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A Review Article about

Hosmer lemeshower test

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Abstract

It is essential to evaluate the goodness and quality of the models` logistic regression fit for ensuring the correctness of the estimated models. To check the goodness in regression models fit with binary dependent variables, various tests and criteria can be used. In this research, one of the most frequently applied tests used for the goodness of fit in models of logistic regression that is, the Hosmer-Lemeshower test, is discussed. For this purpose, a review method was used to examine various aspects of this test, issues related to it, and the opinions of various researchers in this regard. In general, the findings show that in this test, the general importance of the regression coefficients is evaluated through comparing the actual and dependent variables' estimated values in various groups. A substantial discrepancy between the actual and dependent variables' estimated values suggests a lack of adequacy in the model's fit. The review of studies in this field shows that while the most commonly utilized test for the model of logistic regression is the Hosmer-Lemeshow test, it is highly sensitive to the number of samples and may not be suitable for interpreting the fit of the model in high-volume samples. Therefore, there is a need for strengthened and generalized models. By summarizing the existing studies, this research makes a discussion and comparison about the efficiency of the Hosmer-Lemeshower test and clarifies the research line for future researchers.

Keywords: Non parametric, Hosmer lemeshower test, Goodness of fit.

1. Introduction

Logistic regression models are a frequently applied statistical method for analyzing and estimating binary outputs that are used in various research fields such as medicine, social sciences, politics, etc. (Nattino, Pennell and Lemeshow, 2020; Paul, Pennell and Lemeshow, 2012). Researchers are always looking for a model that fits well with the actual data, because model fit assessment is an important and necessary part of any modeling activity. One of the different methods to evaluate the quality of a model's fit is fit testing goodness (Surjanovic and Loughin, 2021). When evaluating a statistical model, fit goodness is a significant step, including logistic regression, which measures the degree of agreement of the model's observed results with the estimated probabilities (Fagerland and Hosmer, 2012). Over the past four decades, numerous graphical methods and tests have been introduced by different researchers for evaluating the goodness of fit in logistic regression. Among these tests, the most frequently used technique is the Hosmer-Lemeshow test (Canary et al., 2016; Lai and Liu, 2018).

The Hosmer-Lemeshow test is a statistical procedure used for assessing the overall goodness-of-fit in a model of logistic regression. It provides a measure of the overall fit quality of the model (Surjanovic and Loughin, 2021). The purpose of the test is to divide the observations into distinct groups and calculate a chi-squared statistic that encapsulates the disparity between the actual and anticipated frequency of occurrences over all possible combinations of groups and result states. This examination involves the comparison of the anticipated frequencies of a dichotomous result with the observed frequencies of such a result across several groups or intervals. Typically, groups are constructed by partitioning the anticipated odds of the result into 10 distinct categories. (Nattino et al., 2020). For logistic regression model fitting, the test of Hosmer-Lemeshow is very popular and of interest due to its wide acceptance by various statistical packages, simplicity of interpretation, and ease of implementation. However, utilizing the test of Hosmer-Lemeshow faces a problem, namely that the number of groups and the sample size have an effect on determining the fit of the model; in other words, the adequacy of the fit may be deemed unacceptable in a large sample size, yet it may be seen as acceptable in a small sample (Nattino et al., 2020; Lai and Liu, 2018; Yu et al., 2017; Pual et al., 2013). This research seeks to review the background of this test and the research opinions of different researchers in this field. For this purpose, this research starts with a review of the concept of logistic regression, the test of Hosmer-Lemeshow as a quality of testing fit, its assumptions, and then a review of the cited research conducted in this field. In the next part, discussion and comparison will be done based on the points raised in the research, and finally, the conclusion will be presented.

2. Literature Review

In this section, the background of Hosmer Lemeshow's fit test goodness, logistic regression, and issues about this test in past research are reviewed.

