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**A Review Article about
Leven's test**

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Non parametric leven's Test

Abstract

Making sure that the variation is homogeneous is a necessary step before judging the equality of means across different groups. Usually, early tests that looked at scale equality based on the idea that things were normal. Levene's non-parametric test has become a useful way to check if the variance between groups is similar, which is helpful when the data is not normal. This piece goes into great detail about Levene's non-parametric test, focusing on how useful and reliable it is.

Secondary sources and a library-based method were used to conduct the article review. Its goal was to put together what was known about Levene's non-parametric test for consistency of variance.

It is mentioned that Levene's non-parametric test is adaptable and can be used in lots of different science and study areas. People like this test because it is accurate and easy to use. It is a good way to look at data. More than one study that looked at how good Levene's test is found that it works most of the time, even when the means of the groups are different. But you should be careful when you use Levene's non-parametric test.

Last but not least, this study helps future researchers who might want to use Levene's non-parametric test for homogeneity of variance in between-group comparisons. People who use it should know how widely it can be used, but they should also know the limits that come with different group means.

Keywords: Homogeneity of variance, Levene's test, Non-parametric

1. Introduction

ANOVA and other statistical tests are very useful in many fields of study, like education, psychology, and the health and psychosocial sciences (Shear, Nordstokke & Zumbo, 2018). It's important to talk about the idea that differences are all the same before you use ANOVA. When you compare two groups of scores, variance variation means that the spread of scores in some groups is larger than in others. This difference changes how the parameters for within-group variance are estimated, which in turn changes how the parameters for population variance are estimated. This changes the test power and the rates of type I and type II errors (Nordstokke and Colp, 2014). So, when you use tests like t-test and ANOVA on datasets with different variance, you might not be able to draw as strong of conclusions (Zhou, Zhu, and Wong, 2023).

This is the first step in a lot of different types of studies that compare means. For example, t-tests and ANOVA (Parra-Frutos, 2009) need this step. A lot of research on the reliability of parametric tests for non-normal data has found a higher type I mistake rate (Cain, Zhang, and Yuan, 2017). Some statistical tests, like t-tests and ANOVA, that measure group means

are skewed when the populations are not the same. This is very important when sample numbers aren't all the same (Shear et al., 2018).

Other than variance equality tests, Levene's (1960) test was made because variance equality tests have some problems. It works well when the material is spread out in a normal way. You have to change the raw scores and then do ANOVA on the new ones to use this method. On the other hand, the mean-based Levene test changes the scores of factors that are dependent by taking away the mean from each score. Data that is not usually spread, on the other hand, can change it (Nordstokke et al., 2011). Levene's test has been used in many ways, and they all work well with both real and fake data (Nordstokke & Zumbo, 2010). One example is the nonparametric Levene (NPL) test by Nordstokke and Zumbo (2007). The NPL test was made to make it harder for things to break the idea of normalcy.

Many fields use Levene's non-parametric test, but some still don't agree on whether it is correct, especially when the sample sizes are small and not evenly distributed. Levene's non-parametric test will be looked at, and different studies that have been done in this area will be summed up. The first part of the study talks about Levene's test and the changes that happened after it. After this, there is a full list of papers that look at homogeneity of variance tests and Levene's non-parametric test. It is then used to talk about and compare the results of these studies. At the end of the piece, opinions based on study are given.

2. Review

2.1. Homogeneity of Variance Tests

Several research has been done on homogeneity of variance tests in the statistics literature. Numerous studies and models have investigated the stability of different tests are at nominal significance levels. In this area, the F test, Levene's test (1960), and Bartlett's test (1937) continue to get a lot of attention (Parra-Frutos, 2013; Parra-Frutos, 2009). In this group, Levene's test has become popular among experts because it is a common and easy way to check if linear variance is equal.

Different kinds of new study have been done to try to make Levene's test better (Parra-Frutos, 2013). Miller (1998) notes that Levene's test is a good way to test things, and Van Valen (2005) says that it is easy and good to use. Hosken et al. (2018) mention that Levene's test is the best way to compare differences because it is conservative, doesn't get thrown off by anything that isn't normal, and can find signs.

Permutation tests also show that they are steady and have relative strength for sample numbers that are pretty big. Based on Zhou et al. (2023), Parra-Frutos (2013) also mentions that Levene's test handles the type I error rate well for all groups, even when sample sizes are small or not spread out evenly.

