0.65	13.5
0.53	7	0.77	22
0.44	2	0.62	12
0.65	13.5	0.48	3.5
0.75	20.5	0.59	9.5
Sum of Ranks	106.000		147.000

Decision rule

$n_1 = n_2 = 11$

$R_1 = 106$

$R_2 = 147$

$U_1 = 81$

$U_2 = 40$

$$\sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{\frac{(11)(11)(11+11+1)}{12}} = 15.23$$

$$\mu_U = E(U) = \frac{n_1 n_2}{2} = \frac{(11)(11)}{2} = 60.5$$

$$z = \frac{U - \mu_U}{\sigma_U} = \frac{40 - 60.5}{15.23} = -1.346$$

For a two-sided test with significance level $\alpha = 0.05$, the critical value is $z_{1-\alpha/2} = -1.96$. Since $|z|$ is less than the critical value,

Decision

we do not reject the null hypothesis and conclude that there is not enough evidence to claim that two groups have different central tendencies.

The one-sample runs test for randomness

What is run test for randomness?

Runs test is a statistical procedure which determines whether a sequence of data within a given distribution have been derived with a random process or not. It may be applied to **test** the **randomness** of data in a survey that collect data from an ordered population.

The one-sample runs test for randomness

In many situations we want to know whether we can conclude that a set of observations constitute a random sample from an infinite population. Test for randomness is of major importance because the assumption of randomness underlies statistical inference (see Ofosu & Hesse, 2011). In addition, tests for randomness are important for time series analysis. The runs test procedure is used to examine whether or not a sequence of sample values is random.

Consider, for example, the following sequence of sample values
21 23 24 27 30 28 27 26 25 23 22 21

Each observation is denoted by a '+' sign if it is more than the previous observation and by a '− ' sign if it is less than the previous observation as shown in the following table.

21	23	24	27	30	28	27	26	25	23	22	21
	+	+	+	+	−	−	−	−	−	−	−
		1						2			

A run is a sequence of signs of the same kind bounded by signs of other kind. In this case, we doubt the sequence's randomness, since there are only two runs . If the order of occurrence were

25	22	27	23	27	28	21	26	23	30	21	24
	−	+	−	+	+	−	+	−	+	−	+
	1	2	3		4	4	6	7	8	9	10

we would doubt the sequence's randomness because there are too many runs (10 in this instance).

Too few runs indicate that the sequence is not random (has persistency) whilst too many runs also indicate that the sequence is not random (is zigzag). Let us now consider the one sample runs test. This procedure helps us to decide whether a sequence of sample values is the result of a random process.

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Too few runs indicate that the sequence is not random (has persistency) whilst too many runs also indicate that the sequence is not random (is zigzag). Let us now consider the one sample runs test. This procedure helps us to decide whether a sequence of sample values is the result of a random process.

Assumptions

The data available for analysis consist of a sequence of sample values, recorded in the order of their occurrence.

Hypotheses test

H_0 : The sequence of sample values is random, against

H_1 : The sequence of sample values is not random.

Test Statistic

The test statistic is R, the total number of runs.

Decision Rule

Since the null hypothesis does not specify the direction, a two-sided test is appropriate. The critical value, r, for a given sample size n and at a desired level of significance α.

If lower $r_c \leq r \leq$ upper r_c accept H_0 Otherwise reject H_0

Tied Values

If an observation is equal to its preceding observation, denote it by zero. While counting the number of runs, ignore it and reduce the value of n accordingly.

Large Sample Sizes

If $\boldsymbol{n > 25}$ then the test statistic can be approximated by

$$Z = \frac{R - \mu_R}{\sqrt{var(R)}}$$

which is $N(0, 1)$, when H_0 is true, $\mu_R = E(R) = \frac{2n-1}{3}$ and $var(R) = \frac{16n-29}{90}$

where. We reject H_0 at the level of significance $\propto$ if $|z| > z_{1-\frac{\propto}{3}}$ where z is the computed value of Z.

Example

The following are the blood glucose levels of 12 patients who attend St. Thomas Hospital: Test, at the 0.05 level of significance whether the sequence is random?

86	99	98	90	109	101	100	110	110	93	108	120

Solution

Test Hypothesis

H_0 The sequence is random, against

H_1 The sequence is not random.

The test statistic is

The test statistic is R, the total number of runs.