2.1. A logistic regression model

Logistic regression models have garnered considerable interest as a statistical technique for assessing the likelihood of a positive outcome for a binary response variable (Surjanovic and Loughin, 2021). As a statistical regression model, logistic regression is often utilized for analyzing dichotomous dependent variables, including the presence or absence of a disease or the occurrence of death or survival. The model under consideration may be observed as a generalized linear model, whereby the error distribution implies a polynomial distribution and the logit function is used as the link function. Bilaterality is the occurrence of a random event in two possible situations (Hosmer, Lemeshow and Sturdivant, 2013).

Provided a set of predictors X1, X2, ...Xp and a binary result, Y is a model of logistic regression in the form $\{P(Y=1 \mid X1, X2, ...Xp)\}=\beta 0+ \beta 1\, X1+ \beta 2\, X2+ ...+ \beta p\, Xp$ where that logit link is described as $logit(p)=\ln(p/(1-p))$. The variables are evaluated using numerical maximization of the probability associated with the specified model of a sample $\{(Yi, Xi1, Xi2, ..., Xip)\}i=1, ..., n$. The probability assigned to target i in the model is denoted as pi and is calculated using the formula $pi = \{1+\exp(-(\beta 0+ \beta 1\, Xi1+ \beta 2\, Xi2+ \ldots+ \beta p\, Xip))\}^{-1}$.

By putting the estimate of $\widehat{\beta 0}$ $\widehat{\beta 1}$, …$\widehat{\beta p}$ in this equation, it is feasible to calculate the likelihood of obtaining a favorable outcome (Y = 1) for every individual within the selected sample (Nattino et al., 2020).

2.2. Hosmer lemeshower goodness-of-fit test

It is important to investigate the adequacy of regression models in terms of their fit to the data and adherence to underlying assumptions. This exam has two primary components: (1) differentiation and (2) discrimination. The metric evaluates the model's capacity to accurately categorize the data into the respective outcome groups. Calibration pertains to the assessment of the concordance between the estimated probability generated by a model and the actual observed outcomes. This evaluation is often conducted by means of a goodness of fit test (Fagerland and Hosmer, 2012). Goodness of fit measures the accuracy of the estimated probabilities. The goodness of fit test is a beneficial tool that makes it possible to evaluate the model's validity without limiting the potential hypothesis to a particular kind of deviance (Surjanovic and Loughin, 2021).

The calibration statistic consists of comparing the estimated possibility of the model with the actual result. One often-used statistic for model calibration is derived from the $\chi 2$ statistic of Pearson. This metric evaluates the adequacy of the model by comparing actual and anticipated results within specified groups, which are determined by projected probability rankings. The test of Hosmer Lemeshow goodness-of-fit is a calibration statistic often used in logistic regression analysis (Guffey, 2012). The test of Hosmer-Lemeshow has been frequently utilized to evaluate models of risk scoring and is a common method for evaluating fit goodness in logistic regression (Paul et al., 2012). In this test, general observations are arranged in ascending order based on their predicted probability and then divided into several groups of equal size (Lai and Liu, 2018).

The test of Hosmer-Lemeshow for goodness-of-fit is applied to evaluate the congruence between the anticipated number of events derived from the logistic regression model and the real number of events seen in the dataset (Nattino et al., 2020). This test compares the expected and actual number of events from data grouped by model fit values. The decision criterion for this test is derived from the comparison between the test statistic and the $\chi 2$ distribution, where the degrees of freedom are contingent upon the groups utilized in constructing the test statistic (Surjanovic and Loughin, 2021).

2.3. Hosmer lemeshower test hypothesis

A model is said to be completely calibrated when the estimated probability of the outcomes, denoted as $\{\pi i = p(Yi = 1 \mid X1i, X2i, \ldots, Xip)\}i=1,\ldots n$, aligns precisely with the actual observed outcome probabilities (Nattino et al., 2020). The test of Hosmer-Lemeshow, like many goodness-of-fit tests, makes two assumptions for deciding goodness of fit: (1) a null hypothesis indicating complete fit, in which the model assumes that the probability it considers is in accordance with the actual probability, and (2) a counter hypothesis involving imperfect fit (Nattino et al., 2020).