2.2. Levens Test

Before using tests like ANOVA and t-tests to compare two groups, it's important to think about the important idea of homogeneity of variance to make sure that the results are correct. So this idea comes from the thought that the differences between groups in the trait being looked at are the same. When you compare two groups, it's important to make sure that the differences are all the same, because different variances can make the results less reliable (Nordstokke et al., 2011).

Box recognized in 1953 that the F test for equality of differences could be wrong when the data was not normal, especially when the Type I error rate was too high. Non-normal distributions also have significant impacts on this test (Nordstokke and Colp, 2014). Following Box's ideas and realizing how important it was to assume that variance was homogeneous, Professor Howard Levene created a groundbreaking test in 1960 that looked at the equality of population variances (k). Notably, Levene's (1960) paper has garnered over 1,000 citations in scientific literature, underscoring its practical importance (Gastwirth, Gel, and Miao, 2009).

Levene's contribution revolves around suggesting an innovative examination wherein an analysis of variance (ANOVA) is executed on the absolute values of the residuals within each specific group. This unique test, formulated by Levene, presupposes the independence and normal distribution of the data Y_{ij} . Notably, the test doesn't hinge on sample variance, rendering it less susceptible to outliers. Its primary objective lies in evaluating the null hypothesis that the compared samples originate from populations possessing identical variances. Essentially, any perceived differences in variances are ascribed to chance, representing minor discrepancies in each sampling. If the p-value derived from Levene's test surpasses 0.05, it signifies that the variances lack significant distinctions (indicating the satisfaction of the assumption of homogeneity of variance) (Zhou et al., 2023).

Levene's initial publication advocated the exclusive use of means. The importance of Levene's examination lies in its relevance to numerous scientific investigations that focus on the variability among k populations rather than the averages or central tendencies of variables. Often, Levene's assessment functions as a preliminary validation for the assumption of uniform variance in traditional ANOVA. Levene's test is used to check if measurement errors are equal when Moore's k population is used (Gastwirth et al., 2009).

There are three main situations in which Levene's test can be used (Shear et al., 2018):

1. Equal Variance Test: The equality of variance test is used as a preliminary test before comparing community values.
2. Direct Variance Comparison: The test is sometimes used to directly look at how different study results are from each other.

3. Effect Size: The equality of variances test between groups can be used to find out the effect size (Nordstokke et al., 2011; Shear et al., 2018).

Levene's first review is based on means that have the same range. To do this, he does an analysis of variance (ANOVA) on the exact difference between the scores that were recorded and the means for each group. The original Levene test is basically an ANOVA on the absolute values of deviations, which are shown as $Z_{ij} = Y_{ij} - \bar{X}_j$. Here, Y_{ij} is the score of observation i in group j and \bar{Y}_j is the group j mean. Many people suggest to use Levene's test to check for variance equality, but it's important to remember that this method depends on symmetric distributions, which means that the data is normal. However, a computer study shows that Levene's original test may have a higher type I error rate when symmetry is broken (Nordstokke et al., 2011).

Levene's test operates under certain assumptions, which are outlined below:

$$(1) \quad H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$$

$$(2) \quad H_1: \sigma_1^2 \neq \sigma_2^2 \neq \dots \neq \sigma_k^2$$

At least for one pair (i,j)

the test statistic of Levene, denoted as W , is established when there exists a variable Y comprising a sample of size N distributed into k subgroups. Here, N_i represents the sample size of subgroup i .

$$(3) \quad W = \frac{(N-K)}{(K-1)} \frac{\sum_{i=1}^k N_i (\bar{Z}_i - \bar{Z}_{..})^2}{\sum_{i=1}^k \sum_{j=1}^{N_i} N_i (\bar{Z}_{ij} - \bar{Z}_i)^2}$$

Z_{ij} equals the mean, median, or trimmed mean as follows:

$$1. \quad Z_{ij} = |Y_{ij} - \bar{Y}_i| \quad (4)$$

In relation (4), \bar{Y}_i is the mean of subgroup i .

$$2. \quad Z_{ij} = |Y_{ij} - \tilde{Y}_i| \quad (5)$$

In relation (5) \tilde{Y}_i is the median of subgroup i .

$$3. \quad Z_{ij} = |Y_{ij} - Y_i^{\sim'}| \quad (6)$$

In relation (6), $Y_i^{\sim'}$ is the cut mean of subgroup i .

\bar{Z}_i is the group mean of Z_{ij} and $\bar{Z}_{..}$ is the overall mean of Z_{ij} .