Desicion rule

We reject H_0 at the 0.05 level of significance if lower $r_c \leq r \leq$ upper r_c accept H_0 Otherwise reject H_0

where r is the observed value of R and r_c is the critical value.

Chapter Four
Three or More Independent Samples
The Kruskal-Wallis one-way analysis of variance by Ranks

The Kruskal Wallis test is the non parametric alternative to the One way ANOVA. The H test is used when the assumptions for ANOVA aren't met (like the assumption of normality). It is sometimes called the *one-way ANOVA on ranks*, as the ranks of the data values are used in the test rather than the actual data points.

The test determines whether the medians of **Three** or more groups are different. Like most statistical tests, you calculate a test statistic and compare it to a distribution cut-off point. The test statistic used in this test is called the **H statistic.**

The Kruskal–Wallis test has its goal to determine if all k populations are identical or if at least one of the populations tends to give observations that are different from those of other populations.

The test is used when we have k samples, with k≥3

Advantages of the H Test

1-The H test is simply an analog to the F test in that the statistic is calculated using ranked data rather than the original observations.

2-it is simple to use, it does not require a computer to calculate, and it is widely available in applied text. Compared with the F test, the H test is quicker and easier to apply and it makes fewer assumptions of the population under study.

3- H test can be calculated for interval- or ratio-level data by transforming it to the ordinal scale through ranking.

The H test is distributed approximately as a chi-square distribution (with df = C- 1),

Assumptions

Regarding Use The hypothetical example presented covered the four main assumptions underlying the use of the H test.

1-It was assumed that the dependent variable under study has continuous distribution.

2- The observations should be independent. In other words, there should be no relationship between the members in each group or between groups. .

3-One independent variable with two or more levels (independent groups). The test is more commonly used when you have three or more levels. For two levels, consider using the Mann Whitney U Test instead.

Ordinal scale, Ratio Scale or Interval scale variables.

The hypotheses Test :

H_0: population medians are equal.

H_1: population medians are not equal.

level of significant : $\propto$

Test Statistic:

1- Sort the data for all groups/samples into ascending order in one combined set

2- Assign ranks to the sorted data points. Give tied values the average rank.

3- Add up the different ranks for each group/sample.

4- Calculate the H statistic:

$$H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{c} \frac{R_i^2}{n_i} \right] - 3(N+1)$$

Where:

n = sum of sample sizes for all samples,

c = number of samples,

R_i = sum of ranks in the ith sample,

n_i = size of the ith sample.

Decision Rule: Find the critical chi-square value, with c-1 degrees of freedom **at specific level of significant**

Decision: Compare the H value from to the critical chi-square value .

If the critical chi-square value is less than the H statistic, reject the null hypothesis that the medians are equal . If the chi-square value is not less than the H statistic, there is not enough evidence to suggest that the medians are unequal.

Example1

A company wants to know if three groups of workers have different salaries at 5% Level of significant

Women	23K	41K	54K	66K	78K
Men	45K	18K	55K	30K	60K
Minorities	34K	70K	40K	72K	44K

Solution :

H_0: The workers salaries are same

H_1: The workers salaries are different

level of significant 5%

Test Statistic

Women n	Men	Minorities	All salaries Ordered Smallest to largest	All Ranks	Women Ranks	Men Ranks	Minorities Ranks
23K	45K	18K	18K	1			1
41K	55K	30K	23K	2	2		
54K	60K	34K	30K	3			3
66K	70K	40K	34K	4			4
78K	72K	44K	40K	5			5
			41K	6	6		
			44K	7			7
			45K	8		8	
			54K	9	9		
			55K	10		10	
			60K	11		11	
			66K	12	12		
			70K	13		13	
			72K	14		14	
			78K	15	15		
					44	56	20

Calculate the H statistic:

$$H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{c} \frac{R_i^2}{n_i}\right] - 3(N+1)$$

$$H = \left[\frac{12}{15(15+1)} \left[\frac{44^2}{5} + \frac{56^2}{5} + \frac{20^2}{5}\right] - 3(15+1)\right]$$

H = 6.72

Find the critical chi-square value, with c-1 degrees of freedom. For 3 – 1 degrees of freedom and an alpha level of .05, the critical chi square value is 5.9915.

Compare the H value from 4 to the critical chi-square value from .
In this case, 5.9915 is less than 6.72, so you can reject the null hypothesis.