The null hypothesis posits that there is a perfect match, whereas the alternative hypothesis suggests that there is no fit:

H_0: $\pi_i = p_i$

H_α: $\pi_i \neq p_i$

The null hypothesis posits that there exists no statistically noteworthy variance between the projected values and the actual values, hence implying an ideal alignment with the model. The contrary assumption demonstrates a contrasting perspective and suggests the model's inadequate alignment (Lai and Liu, 2018). In order to calculate the HL statistic, the data are arranged in ascending order and then partitioned into G distinct groups. The frequency of occurrences and non-events for each group, denoted as O1g and O0g, respectively, is calculated. This statistical measure provides a summary of the disparities between the actual and anticipated quantities of occurrences and non-occurrences throughout the whole of the G groups:

(1)

$$\widehat{C} = \sum_{g=1}^{G} \left(\frac{(O_{1g} - E_{1g})^2}{E_{1g}} + \frac{(O_{0g} - E_{0g})^2}{E_{0g}} \right)$$

where O_{0g}, E_{0g}, O_{1g}, E_{1g} represent actual Y=0 events, predicted Y=0 events, actual Y=1 events, and predicted Y=1 events for the gth risk decile group, and G is number of the groups.

According to the perfect fit null hypothesis, it may be inferred that the statistic follows a distribution characterized by a variable of square random with a degree of freedom of G-2 (Hosmer

& Lemesbow, 1980). In the event that the null hypothesis is false, the statistical distribution is in the form of a non-centralized chi-square with a non-centralized parameter λ and an identical degree of freedom. The greater the distinction between the estimates of a model and the observed possibilities of the result, the greater λ is (Nattino et al., 2020).

An overview of the research about Hosmer lemeshower test

In 2021, Surjanovic and Loughin showed in a paper that with increasing complexity of the model, the error rate of type 1 (and thus the power) of the test of Hosmer Lemeshow declines. They found that the test of Hosmer-Lemeshow is not efficient enough in relatively large models of logistic regression corresponding to repetitions of exact values or clusters in the space of covariates. This research shows that Surjanovic et al.`s (2020) generalized type of the Hosmer Lemeshow test could prevent this power loss in estimation. In this generalized model, the use of the distribution of chi-squared with G-2 freedom degree could lead to a reduction in the error rate of type 1 and, as a result, a lower probability of misspecification of the model.

In 2020, Nattino, Pennell and Lemeshow did a paper in large samples on solving the test problem of Hosmer-Lemeshower goodness-of-fit. In order to overcome this constraint, the authors put out a ten-modification strategy for the test of Hosmer Lemeshow. The introduction of a standardized decentralized parameter for the alternative distribution of the test statistic of Hosmer Lemeshow has resulted in the incorporation of an indicator of model goodness of fit that is independent of the sample size. The author introduced a technique for estimating this parameter and constructing ranges of confidence for it. Finally, they implemented and compared their proposed method in a simulation study in a group with a sample size of more than 300,000 observations.

In 2018, Lai and Liu conducted research with a large sample size on the response to the Hosmer-Lemeshow test problem. In their article, the researchers put up a straightforward approach for assessing the sufficiency of the model of logistic regression in the context of a substantial sample size. This method involves the application of bootstrapping techniques. The results of the simulation tests demonstrate that the suggested methodology is capable of successfully standardizing the power of the test while maintaining type I error.