2.3. Non parametric levens test Process

Considering the diminished efficacy of Levene's test when sampling from skewed and non-normally distributed populations (Nordstokke & Zumbo, 2010; Shear et al., 2018), Conover (1971) introduced a nonparametric variant of Levene's parametric tests to evaluate the equality of variances.

$$C_{ij} = (\text{Rank}(|y_{ij} - \bar{y}_j|))^2$$

Conover and Iman (1987) furnished extensive tables for applying the squared rank technique. However, a more commonly adopted approach is to perform a test resembling the Wilcoxon rank sum test on these squared ranks (Gaonkar and Beasley, 2023):

$$CN = \frac{(N-K)}{(K-1)} \frac{\sum_{i=1}^J n_i (\bar{c}_i - \bar{c}^*)^2}{\sum_{i=1}^K \sum_{j=1}^N (\bar{c}_{ij} - \bar{c}^*)^2 / (N-1)}$$

Here, C_{ij} represents the transformed value for the i -th subject in group j , while \bar{C}_j denotes the mean for group j . Additionally, \bar{C}^* signifies the overall mean of the transformed scores, and N corresponds to the total sample size, where N is the sum of individual subgroup sizes ($\sum n_j$).

Brown and Forsythe (1974) significantly advanced Levene's test by incorporating not only the mean but also the trimmed mean in their approach.

In an alternative rendition of Levene's test proposed by Brown and Forsythe (1974), the calculation of deviations involves the use of sample medians instead of sample means. Their in-depth Monte Carlo studies showed that this change worked. It shows that when the data was distributed in a Cauchy way, using the adjusted mean gave the best results. But it turned out that the median was the best choice for skewed statistics. It's interesting that finding the mean worked better when the distributions were even and had a mean tail.

A clever way to change Levene's test without using parametric data was found by Nordstokke and Zumbo in 2010. Instead of raw numbers, they used ranks (Shear et al., 2018). There are a lot of studies that have shown that this test is more accurate and has a lot of statistical power (Nordstokke & Zumbo, 2010). It does this by keeping the theoretical type I error. This review is more reliable across a wider range of data sets when ranks are used instead of raw scores. It is also more useful when standard parametric assumptions might not hold true.

When Levene's test was first made, X_{ij} stood for the scores of the notes. In the nonparametric version of Levene's test, Z_{ij} stands for the ranks that were added together earlier. The null hypothesis is tested using the F statistic that is found by this nonparametric test. Nordstokke and Zumbo (2010) explain that the nonparametric Levene test combines data from both groups (in a two-group situation), ranks the scores, and assigns the rank

values to the original groups. Then it uses these ranks to run the Levene test. This test can be put into formulation in the following way:

$$\text{ANOVA } (|R_{ij} - \bar{X}_j|)$$

In this case, R_{ij} is found by adding up the numbers in each group (j) and giving each data point a rank. After that, an analysis of variance is done on the absolute value of the average ranks for each group (\bar{X}_j), which is found by taking the rank of each R_{ij} away. From a computational perspective, this nonparametric Levene test incorporates the rank transformation concept introduced by Conover and Iman (1981) to bridge the gap between parametric and nonparametric statistics. The process encompasses three distinct steps, as outlined by Nordstokke et al. (2011):

- 1) Aggregating the data and replacing the original scores with their corresponding ranks.
- 2) Segmenting the data into individual groups.
- 3) Employing Levene's mean-based normality test for ranks.

This test can be conveniently executed using readily available software such as SPSS or SAS.

Nordstokke and Zumbo (2010) introduced the nonparametric version of Levene's test designed to evaluate the equality of population (or scale) variances. This test is particularly beneficial in scenarios where samples display non-normality, such as those derived from skewed distributions. The nonparametric iteration of Levene's test involves the ranking of observed scores, followed by the application of Levene's original mean-based test (Levene, 1960) for equal variances on the ranked data. Significantly, this nonparametric test exhibits favorable characteristics in terms of type I error and power, especially when dealing with highly skewed populations, as evidenced in both simulated and real data scenarios (Nordstokke and Zumbo, 2010).

In the particular scenario of comparing variances between two groups, Nordstokke and Zumbo (2010) delineate the procedural steps for Levene's test as outlined below:

1. Amass the observed data and allocate ranks to all scores, commencing with the lowest score receiving a rank of 1.
2. Categorize the ranked data into their respective primary groups .
3. Employ the mean-based Levene's test for equal variances (described subsequently) on the ranked data.

