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A review article about

Anderson-Darling Test

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Abstract

There are many different methods to analyze the non-parametric tests to determine the normality of the data that are collected. some of the give accurate shape in the results and some of the lack some accuracy due to their nature of where they focus in the analysis of the data. In this article we are going to review one of the non-parametric tests which is Anderson-Darling test. The definition of the method and the meaning of its basics with the history from the beginning to the present and how its developed along with the uses of the test will be discussed with the support of several scientific references and articles which provide excellent information about it. How scientists describe the goodness of the method compared with other methods and its accuracy will be shown with evidences and experimental reports will be derived from their statistical reports. the articles will be discussed and a comparison between how each author conducted the studies and gave results will be provided as well as a conclusion on all of the articles at the end of this paper.

Keywords: non-parametric, Anderson-Darling test, goodness of fit, normality.

Introduction

There are two main sorts of statistical test methods. The parametric model is employed when all population parameter information is known, the data is normally distributed, and the curve has a bell shape. The second strategy uses nonparametric tests, which are statistical analysis approaches that don't need a distribution to satisfy specific presumptions in order to be applied (especially when working with nonnormally distributed data). They are also known as distribution-free tests because of this. Numerous tests are provided in the non-parametric model to aid in the interpretation of the data received from a population. The Kolmogorov-Smirnov test (K-S) is a technique applied to examine if a sample is representative of a population with a given distribution. This test has the appealing property of not requiring knowledge of the underlying cumulative distribution function to compute the distribution of the K-S test statistic. An additional benefit is that the test is accurate (the validity of the approximations in the chi-square goodness-of-fit test is dependent upon a suitable sample size). Notwithstanding these benefits, the K-S test has a number of important downsides.

It only involves continuous distributions.

1. It tends to be more sensitive at the middle of the distribution than at the tails.

2. Perhaps the most severe limitation is that the distribution must be completely described. That is, if location, scale, and shape parameters are guessed from the data, the critical region of the K-S test is no longer valid. It normally must be determined by simulation.

The Kolmogorov-Smirnov test was improved into the Anderson-Darling and Cramer Von-Mises tests, which are regarded to be more effective than the original K-S test. A variation of the K-S test called the Anderson-Darling test is used to ascertain if a given sample of data is representative of a population with a certain distribution. Compared to the distribution-free K-S test, it provides larger weight to the tails. The Anderson-Darling test is more sensitive because it computes critical values based on the individual distribution, but it demands calculations for each distribution. There are critical value tables available for the following distributions: logistic, normal, uniform, lognormal, exponential, Weibull, extreme value type I, and generalized Pareto. Nevertheless, while critical values are frequently applied using statistical software packages that display the pertinent critical values, tables of critical values are not included in this Handbook. The Anderson-Darling test is an alternative to the chi-square and Kolmogorov-Smirnov goodness-of-fit tests.

The Anderson-Darling test is defined as:

H_0: The data follow a specified distribution.

H_a: The data do not follow the specified distribution

Test Statistic: The Anderson-Darling test statistic is defined as

$$A^2 = -N - S$$

where:

$$S = \sum_{i=1}^{N} \frac{(2i-1)}{N} [lnF(Yi) + \ln(1 - F(Y_{N+1-i}))]$$

F is the cumulative distribution function of the specified distribution. Note that the Yi are the ordered data.

Significance Level: α

The critical values for the Anderson-Darling test change based on whatever particular distribution is being evaluated. According to multiple publications by Stephens, it is a one-sided test that, if the test statistic A^2 is larger than the critical value, rejects the hypothesis that the distribution is of a particular kind.

Main Body

1. **In 2009**, Ciprian NECULA wrote a research paper. In his paper he included that using non-parametric econometric techniques, the study examined the parameters of the Generalized Hyperbolic Distribution for a number of stock index returns, including the Romanian BETC, and compared it to the actual distribution. The outcomes demonstrated that, in comparison to the normal distribution, the Generalized Hyperbolic Distribution provides a more accurate approximation of the actual distribution. The findings demonstrated that there is no difference between the actual values and the primary statistical markers used in finance theory, such as mean, variance, skewness, and kurtosis. The study also discovered that the normal distribution is not as good at simulating returns on financial assets as the Generalized Hyperbolic Distribution.