Example 2

In a manufacturing unit, four teams of operators were randomly selected and sent to four different facilities for machining techniques training. After the training, the supervisor conducted the exam and recorded the test scores.

At 5% level of significant does the scores are same in all four facilities?

Facility 1	Facility 2	Facility 3	Facility 4
64	77	71	52
81	76	86	65
56	84	88	68
87	59	51	82

Solution :

The hypothesis test

H_0: The distribution of operator scores are same

H_1: The scores may vary in four facilities

level of significant 5%

Test Statistic: $H = \left[\frac{12}{N(N+1)}\sum_{i=1}^{c}\frac{R_i^2}{n_i}\right] - 3(N+1)$

Compute Test Statistic

Facility1	Facility2	Facility3	Facility4	all Facility Ordered Smallest to Largest	Ranks			
					F1	F2	F3	F4
64	77	71	52	51	5	10	8	2
81	76	86	65	52	11	9	14	6
56	84	88	68	56	3	13	16	7
87	59	51	82	59	15	4	1	12
				64				
				65				
				68				
				71				
				76				
				77				
				81				
				82				
				84				
				86				
				87				
				88				
Total					34	36	39	27

$$H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{c} \frac{R_i^2}{n_i}\right] - 3(N+1)$$

$$H = \left[\frac{12}{16(17)}\right] \left(\frac{34^2 + 36^2 + 39^2 + 27^2}{4}\right) - 3(17) = 0.7$$

While for a right tailed chi-square test with 5% significant level, and df =3, critical χ^2 value is 7.815

Critical values of Chi-Square Distribution

							Area in the Right Tail			
	0.999	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010
Degrees of Freedom										
1	0.000	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.002	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.024	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.091	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.210	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.381	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812

Calculated χ^2 value is smaller than the critical value of χ^2 for a 0.05 significance level. H ($\chi^2_{critical} > \chi^2_{calculated}$) hence can not reject the null hypothesis So, there is not enough evidence to conclude that difference in test scores exists for four teaching methods at different facilities.

Adjusting for tied ranks in the Kruskal-Wallis test

When you have tied values, you will get tied ranks. In these circumstances you should apply a correction to your calculated test statistic. The notes show you how this can be done.

Calculating the adjustment factor

In order to correct for tied ranks you first need to know which values are tied. Then you need to know how many ties there are for each rank value. Finally, you'll need to know how many replicates there are in the dataset.

Once you've ascertained these things you can use the following formula to work out a correction, or adjustment, factor:

$$D = 1 - \frac{\Sigma(t^3 - t)}{(N-1)N(N+1)}$$

In the formula t is the number of ties for each rank value. For each value of t, you evaluate the t-cubed minus t part. This is then summed for all the tied values (values without ties can also be evaluated but $1^3 - 1 = 0$). Once you have the numerator you work out the denominator using n, the number of replicates in the original dataset.

The final value of D is then 1 – your fraction.

Adjusting the KW test statistic

Once you have the value of D, the correction factor, you can use it to adjust the original Kruakal-Wallis test statistic (H) $H_{adj} = \frac{H}{D}$

Formula for adjustment of the Kruskal-Wallis statistic in the case of tied ranks. The correction is simple: H/D.
You then use the Hadj value in place of the original to determine the final test significance (using critical values tables –

Example

Upper	Mid	Lower
3	4	11
4	3	12
5	7	9
9	9	10
8	11	11
10		
9		

The first step is to evaluate the ranks. Each value is replaced by its rank in the overall dataset. Sample data shown as ranks instead of original values. Samples are combined for ranking.

	Upper ranks	Mid ranks	Lower ranks
	1.5	3.5	15
	3.5	1.5	17
	5	6	9.5
	9.5	9.5	12.5
	7	15	15
	12.5		
	9.5		
$\sum$	48.5	35.5	69

The Kruskal-Wallis test looks at the sum of the ranks from each sample. If the $\sum$rank is different between samples there is a good chance that differences are statistically significant. If the $\sum$rank are close, then differences are less likely to be significant.

You can now calculate the Kruskal-Wallis test statistic, H.

Original H value

Once you have the rank sums you can compute the Kruskal-Wallis test statistic:

$$H = \left[\frac{12}{N(N+1)} \sum_{i=1}^{c} \frac{R_i^2}{n_i} \right] - 3(N+1)$$

Formula for calculating the Kruskal-Wallis statistic for non-parametric samples.