An Overview of the Research about Non parametric Leven’s Test

1. Bartlett's test (1937) and Levene's test (1960), Among the tests scrutinized, the F test (two samples), received notable attention. Their findings underscored the F test (two samples) as generally highly sensitive to the assumption of normality. In contrast, Bartlett's test demonstrated considerable resilience in the face of non-normality.

2. Robust test (1960), Levene, H. for equality of variances. In I. Olkin. shows that the simulations show that heterogeneity tests work better than Levene's test, especially when the Keyes-Levy fix is used.

3. Conover et al. (1981) conducted a comprehensive investigation into tests evaluating the homogeneity of variances. Numerous tests were subjected to scrutiny and simulation to evaluate their robustness at nominal significance levels.

4. Parra-Frutos (2009) conducted a simulation study to investigate the resilience characteristics of tests examining the equality of variances, with a specific emphasis on the type I error rate and test power. The study explored the established Levene's and Bartlett's tests, alongside several suggested modifications for Levene's test—both with and without the application of bootstrapping. The results indicated that certain adjustments, when implemented without bootstrapping, have the potential to improve the effectiveness of homogeneity tests.

Remarkably, when applying bootstrapping to Levene's test using the median and concurrently applying the Keyes–Levy adjustment and Satterthwaite correction, resilience to the significance level was consistently noted across a variety of distributions and permutations of sample sizes. However, it's crucial to acknowledge that none of the examined tests demonstrated the capability to safeguard against extreme values in symmetric distributions.

5. Nordstokke et al. (2011) authored a paper delineating the functional attributes of Levene's nonparametric test. The primary aim was twofold: firstly, to clarify the features of the innovative nonparametric Levene test for equal variances, adaptable to commonly used statistical software such as SPSS or SAS; and secondly, to examine its operational traits, type I error, and statistical power using authentic assessment data.

This investigation delved into the operational aspects of Levene's non-parametric test through computer experiments involving mathematical distributions. The outcomes underscored the commendable validity and power of Levene's non-parametric test when applied to non-normal data. However, the study also identified that the presence of correlations within the data could diminish the test's efficacy.

6.A study by Parra-Frutos (2013) looked at how well common alternatives to the ANOVA F test dealt with variation. This method turns into a collinearity test like Levene's test from 1960 on its own. The simulations show that heterogeneity tests work better than

Levene's test, especially when the Keyes-Levy fix is used. Predicted key values are used instead of structure zeros as part of this change.

7. Nordstokke and Colp (2014) investigated how well Levene's non-parametric test worked when there were three or more groups. The results showed that Levene's nonparametric test has good statistical qualities when samples are very different from how the population is distributed. It happened a lot when the sample size was small and the distributions of the groups were not fair. These results not only show that Levene's test works well in tough situations, but they also make it possible to compare the test to other tests. The mean can be used to check if the differences are the same across different situations.

8. Shear et al. (2018) tested how well Levene's nonparametric test for equal variances worked when samples were taken from groups whose means were different and unknown. The main goal of this simulation study was to find out how changes in population means affect how well Levene's nonparametric test for equal variances works.

A hard Monte Carlo computer tool was used in the study to show that Type I error rates in Levene's nonparametric test might be too high when samples are taken from groups whose means are unknown or are different. In addition, the Type I error rate could not be fixed by putting samples in the middle using sample means or medians.

9. Hosken et al. (2018) evaluated the jackknifing, Levene's, Box-Anderson, and Smith tests all at the same time. The results showed that each test has its own pros and cons that make it better or worse at solving the issues that the F test has when trying to find differences that are equal. The results showed that both Levene's and Box-Anderson tests were more accurate than the other tests when the sample sizes were small.(2)

10. Zhou et al. (2023) studied all the various types of consistency of variance tests that are used in clinical studies. Based on modeling results, the main goal of this review was to give broad ideas on how to use different methods to look for differences in clinical data.

It was clear from a close study of the computer data that the Jackknife method worked better than the other tests talked about in the article when there were only two samples. Many thanks to Cochran and Bartlett for their help. They worked better when there were more than two groups and the data was close to a normal distribution. But Levene's test seemed to be a better choice when the data wasn't quite right.

Discussion and Comparison

One important new idea is the nonparametric Levene test that Nordstokke and Zumbo (2010) came up with. Both Shear et al. (2018) and Nordstokke and Zumbo (2010) agree that this test has a lot of power when dealing with non-normal data. In particular, Levene's nonparametric test does an excellent task of keeping its Type I error rate low when dealing with normally distributed data, which had middling to high statistical power for finding changes in differences. These results strongly support Levene's nonparametric test being

useful for a wide range of ANOVA designs, especially when sample numbers are small and population distributions are skewed (Nordstokke et al., 2011).