In 2017, Yu, Xu and Zhu wrote a paper in large samples on solving the Hosmer-Lemeshower goodness-of-fit test evaluation problem. An adjusted model of the test of Hosmer-Lemeshow was presented, whereby the estimate and normalization of the distribution variable of the Hosmer-Lemeshow statistic were used. The presenter provided a mathematical derivation that enables the calculation of the power of the test and the critical value. To evaluate the proposed model, they performed a simulation in the data set of bank marketing, which satisfactorily confirms the power of the test of Hosmer-Lemeshow. He claimed that the power of his recommended test is still stable even in large sample sizes.

An evaluation conducted by Canary et al. (2016) aimed to examine the associations among the Tsiatis, Pigeon-Heyse, and Hosmer-Lemeshow methods in the context of binary models of logistic regression with constant variables. The researchers evaluated the performance of these methods using simulation techniques. Based on his research, it was found that, in general, all three tests resembled strength. Among them, Tsiatis performed better, having an error rate of Type I and a consistent distribution over variable and number features. But in the "risk deciles" method, the Hosmer-Lemeshow test performed better.

In 2012, Pual et al. carried out an article on Hosmer-Lemeshow and its power reduction in large sample sizes. The current study examines the relationship between the number of groups and the power of the Hosmer-Lemeshow test utilized. To investigate this, mathematical derivations were conducted and then validated by data analysis and simulation. The dataset utilized in this research consisted of 31,713 children from the Collaborative Perinatal Project. Two suggestions were provided in the Hosmer-Lemeshow test for the determination of the number of groups, with consideration given to the size of the sample. (1) The first approach involves the selection of random subsamples of a standardized size, such as 1,000, from a very large sample ($n > 25{,}000$). The evaluation of model fit is then conducted utilizing these subsamples, with a value of $g = 10$. The second proposed technique involves assessing the model's performance on a substantial sample using various group sizes of no less than five. If the goodness of fit is deemed acceptable for a small value of g, such as $g < 10$, then it may be inferred with confidence that the model sufficiently represents the observed data.

Guffey (2012) conducted a comprehensive examination of tests that bear resemblance to the Hosmer-Lemeshow goodness-of-fit test, which is often used for the purpose of adjusting survival statistics. When examining event-to-time data, there is generally a preference for using a nonparametric estimation method to determine the survival function. The Kaplan-Meier estimator, sometimes referred to as the product limit estimator, offers an estimation of the survival function by utilizing data on event timing and censored periods. The graphical representation of Kaplan-Meier survival estimates frequently shows a stepwise decline in survival as a function of time.

Discussion and Comparison

Based on the research review, it was found that in the model of logistic regression, the test of Hosmer-Lemeshow GOF is the most utilized test (Zhang, 2016), because in logistic regression, the test of Hosmer-Lemeshow can be easily implemented in many statistical packages and is understandable for users (Guffey, 2012; Yu et al., 2017; Surjanovic and Loughin, 2021). However, this test also has disadvantages that will be reviewed below:

(1) Inefficiency in a large sample size: in model fit tests, "power" means the possibility of not accepting a model that has a poor fit. In general, the statistical significance of the chi-square test is positively correlated with the size of the sample, and this relationship holds true for the Hosmer-Lemeshow test as well. This particular matter does not constitute a desirable characteristic for the goodness of fit test. Ideally, the likelihood of erroneously refusing a regression model that is deemed to be acceptable, while not flawless, ought to be unaffected by variations in the size of the sample. Kramer and Zimmerman (2007), Pual et al. (2013), and Nattino et al. (2020) have shown an adverse characteristic of the test of Hosmer-Lemeshow in their respective investigations. According to the author, assessing the adequacy of the Hosmer-Lemeshow test for logistic regression with high sample sizes presents challenges. The power of a statistical test is known to have a positive relationship with the sample size in any given scenario. Goodness-of-fit tests may encounter challenges in the existence of huge data sets, since even little deviations from the model that was suggested might be deemed significant (Paul et al., 2012). In larger samples, the estimated and actual probabilities exhibit a growing tendency to refute the concept of perfect fit. The prevalence of this phenomenon has been extensively recorded in relation to generic goodness-of-fit tests, such as the test of Hosmer-Lemeshow (Nattino et al., 2020).