The Kruskal–Wallis formula looks pretty horrendous but actually it is not that bad. The numbers 12 and 3 are constants. Uppercase N is the total number of observations. The R refers to the ranks of the observations in each sample and n is the number of observations per sample.

In the example the final value of the Kruskal-Wallis statistic works out to be
$H = 6.403$.

Tied ranks

The formula for working out the correction factor was given earlier. You need to work out, for each rank value, the number of repeats.

Rearranging ranked data to make calculation of tied ranks easier.

Rank	Ord	Ties	$T^3 - T$
1.5	1.5	2	6
3.5	1.5		
5	3.5	2	6
9.5	3.5		
7	5		
12.5	6		
9.5	7		
3.5	9.5	4	60
1.5	9.5		
6	9.5		
9.5	9.5		
15	12.5	2	6
17	12.5		
9.5	15	3	24
12.5	15		
15	15		
15	17		
			102

Once you have the ranks in order it is easy to work out the number of repeats by inspection. You can simply fill in the values as you work down the column. In the preceding table the 3rd column shows the tied rank repeats. So, for example the rank 1.5 is repeated 2 times. The rank 9.5 has 4 repeats.

The final column shows the $T^3 - T$ values. In other words, you take the number of repeats and cube it, then subtract the number of repeats.

Once you have these values you can sum them to get an overall value, in this case 102.

Final H adjustment

Once you have your final sum of $T^3 - T$ values you can work out the value of D using the formula given earlier. You need to know the total number of replicates in the original dataset (in this case 17).

The final value of D works out at: 0.9792.

The adjusted H value is then $H/D = 6.403 / 0.9792 = 6.540$.

You can now use the adjusted H-value to compare to critical value tables to see if your result is statistically significant.

Chapter Five
5.1Chi-Square Test of Independence

The chi-square test of independence is used to determine whether there is a relationship between two categorical variables. This lesson explains how to conduct a **chi-square test for independence**. The test is applied when you have two categorical variables from a single population. It is used to determine whether there is a significant association between the two variables.

For example, in an election survey, voters might be classified by gender (male or female) and voting preference (Democrat, Republican, or Independent). We could use a chi-square test for independence to determine whether gender is related to voting preference.

When to Use Chi-Square Test for Independence the test procedure described is appropriate when the following conditions are met:

-The sampling method is simple random sampling.

-The variables under study are each categorical.

-If sample data are displayed in a contingency table,

This approach consists of four steps:

1-State the Hypotheses

Suppose that Variable A has r levels, and Variable B has c levels
The [null hypothesis](#) states that knowing the level of Variable A does not help you predict the level of Variable B., the variables are independent.
H_o: Variable A and Variable B are independent.
H_1: Variable A and Variable B are not independent.

2-Formulate an Analysis Plan

The analysis plan describes how to use sample data to accept or reject the null hypothesis. The plan should specify the following elements
Significance level. Often, researchers choose [significance levels](#) equal to • 0.01, 0.05, or 0.10; but any value between 0 and 1 can be used.
Test method. Use the [chi-square test for independence](#) to determine • whether there is a significant relationship between two categorical variables.

3-Analyze Sample Data

Using sample data, find the degrees of freedom, expected frequencies, test statistic, and the P-value associated with the test statistic..

4- Interpret results

- **Degrees of freedom.** The <u>degrees of freedom</u> $[df = (r - 1) * (c - 1)]$
- where r is the number of levels for one categorical variable, and c is the number of levels for the other categorical variable.
- **Expected frequencies.** The expected frequency counts are computed separately for each level of one categorical variable at each level of the other categorical variable.
- expected frequencies, according to the following formula $. E_{r,c} = (n_r * n_c) / n$
- where is the expected frequency count for level r of Variable A and level c of Variable B, n_r is the total number of sample observations at level r of Variable A, n_c is the total number of sample observations at level c of Variable B, and n is the total sample size.
- **Test statistic.** The test statistic is a chi-square random variable $(\mathcal{X}^2)$ defined by the following equation. $\mathcal{X}^2 = \Sigma [(O_{r,c} - E_{r,c})^2 / E_{r,c}]$ where:
- $O_{r,c}$ is the observed frequency count at level r of Variable A and level c of Variable B,
- $E_{r,c}$ is the expected frequency count at level r of Variable A and level c of Variable B.
- **P-value.** The P-value is the probability of observing a sample statistic as extreme as the test statistic. Since the test statistic is a chi-square, use the <u>Chi-Square Distribution Calculator</u> to assess the probability associated with the test statistic. Use the degrees of freedom computed above. $\mathcal{X}^2_\alpha$