However, different people have different thoughts on how to grade this test. In some simulations, Nordstokke and Colp (2014) mention that the non-parametric Levene test had a slightly higher Type I error rate than its median-based equivalent. This difference shows how complicated the test's result was. While Levene's test based on the median showcases commendable statistical characteristics and robustness, a cautious note is warranted—sole reliance on this comparison test may curtail the generalizability of results.

Additionally, if there are a lot of "outliers" in the dataset, the ranking method might not be able to tell the difference between changes in variances (Nordstokke et al., 2011). In light of this, Nordstokke et al. (2011) advocate for a cautious approach, suggesting that in instances where concerns arise regarding associations within the sample, the consideration of critical values for the test statistic should involve rigorous nonparametric tests employing computer-intensive methods. These methods encompass strategies such as replacement, randomization, bootstrap, or Jackknifing methods.

Jackknifing, in particular, stands out as an additional randomization technique necessitating a sizable sample size (greater than 20). While exhibiting a conservative nature in handling strong outliers, it surpasses Levene's test in power at reasonable sample sizes. However, it does encounter diminished power in scenarios with smaller sample sizes when working with data subsets. Notably, this method possesses a unique capability—it is the sole test capable of providing confidence intervals on variance estimates (Hosken et al., 2018). By leveraging these methodologies, critical values for the test are computed, tailored to specific instances of associations, thus partially mitigating the challenges posed by links within the sample.

A study by Shear et al. (2018) found that the non-parametric Levene test was more likely to make type I mistakes as the difference between sample means and the amount of skewness in the samples were wider. This means that experts might get it wrong when they think there are differences in group variances. This means that nonparametric scale tests like Levene's nonparametric test can be messed up when samples are taken from groups with uneven and unknown means. More specifically, Levene's non-parametric test and other rank-based tests may not work as well when group means are not equal (Shear et al., 2018).

It is possible to account for known changes in group means by taking the right number away from each sample. But if you don't know what the overall mean is, you might not want to use group means or medians to make changes. Researchers should be careful when using Levene's nonparametric test, say Shear et al. (2018). Because of this, problems with variation in the factors come up when sample numbers are not similar because of absolute differences from the mean.

To fix the problem of variable variability in Levene's test, Parra-Frutos (2013) suggests using the Welch (1951), James (1951), Alexander, and Govern (1994) tests, especially when sample numbers are small and not all of them are the same.

Nordstokke and Zumbo (2010) point out that the primary issue with the Levene non-parametric test is that it is not easy to find in statistics tools. Because it uses a new method, this test is not yet included in any paid statistics software. Because of this, experts have to come up with their own formulas to carry out this process. This lack of software interaction makes it hard for researchers and analysts to use Levene's non-parametric test in real life.

Nordstokke and Colp (2014) also say that in future studies, a wider range of variance tests, such as bootstrap methods, should be looked into. Many times, bootstrap methods are used to see if the variances are similar. Random selection with replacement is used in these methods. But remember that not all bootstrapping methods will work well or be effective. This method can give wrong results when distributions are very skewed, but these results become stronger as the sample size increases (Hosken et al., 2018). Another way to resample is the Jackknifing method, which was suggested by Hosken et al. (2018) and Zhou et al.(2023) .

To sum up, homogeneity of variance tests are useful in many study situations. However, Levene's non-parametric test hasn't been written about much lately, and the results aren't always clear or consistent. Levene's random test needs more research and study, according to research that has already been done. Because of this, it is a useful statistical tool that can be used in many study situations, especially when the data is spread out unevenly or the sample size is small.

Conclusion

The primary goal of this study was to use secondary data and library sources to look at Levene's non-metric test and everything that goes with it. We know that there isn't just one test that works best for all group sizes and situations because of past study. However, Levene's non-parametric test is still one of the most reliable options, especially when working with data that isn't spread out regularly. One of the best things about Levene's nonparametric test is that it can be used even when the data isn't normal. You can use it to do statistics studies because of this. The statistical power of this test is also higher when it is used on very skewed data. Levene's non-parametric test is often mentioned as a way to check for homogeneity of variance because of this. It is used a lot in many areas of statistical study.

But you should know about the issues that various computer studies have found, especially when it comes to groups of people whose means aren't equal or are unknown. More research needs to be done to find out how Levene's null test works in various social situations. The more we study this, the more we'll understand how the test works in different scenarios.

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