In this context, Pual et al. (2013) did not present a new method but, in various sample sizes, tried to keep the test power constant by adjusting the parameter g (number of groups). But they admitted that their method is limited by the upper limit of the sample (≤25000). It means that it does not work for more than 25,000 samples. Yu et al. (2017) also found that the model of Pual et al. (2013) works poorly for some models. Yu et al. (2017) recommended an adjusted test method for standardizing the power of the Hosmer-Lemeshow test that could be utilized in sample sizes larger than 25,000. But this model also needs to evaluate a complex integral, which makes it difficult to use this model. In response to these limitations, Nattino et al. (2020) proposed an adjustment in the test of Hosmer-Lemeshow to permit an accurate evaluation of goodness-of-fit without the effect of sample size. They show that this model, for sample sizes ranging from 500,000 to 5 million, does not change qualitatively. Therefore, this approach seems to be suitable for large volumes of data. However, further investigation is required in order to validate the efficacy of the model.

(2) Inefficiency in data replication: When the observations have the same explanatory variable patterns, the answers can be summarized in binomial numbers. In some instances, the combined distribution of the variables might result in the observed values being clustered in a manner that closely resembles duplication, so that the responses may be considered nearly binomial. These cases are not a problem for fitting the model and predicting the probabilities. Nevertheless, it has been shown that the presence of clustering in the covariate space can significantly affect the reliability of the Hosmer-Lemeshow test. In the case of these specific data structures, it can be shown that the chi-square distribution does not accurately depict the null distribution of the test statistic of Hosmer-Lemeshow when dealing with small sample sizes (Surjanovic and Loughin, 2021). In this context, Surjanovic and Loughin (2021) extended the standard Hosmer-Lemeshow test to allow its use with Poisson regression models. In this generalized model, it was observed that the predicted error of type 1 was reduced. In the generalized Hosmer-Lemeshow test statistic, Σn, as the central matrix, provides a kind of correction of the test statistic of Hosmer-Lemeshow by standardizing the correlation between group residuals.

(3) different test values based on different groupings: Hosmer et al. (1997) acknowledged that it should be kept in mind that various software packages might estimate various values of Hosmer-Lemeshow; according to them, this difference happens because of how the deciles are defined by

the algorithms. Bertolini et al. (2000) also stated that if there are associations among the estimated probabilities and these associations manifest at the boundary separating the ten equal-sized groups, placing various groupings leads to the production of different values of Hosmer-Lemeshow. This issue arises due to the variability in the rankings of correlated observations, resulting in the potential for various observations to be assigned to distinct groups with each iteration of the program (Canary et al., 2016).

In addition, according to Zhang et al. (2016), although in the model of logistic regression, the test of Hosmer-Lemeshow is the most frequently applied test, this is a summary statistic for examining model fit. According to Guffey (2012), the sensitivity of this test is influenced by both the number of groups and the boundaries that are utilized for the purpose of grouping. The inclusion of a constant variable in the model is essential. The ability to identify the elimination of an interaction or a misspecified linking function is limited, as is the power to identify the existence of small sample sizes (Hosmer, Hosmer, Le Cessie, & Lemeshow, 1997). One notable limitation is the absence of explanatory insights when testing reveals inadequate model fit (Guffey, 2012).