Interpret Results

If the sample findings are unlikely, given the null hypothesis, the researcher rejects the null hypothesis. Typically, this involves comparing the P-value to the <u>significance level</u>, and rejecting the null hypothesis when the P-value is less than the significance level.

Example:

A public opinion poll surveyed a simple random sample of 1000 voters. Respondents were classified by gender (male or female) and by voting preference (Republican, Democrat, or Independent). Results are shown in the [contingency table](#) below. Do the men's voting preferences differ significantly from the women's preferences? Use a 0.05 level of significance.

	Voting preference			Row Total (n_r)
	Repu.	Demo.	Inde.	
Male	200	150	50	400
Female	250	300	50	600
Column Total (n_c)	450	450	100	1000 (n)

Solution

State the hypotheses.

H_o: Gender and voting preferences are independent.

H_1: Gender and voting preferences are not independent.

Formulate an analysis plan. the significance level is 0.05.

Analyze sample data. Applying the chi-square test for independence to sample data,
 we compute the degrees of freedom,.
df = (r - 1) * (c - 1) = (2 - 1) * (3 - 1) = 2
The expected frequency counts:

$$E_{r,c} = (n_r * n_c) / n$$

$E_{1,1}$	=	(400	*	450)	/	1000	=	180000/1000	=	180
$E_{1,2}$	=	(400	*	450)	/	1000	=	180000/1000	=	180
$E_{1,3}$	=	(400	*	100)	/	1000	=	40000/1000	=	40
$E_{2,1}$	=	(600	*	450)	/	1000	=	270000/1000	=	270
$E_{2,2}$	=	(600	*	450)	/	1000	=	270000/1000	=	270

$E_{2,3}$ = (600 * 100) / 1000 = 60000/1000 = 60

the chi-square test statistic. Based on the chi-square statistic and the [degrees of freedom](),

$$\mathcal{X}^2 = \Sigma [(O_{r,c} - E_{r,c})^2 / E_{r,c}]$$

$$\mathcal{X}^2 = (200 - 180)^2/180 + (150 - 180)^2/180 + (50 - 40)^2/40$$
$$+ (250 - 270)^2/270 + (300 - 270)^2/270 + (50 - 60)^2/60$$
$$\mathcal{X}^2 = 400/180 + 900/180 + 100/40 + 400/270 + 900/270 + 100/60$$
$$\mathcal{X}^2 = 2.22 + 5.00 + 2.50 + 1.48 + 3.33 + 1.67 = 16.2$$

The P-value is the probability that a chi-square statistic having 2 degrees of freedom is more extreme than 16.2.

We use the [Chi-Square Distribution Calculator](#) to find P($\mathcal{X}^2 > 16.2$) = 0.0003.

Interpret results. Since the P-value (0.0003) is less than the significance level (0.05), we cannot accept the null hypothesis. Thus, we conclude that there is a relationship between gender and voting preference.

Chapter Five
5.2 Chi-Square Test of Homogeneity

nonparametric test used in a situation where the dependent variable is categorical. Data can be presented using a contingency table in which populations and categories of the variable are the row and column labels.

Example

In a study of the television viewing habits of children, a developmental psychologist selects a random sample of 300 first graders - 100 boys and 200 girls. Each child is asked which of the following TV programs they like best: The Lone Ranger, Sesame Street, or The Simpsons. Results are

shown in the contingency table below.

Do the boys' preferences for these TV programs differ significantly from the girls' preferences? Use a 0.05 level of significance.

	Viewing Preferences			Total
	Like alone Ranger	Sesame Street	The Simpsons	
Boys	50	30	20	100
Girls	50	80	70	200
Total	100	110	90	300

Solution

State the hypotheses.