2. **In 2011**, Sonja ENGMANN and Denis COUSINEAU conducted researches to discover the distinctions between the K-S and A-D Test. In the research they concluded that the Anderson-Darling test is very effective and is very sensitive to the extreme end of the distribution. They also observed that the AD is more powerful when comparing two distributions that change in shift only, in scale alone, in symmetry only or that have the same mean and standard deviation but differ on the tail ends only. They also observed that the AD test is more powerful than the K-S in detecting the differences. Finally, according to their paper the AD test needs fewer data compared to the K-S test to have enough statistical power.

3. **In 2015**, Matteo Formenti, Luca Spadafora, Marcello Terraneo and Fabio Ramponi showed in research investigates the ability of the Anderson-Darling (AD) test to discover volatility underestimation and derives an extension for risk management. The extension concentrates on rejecting larger distribution variations while maintaining low rejection rates due to the moderate sample size uncertainty. When the model's volatility is underestimated, the asymmetric impact is amplified and results in a more noticeable non-linear behavior, which raises the test rejection power. In smaller sample sizes, the modified test outperformed the uniform standard test in terms of rejection when the forecasting distribution was incorrect. When compared to other uniform tests like the Kolmogorov-Smirnov test and the AD test, the modified test performed better when it came to rejection.

4. **In 2015**, Fiaz Ahmad and Rehan Ahmad Khan Sherwani conducted research to test some data using 12 standard normality tests using simulated data which are generated from 4 distributions. In the research they used the Anderson-Darling test to compare it with the other tests and it showed that the A-D test significantly

fucoses on the tail of the distribution. In the study they observed that size of sample data affects the result of the tests. They concluded that the A-D test is less powerful than the Shapiro Francia test but better than the rest of the tests.

5. **In 2017**, A. A. Makarov and G. I. Simonova, wrote a paper. In their paper they used models for contamination, the Kolmogorov-Smirnov test has poor power and leaves outliers after trimming when applied to conditional distributions. For the same model samples, the Anderson-Darling test was employed to compare its attributes. In order to demonstrate that several goodness-of-fit tests may be used based on the issue description and research objective, the paper examined the characteristics of the two tests for certain options. Comparing these tests' characteristics for particular alternatives is the goal of the study.
   The problem of categorizing nonparametric goodness-of-fit tests according to their power in comparison to certain alternatives may present a barrier for future research. Classes such as these could have distributions with certain forms, like strongly and weakly asymmetric, positive or negative excess, and the length of the "tail length." Initial data processing can assist in selecting the best test to meet the objectives of the study.

6. **In 2020**, Muhammad Aslam and Ali Algarni wrote a paper is introduces an extension of the Anderson-Darling (AD) test, which is used to assess the normalcy of data within intervals, taking into account neutrosophy and uncertainty. This article presents the design and operational technique of the neutrosophic Anderson-Darling test, which is used to assess the quality of interest based on the neutrosophic normal distribution. The test is administered on data pertaining to the renewable energy sector, and it proves to be efficacious in addressing ambiguity. The test yields findings at unpredictable intervals, a requirement for managing intricate systems in the renewable energy sector. The report suggests that renewable energy specialists should implement the test in an uncertain environment and explore the creation of software to execute the test. Future study also examines the applicability of the test for huge data.

7. **In 2021**, Sang-Hyuk Lee, Jong-Won Lee, Moon-Kyung Kim and Hee-Mun Park wrote a paper in their paper test The effect of TiO2 on lowering the content of NOx in asphalt pavement is investigated in this study. A large-scale testbed was constructed at an underpass portion, with the test section receiving TiO2 treatment and the reference section receiving none. For ten days, NOx concentration was monitored and recorded. The statistical verification approach known as the Anderson-Darling test was employed to examine the reduction effects.

With an average of 28.1% and 18.8% higher NOx concentrations in the reference part and 46.2% high in the test section, the study showed that the test section had a larger chance of NOx concentration than the reference section. This shows that the right weather contributed to the photochemical reaction of the TiO2 material on the asphalt surface.