Conclusion

In this research, a review of the test of Hosmer-Lemeshow, as one of the most common goodness of fit tests in logistic regression, was done. This is a frequent test among different researchers whose performance is well documented. But based on the review of past research, it was found that despite the efficiency of this test, in relatively large logistic regression models or exact repetitions in auxiliary variable space, this test has a limited ability to detect incorrect indicators of the model. Failure to identify incorrect specifications of the model can lead to the creation of an inappropriate model, which will have negative consequences for research. Therefore, it is advisable to use care when evaluating the Hosmer-Lemeshow test results in the context of a large sample size. In such cases, it is recommended to use generalized Hosmer-Lemeshow tests. Therefore, there is a need to improve and adjust the test for large samples, as suggested by several studies. However, it is important to mention that the power of most of the proposed methods is low for small sample sizes. Therefore, it is better to continue to use the Hosmer-Lemeshow test in small samples (for example, less than 1000) and to use the suggested Hosmer-Lemeshow tests in larger samples (for example, more than 1000). Also, in the field of adjusted tests introduced by previous researchers, it is better to conduct more research to simplify and reduce their complexity so that the efficiency and reliability of this test can be confirmed in large samples.

References

1. BERTOLINI, G., D'AMICO, R., NARDI, D., TINAZZI, A. AND APOLONE, G., 2000. One model, several results: the paradox of the Hosmer-Lemeshow goodness-of-fit test for the logistic regression model. *Journal of epidemiology and biostatistics*, *5*(4), pp.251-253.
2. CANARY, J.D., BLIZZARD, L., BARRY, R.P., HOSMER, D.W. AND QUINN, S.J., 2016. A comparison of the Hosmer–Lemeshow, Pigeon–Heyse, and Tsiatis goodness-of-fit tests for binary logistic regression under two grouping methods. *Communications in Statistics-Simulation and Computation*, *46*(3), pp.1871-1894.
3. FAGERLAND, M.W. AND HOSMER, D.W., 2012. A generalized Hosmer–Lemeshow goodness-of-fit test for multinomial logistic regression models. *The Stata Journal*, *12*(3), pp.447-453.
4. GUFFEY, D., 2012. *Hosmer-lemeshow goodness-of-fit test: Translations to the cox proportional hazards model*. University of Washington.
5. HOSMER JR, D.W., LEMESHOW, S. AND STURDIVANT, R.X., 2013. *Applied logistic regression* (Vol. 398). John Wiley & Sons.
6. HOSMER, D.W. AND LEMESBOW, S., 1980. Goodness of fit tests for the multiple logistic regression model. *Communications in statistics-Theory and Methods*, *9*(10), pp.1043-1069.
7. HOSMER, D.W., HOSMER, T., LE CESSIE, S. AND LEMESHOW, S., 1997. A comparison of goodness-of-fit tests for the logistic regression model. *Statistics in medicine*, *16*(9), pp.965-980.
8. KRAMER, A.A. AND ZIMMERMAN, J.E., 2007. Assessing the calibration of mortality benchmarks in critical care: the Hosmer-Lemeshow test revisited. *Critical care medicine*, *35*(9), pp.2052-2056.
9. LAI, X. AND LIU, L., 2018. A simple test procedure in standardizing the power of Hosmer–Lemeshow test in large data sets. *Journal of Statistical Computation and Simulation*, *88*(13), pp.2463-2472.
10. NATTINO, G., PENNELL, M.L. AND LEMESHOW, S., 2020. Assessing the goodness of fit of logistic regression models in large samples: A modification of the Hosmer-Lemeshow test. *Biometrics*, *76*(2), pp.549-560.
11. PAUL, P., PENNELL, M.L. AND LEMESHOW, S., 2013. Standardizing the power of the Hosmer–Lemeshow goodness of fit test in large data sets. *Statistics in medicine*, *32*(1), pp.67-80.
12. SURJANOVIC, N. AND LOUGHIN, T.M., 2021. Improving the Hosmer-Lemeshow Goodness-of-Fit Test in Large Models with Replicated Trials. *arXiv preprint arXiv:2102.12698*.
13. YU, W., XU, W. AND ZHU, L., 2017. A modified Hosmer–Lemeshow test for large data sets. *Communications in Statistics-Theory and Methods*, *46*(23), pp.11813-11825.
14. ZHANG, Z., 2016. Model building strategy for logistic regression: purposeful selection. *Annals of translational medicine*, *4*(6).

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