Null hypothesis: The null hypothesis states that the proportion of boys who prefer the Lone Ranger is identical to the proportion of girls. Similarly, for the other programs. Thus,

H_o: $P_{\text{boys like Lone Ranger}} = P_{\text{girls like Lone Ranger}}$

H_o: $P_{\text{boys like Sesame Street}} = P_{\text{girls like Sesame Street}}$

H_o: $P_{\text{boys like Simpsons}} = P_{\text{girls like Simpsons}}$

H_1: At least one of the null hypothesis statements is false.

Formulate an analysis plan. For this analysis, the significance level is 0.05. Using sample data, we will conduct a [chi-square test for homogeneity](#)

Analyze sample data. Applying the chi-square test for homogeneity to sample data, we compute the degrees of freedom, the expected frequency counts, and the chi-square test statistic. Based on the chi-square statistic and the degrees of freedom, we determine the P-value.

df=(r-1)*(c-1)

df = (r - 1) * (c - 1) = (2 - 1) * (3 - 1) = 2

$E_{r,c}$ =(n_r *n_c)/n

$E_{1,1}$ = (100 * 100) / 300 = 10000/300 = 33.3

$E_{1,2}$ = (100 * 110) / 300 = 11000/300 = 36.7

$E_{1,3}$ = (100 * 90) / 300 = 9000/300 = 30.0

$E_{2,1}$ = (200 * 100) / 300 = 20000/300 = 66.7

$E_{2,2}$ = (200 * 110) / 300 = 22000/300 = 73.3

$E_{2,3}$ = (200 * 90) / 300 = 18000/300 = 60.0

$$\mathcal{X}^2 = \Sigma \left[(O_{r,c} - E_{r,c})^2 / E_{r,c} \right]$$

$$\mathcal{X}^2 = (50 - 33.3)^2 / 33.3 + (30 - 36.7)^2 / 36.7 + (20 - 30)^2 / 30 + (50 - 66.7)^2 / 66.7 + (80 - 73.3)^2 / 73.3 + (70 - 60)^2 / 60$$

$$\mathcal{X}^2 = (16.7)^2 / 33.3 + (-6.7)^2 / 36.7 + (-10.0)^2 / 30 + (-16.7)^2 / 66.7 + (6.7)^2 / 73.3 + (10)^2 / 60$$

$$\mathcal{X}^2 = 8.38 + 1.22 + 3.33 + 4.18 + 0.61 + 1.67 = 19.39$$

The P-value is the probability that a chi-square statistic having 2 degrees of freedom is more extreme than 19.39.
We use the Chi-Square Distribution Calculator to find
$P(\mathcal{X}^2 > 19.39) = 0.0001$ (The actual P-value, of course, is not exactly zero. If the Chi-Square Distribution Calculator reported more than four decimal places, we would find that the actual P-value is a very small number that is less than 0.05 and greater than zero.)
Interpret results. Since the P-value (0.0001) is less than the significance level (0.05), we reject the null hypothesis.

Chapter Six
Runs Test to Test for Randomness of Observations

1. What is Run Test

Run test is a statistical test used to determine of the data obtained from a sample is random. That is why it is called Run Test for Randomness. Randomness of the data is determined based on the number and nature of runs present in the data of interest.

What is a Run?

A run is a sequence of similar or like events, items or symbols that is preceded by and followed by an event, item or symbol of a different type, or by none at all. Randomness of of the series is unlikely when there appear to be either too many or two few runs. In this case, a run test need to be carried out to determine the randomness.

The Run Test when performed helps us to decide whether a sequence of events, items or symbol is the result of a random process.

Example of Runs

A data scientist carrying out a research interviewed 10 persons during a survey. We denote the genders of the poepl by M for male and F for Female.
Assuming the respondents were chosen as follows:

Scenario 1
M M M M M F F F F F
Scenario 1 has only 2 runs and therefore the scenario cannot be considered

Scenario 2
F M F M F M F M F M
Scenario 2 has too many runs and therefore would not be considered as random

Scenario 3
F F F M M F M M F F
random because there are to few runs
Scenario 3 has 5 runs and therefore we need to perform a test to determine the randomness of the data.

Run Test Procedure

First we need to assume that the data available for the analysis consists of a sequence of observations, recorded in order of occurrence, which we can categorize into two mutually exclusive types.