8. **In 2021**, Yasser Al Zaim and Mohammad Reza Faridrohani in this paper introduces the Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) statistics for measuring the goodness of fit of both stationary and non-stationary random fields. The method uses a direct method by engaging a random projection of a Hilbert-valued random field to the actual line R. It then utilizes the AD and KS goodness-of-fit tests. The proposed methodology is investigated using simulated tests and a case study encompassing persons with autism and those without any health issues. The research indicates that the projected AD outperforms the projected KS, and it is possible to build the Cramér-von Mises projection-based test statistic for random fields and all other EDF tests. Future study aims include establishing the predicted Koziol-Green statistic for randomly censored data for random fields. Additional research is required to comprehend the behavior of the approach in functional random fields.

9. **In 2022**, Chuanhai Liu wrote a paper about Reweighted Anderson-Darling Tests of Goodness-of-Fit. In practical and theoretical statistics, evaluating goodness-of-fit is an important problem, especially in data-driven large data research. The Zhang test and the Anderson-Darling test are regarded as significant and potent goodness-of-fit tests. According to Anderson and Darling, the Anderson-Darling test is better than other tests such as Kolmogorov-Smirnov and Cramér-von Mises because it gives the distribution's tails adequate weight. The intention of this study was to use theoretical studies and intuitive reasons to address the question of what would be an all-purpose or default test for this problem. It was discovered that while uniforms in the core area are more associated than those in the tail sections, weights should be assigned to the distribution's tails much more than Anderson-Darling.

10. **In 2023**, Jiahao Liao, Shan Liu wrote a paper about using the Anderson-Darling test in the ground radar field. The article introduces an AD matching test algorithm to the field of ground radar interference identification. Critical values for various clutter distributions and significance levels are calculated by Monte Carlo simulation. Ground radar ground echo data were collected and analyzed using AD test algorithms. To address the problems caused by traditional AD test algorithms with large variations in AD test values and sample parameters of

ground radar echo data following multiple distribution types, cell averaging method and critical value unification method are proposed. For improvement. The results show that the improved AD test algorithm can better identify the clutter distribution of ground radar echoes at each distance unit.
This method can be applied together with an active environment-aware component of location-adaptive optimization.

11. **In 2022**, Parul Khatri, Tripti Arjariya and Nikita Shivhare Mitra made a research about Climate change forecasting using data mining algorithms and used methods for normality tests and showed that Normalcy and stationarity are the fundamental requirements for using the model. There is stationary rainfall data available. The original data's normality is examined using the Anderson-Darling test. This test is used to determine if the provided sample is representative of a certain distribution. The distribution's tails are given greater weight in this test than they are in Kolmogorov-Smirnov (K-S) test. In this test, the critical values are computed using the specific distribution. Despite the drawback that critical values must be determined for each distribution, the Anderson-Darling test employs that specific distribution to derive the critical values. However, since the Anderson-Darling test is thought to be more accurate than the chi-square and Kolmogorov-Smirnov goodness-of-fit tests, it won't be too tough.

12. **In 2023,** Awaz Mohamed Saleem1, Hazheen Mardan Mustafa1 , Zeravan Abdulmuhsen Asaad2*, Amjad Saber Al-Delawi used normality tests for their research. Based on normality and the random walk hypothesis, the study seeks to ascertain the Arab Federation of Exchanges' (SandP AFE 40 Index Return) efficiency at weak-form. Six non-parametric statistical approaches, such as the Augmented Dickey-Fuller test and the Jarque-Bera test, are used to support the results. Following the Covid-19 immunization campaign, these tests were performed on daily data from March 31, 2020, to March 31, 2023. The findings imply that investors might utilize previous data to forecast future stock prices and make extraordinary gains, as the market index return is non-efficient at the weak form. For investors to avoid making atypical profits by forecasting market fluctuations, policymakers should make information more accessible. This defies earlier findings, offering a useful direction for more