First, you need to determine the total sample size, then the number of observation of each type as presented below:

n = total sample size
n_1 = the number of observation of one type
n_2 = the number of observations of the other type

Hypothesis

Then State the null and alternate hypothesis

A. TWO-SIDED
H_0: the pattern of occurrence is random
H_1: the pattern of occurrence is not random

B. ONE-SIDED
H_0: the pattern of occurrence is random
H_1: the pattern of occurrence is not random (because there are too few runs to be attributed as random)

C. ONE-SIDED
H_0: the pattern of occurrence is random
H_1: the pattern of occurrence is not random (because there are too few runs to be attributed as random)

Test Statistic and Decision Rule

A. TWO-SIDED	B. ONE-SIDED (H_1 has few runs)	C. ONE-SIDED (H_1 has to many runs)
Reject H_0:	Reject H_0:	Reject H_0:
R ≤ lower critical value OR R ≥ upper critical value	R ≤ lower critical value	R ≥ upper critical value

Critical Value
Critical value is determined from statistical table using n_1 and n_2

Example 1

On a commuter train, the conductor want to see whether the passengers entering a train enter in a random manner. He observes the first 25 people, with the following sequence of Males(M) and Females(F).

F F F M M F F F F M F M M M F F F F M M F F F M M

Test for randomness at $\alpha = 0.05$

Solution

test hypothesis :

H_0: The patter of occurrence of males and females enter the train is random
H_1: The pattern of occurrence of males and females entering the train is not random

test statistic : (number of runs)

You can easily get this by grouping each run as shown below:

FFF MM FFFF M F MMM FFFF MM FFF MM

Test statistic, r = 10
n_1 = number of females = 15
n_2 = number of males = 10

critical value

We can find the lower and upper critical value from statistical run table
n_1 = 15 , n_2 = 10
Lower critical value = 7
Upper critical value = 18

Decision Rule :
Since r = 10 which is between 7 and 18, we accept the null hypothesis (we fail to reject it)

Decision : Draw a Conclusion
There are not enough evidence to reject the claim hat the pattern of occurrence of males and females enter the train is determined by a random process

Example 2

We have 20 people that enrolled in a drug abuse program. Test the claim that the ages of the people, according to the order in which they enroll occur at random, at $\alpha = 0.05$.

The data are as follows:
18, 36, 19, 22, 25, 44, 23, 27, 27, 35, 19, 43, 37, 32, 28, 43, 46, 19, 20, 22

Solution

Test the hypothesis :

H_0: The pattern of occurrence of ages of the people enrolled in a drug abuse program is determined by a random process

H_1: The pattern of occurrence of ages of people enrolled in a drug abuse program is not random

Test statistic (number of runs)

To find the number of runs we first arrange the data in ascending order and find the median of the data set.

18, 19, 19, 19, , 20, 22, 22, 23, 25, 27, 27, 28, 32, 35, 36, 37, 43, 43, 44, 46

the median of the data set = **27**

Then compare the original data with the median. The replace the above median in the original sequence with an

 A if it is above the median and with

B if it is below the median.(you can also use the mean instead of median)

18	36	19	22	25	44	23	27	27	35	19	43	37	32	28	43	46	19	20	22
B	A	B	B	B	A	B			A	B	A	A	A	A	A	A	B	B	B

We can now arrange the data according to runs and we would have the output below:

B A BBB   A B A B AAAAAA   BBB

From the above we have

Test statistic, $r = 9$

n_1 = number or A runs = 9

n_2 = number of B runs = 9

Critical Value

$n_1 = 9$, $n_2 = 9$

From statistical table of Runs Test, we get the critical values

Lower critical value = 5
Upper critical value =15

Decision Rule

Since the statistic r is between the upper and lower critical values, we accept H_0

Decision

There is not enough evidence to reject the claim that the patter of occurrence of ages of people in the program is determined by a random process

Example 3

This table shows the departures from normal of daily temperatures recorded at Korek January 2016. We would like to know whether we may conclude that the pattern of departures above and below norm al is the result of a non- random process.

Day	temperatures recorded	Day	temperatures recorded	Day	temperatures recorded	Day	temperatures recorded
1	12	9	-1	17	6	25	-2
2	13	10	3	18	7	26	-6
3	12	11	2	19	10	27	-6
4	11	12	-6	20	10	28	-5
5	5	13	-7	21	1	29	-2
6	2	14	-7	22	1	30	-2
7	-1	15	-12	23	3	31	-2
8	2	16	-9	24	7		

Solution

Test the hypothesis:

H_0: The pattern of occurrence of negative and positive deviations from normal of daily temperatures is determined by a random process

H_1: The pattern of occurrences of negative and positive deviations from normal of daily temperatures is not determined by a random process (claim)

Test statistics (number of runs)

To get the number of runs, we need to find the departures from normal above and below zero.