Discussion and Comparison

The current review paper shows the different capabilities and uses of the Anderson-Darling experimentation in Many fields such as finance, statistics, radar technology and climate change predicting. Every paper highlights the advantages and details of the AD test in a dedicated context. The articles include a wide range of statistical examination applications and assessments, predominantly the Anderson-Darling (AD) test, in some different fields. Ciprian Necula demonstrates the dominance of the Generalized Hyperbolic Distribution over the normal spreading in explaining market earnings. In 2011, Denis Cousineau and Sonja Engmann studied the assessment of the efficiency of the AD test, emphasizing its greater statistical even power with limited data and its capability to categorize distributional variances, particularly at the extremes. The improvement of the AD test for underestimation of volatility, Matteo Formenti and his colleagues in their paper in 2015 show that the AD has more ability to reject huge variations in distribution, specifically in lesser sample sizes. when normality is considered for assessment, Fiaz Ahmad and Rehan Ahmad Khan Sherwani (2015) highlighted the importance of the AD test's precise focus on the tails. While acknowledging that it is less effective than some other tests, they rank it higher than some others. A. A. Makarov and G. I. Simonova's 2017 study gives a result for both the Anderson-Darling and Kolmogorov-Smirnov tests and concludes the ways they can be utilized to classify nonparametric goodness-of-fit tests. Sang-Hyuk Lee et al. In the year of 2021, used the AD test to examine the influence of TiO2 on NOx reduction, demonstrated the usefulness of the test in environmental contexts. Chuanhai Liu in 2022 claims that it is imperious to give the tail weights more weight and advocates for the implementation of reweighted AD tests. This claim provides insightful instruction for running thorough goodness-of-fit testing. By addressing problems with conservative AD test instructions, Jiahao Liao and Shan Liu conducted a work from innovations in 2023 by offering an improved AD algorithm for ground radar interference identify. The research done in 2023 by Parul Khatri et al. summarizes the importance of the AD test in forecasting climatic change. Its showing that the test is precise in determining normality, although it needs critical parameters exclusive to each distribution. finally, the study by Awaz Mohamed Saleem et al. (2023) disproves previous conclusions by claiming that the market index return is not efficient in the weak form, which need encourage policy-makers to rise the accessibility of information.

All together, these studies establish how multipurpose the AD test is, highlighting its use in a variety of settings, for example comprehensive data analysis, financial modeling, environmental examination, and climate change estimations. The nuanced outcomes add to a complete understanding of the advantages and disadvantages of the AD test in difference backgrounds. Together, these research studies emphasize the Anderson-Darling (AD) test's usability and reputation in statistical analysis across a widespread range of

disciplines. In addition to argue that the Generalized Hyperbolic Distribution for store returns is more accurate than the normal distribution, Ciprian Necula's 2009 study confronts accepted conventions in finance theory.

All in all, these studies demonstrate the robustness and adaptability of the Anderson-Darling test across a variety of fields, including large-scale data analysis, climate change forecasting, and environmental science and finance. The inclusive insights providing by this study add to a better understanding of the limitations and usefulness of the AD test under various conditions. everyday there are more researches that are done using the AD test which prove the significance of the test in different fields. The current mentioned articles show a brief data about the test. however, the test may subject to changes and improvement according to the needs of future analysis which may only give good results by using this test.

Conclusion:

In summary, the comparative study gives a conclusion on how compliant and useful the Anderson-Darling (AD) test is across a number of different academic disciplines. In many unlike circumstances, for example economic calculations, statistical assessments, theoretical research, ecological influence studies, statistical comparisons, and climate change estimations, the AD test endlessly proves its effectiveness as a powerful statistical tool. The reflective understandings collected from these examinations importantly advance our knowledge of the multi-use and restrictions of the AD test in a number of contexts, indicating its adaptability and reliability in managing complex research questionings. The Anderson-Darling test, a non-parametric statistical method frequently used for normality and goodness-of-fit evaluations, is systematically studied in the academic literature. The first section of the introduction draws a clear line between parametric and non-parametric statistical techniques, highlighting the latter's flexibility in useful data that is not regularly distributed without the need for much assumptions. The article studies the test's evolution over time and clarifies its fundamental principles in order to highpoint the ways in which the A-D test differs from the K-S test.

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