The daily temperatures that is above 0 are recorded as A

 and

 The daily temperatures that are below 0 are recorded as B.
If we do this we would have the arrangement as follows:

AAAAAA B A B AA BBBBB AAAAAAAA BBBBBBB

Test Statistic (number of runs)

$r = 8$

n_1 = number of A = 17

n_2 = number of B = 14

Find the critical value

$n_1 = 17$, $n_2 = 14$

Using statistical table we find the:

Lower critical value = 10

Upper critical value = 22

The Decision Rule

Since r = 8, which is lower than the critical value, we reject the null hypothesis (H_0)

Decision

There is enough evidence to support the claim that the pattern of occurence of positive and negative departures from normal is not random

Chapter Seven
Confidence interval for the median based on the sign test

The $100(1 - \alpha)$ confidence interval for consists of those values of for which we would not reject a two-sided null hypothesis $H_0 : \tilde{\mu} = \tilde{\mu}_0$, at the level of significance(α).

□ We designate the lower limit of our confidence interval by($\tilde{\mu}_L$) and the upper limit by($\tilde{\mu}_U$).

□ We determine the largest positive or negative signs, (i.e. the value $\acute{s}$ such that

$$P(S \leq \acute{s}|n) = \frac{\propto}{2}$$

When the data values are arranged in order of magnitude, the $(\acute{s} + 1)^{th}$ observation is To find the upper limit of the confidence interval, we count the ordered sample values backwards from the largest. The $(\acute{s} + 1)^{th}$ observation from the largest value locates $\tilde{\mu}_U$ i.e.

$\tilde{\mu}_U = (n - \acute{s})^{th}$ value.

Example

Construct a 95% confidence interval for the median of the population from which the following sample data have been drawn, using the sign test.

0.07	0.69	1.74	1.90	1.99	2.41	3.07	3.08
3.10	3.57	3.71	4.01	8.11	8.23	9.10	10.16

-- arrange the data ascending

0.07	0.69	1.74	1.90	1.99	2.41	3.07	3.08
3.10	3.57	3.71	4.01	8.11	8.23	9.10	10.16

The point estimate of the population median is the sample median which is the mean of the two middle values in the ordered array. Thus,

the sample median $= \dfrac{3.08+3.10}{2} = 3.09$

- To find $(\tilde{\mu}_L)$ we consult a table of the sign test and find that, and $P(S \leq 3|16) = 0.0106$ and $P(S \leq 4|16) = 0.0384$

- Thus, we note that we cannot obtain an exact 95% confidence interval for the median. Since $100[1 - 2(0.0106)] = 97.88\%$, which is larger than 95 and $100[1 - 2(0.0384)] = 92.32\%$, which is smaller than 95.

- This method of constructing confidence intervals for the median does not usually yield intervals with exactly the usual coefficients of (0.90) ,(0.95) , and(0.99).

☐ In practice, we choose between a wider interval and a higher confidence or the narrower interval and lower confidence.

Suppose we choose $\acute{s} = 4$ then $\acute{s} + 1 = 5$ Therefore the 5th value in the ordered array is and the 12th (i.e. $16 - 4$) value in the ordered array is ($\tilde{\mu}_U$)
Thus $\tilde{\mu}_L = 1.99$ and $\tilde{\mu}_U = 4.01$

☐ The confidence coefficient is therefore $100[1 - 2(0.0384)] = 92.32$.
We say that we are 92.32% confident that the population median is between 1.99 and 4.01.

Chapter Nine
Computer Applications

9.1Rank Correlation

9.1.1 Spearman's rank correlation

9.2 Procedures Using Data from Three or More Related Samples

9.2.1 Friedman two-way analysis of variance by ranks

9.2.2 Kendall's rank correlation coefficient

9.2.3 cochran

9.3 Two related Samples

9.3 .1-McNemar

9.3 .2The Wald-Wolfowitz

9.4 Several Independent Samples

9.4.1 Jonckheere terpstra

9.5 one sample Kolmogorov–Smirnov