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Sp-open set in topological space

Submitted to the department of (mathematic) in partial fulfilment of the Requirements for the degree of BSc. In mathematic

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Aknowledgment

I would like to thanks Allah for giving me the power to complete this work And I would like to present my profound thanks to supervisor and lecturer **Assist.Prof.Dr.Nehmat K. Ahmad** for his kind valuable suggestions that assisted me to accomplish this work I would also like to extend my gratitude to the head of mathematic department **Assist.Prof.Dr.Rashad Rashid Haji,** and especially thanks for my family to support me and make me what am I today ,and thanks all my friends

Abstract

In this report we have of set which contain three and four elements X={a,b,c)
And x={a,b,c,d}

Which the 3 element has 9 comparable topology elements and four element has 33 comparable topology and we try to obtain the S_p *open set in topological space*

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Introduction

Semi-open sets, preopen sets, α-open sets, b-open sets and β-open sets play an impor- tant for the study and investigation in topological spaces. In 1963, Levine [6] introduced the concept of semi-open sets in topological spaces. After the work of Levine on semi- open sets, several mathematicians turned their attention to the generalizations of various concepts of topology by considering semi-open sets instead of open sets. While open sets are replaced by semi-open sets, new results are obtained in some occasions and in other occasions substantial generalizations are exhibited. In this direction, in 1975, Mahesh- wari and Prasad [7], used semi-open sets to define and investigate three new separation axioms called semi-T0, semi-T1 and semi-T2. Later, in 1987, Bhattacharya and Lahiri [2] generalized the concept of closed sets to semi-generalized closed sets with the help of semi-openness. The notion of α-open sets (originally called α-sets) in topological spaces was introduced by Njåstad [10] in 1965. By using α-open sets, Mashhour et al. [9] defined and studied the notions of α-continuity and α-openness in topological spaces. In 1982, Mashhour et al. [8] introduced and investigated the concepts of preopen sets and precon- tinuous functions in topological spaces. In 1983, Abd El-Monsef et al. [4] introduced a weak form of open sets called β-open sets. The concept of β-open sets is equivalent to that

Definition1.1:Topology((KURONYA January 24, 2010)

A topological space is ordered pair (X, π),where X is a set, π a collection of subsets of X satisfying the following properties

∅1) X ∈ π,

2)if U, V ∈ π implies $U \cap$V,

3)$\{U_\alpha | \propto \in I\}$ implies $U_{\propto \in I}\, U_\propto \in \pi$.

The collection π is called topology on X, the pair (X, π) a topological space, the elements of π are called open sets.

Definition1.4: Semi-open (A.A.Nasef Volume 2,2009)

A subset A of a (T.S) is said to be S-open set if $A \subseteq clintA$

Definition1.5: Pre-open (A.A.Nasef Volume 2,2009)

A subset A of a (T.S) is said to be Pre-open set if $A \subseteq intclA$

Definition1.6:interior (أ. م. د. يوسف يعكوب يوسف و 2019-2018)

Int A is the largest open set containing

Definition1.7:closure (أ. م. د. يوسف يعكوب يوسف و 2019-2018)

ClA is the smallest closed set contained in A

Definition.1.8: [1] A subset A of a topological space X is said to be S– open iff A⊆cl int A.

Definition 1.9: [2] A subset A of a topological space X is said to be p– open iff A⊆ int cl A.

Definition 1.10: gsp-closed (A.A.Nasef Volume 2,2009)

A subset A of pre – open set is said to be $\boldsymbol{gsp}$ – open set of X if for each x∈A there exist a closed set F such that x∈F⊆A.

Let X={a,b,c}

$\boldsymbol{\pi_1}$**={∅,X}**

$\boldsymbol{\pi_2} = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}\}$

$\boldsymbol{\pi_3}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$

$\boldsymbol{\pi_4} = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

$\boldsymbol{\pi_5}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$

$\boldsymbol{\pi_6}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$

$\boldsymbol{\pi_7}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}\}$

$\boldsymbol{\pi_8}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

$\boldsymbol{\pi_9}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

$\boldsymbol{\pi_1}$**={∅,X}**

SO(x) = {∅,X}

PO(x) = {∅,X,{a},{b},{c},{a,b},{a,c},{b,c}}

PC(x) = {∅,X,{ a, b},{a, c},{b, c}{a},{b},{c}}

SpO(x) = {∅,X}

SpC(x) = {∅,X}

gSpC(x)= gSpO = p(X)

$\boldsymbol{\pi_2 = \{\emptyset, X, \{a\}\}}$

SO(x) = {∅, X, {a},{a,c}}

PO(x) = {∅, X,{a},{a,c}}

PC(x) = {∅, X,{b,c},{b},{c}}

SpO(x) = {∅,X}

SpC(x) = {∅,X}

gSpC(x) = gSpO= p(X)

$\boldsymbol{\pi_3}$**={∅,** $\boldsymbol{X, \{a, b\}\}}$

PO(x) = {∅, X, {a},{b},{a,b},{a,c},{b,c}}

PC(x) = {∅, X, {b,c},{a,c},{c},{b},{a}}

SpO(x) = {∅, X,{a,b}}

SpC(x) = {∅, X,{c}}

gSpC(x) = {∅, X,{c},{a,c},{b,c}}

gSpO(x) = {∅, X,{a,b},{b},{a}}

$\boldsymbol{\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}}$

SO(x) = {∅, X,{a},{b,c}}

PO(x) = {∅, X ,{a},{b},{c},{a,b},{a,c},{b,c}}

PC(x) = {∅, X, {b,c},{a,c},{a,b},{c},{b},{a}}

SpO(x) = {∅, X,{a},{b,c}}

SpC(x) = {∅, X,{b,c},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c},{b,c}} = gSpO=p(x)

$\boldsymbol{\pi_5}$**=**$\boldsymbol{\{\emptyset, X, \{a\}, \{a, b\}\}}$

SO(x) = {∅, X,{a},{a,b},{a,c}}

PO(x) = {∅, X,{a},{a,b},{a,c}}

PC(x) = {∅, X, {b,c},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c},{b,c}} = gSpO = p(x)

$\boldsymbol{\pi_6}$**=**$\boldsymbol{\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}}$

SO(x) = {∅, X, {a},{b},{a,b},{a,c},{b,c}}

PO(x) = {∅, X, {a},{b},{a,c}}

PC(x) = {∅, X, {b,c},{a,c},{b}}

SpO(x) = {∅, X,{b},{a,c},{b,c}}

SpC(x) = {∅, X,{a,c},{b},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c}}

= gSp**O =**{∅, X,{b,c},{ a,c },{a,b}{c},{d}}

$\boldsymbol{\pi_7}$=$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}\}$

SO(x) = {∅, X, {a},{ a,b},{a,c}}

PO(x) = {∅, X, {a},{a,b},{a,c}}

PC(x) = {∅, X, {b,c},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c},{b,c}}= gSpO = p(x)

$\boldsymbol{\pi_8}$=$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = {∅, X, {a},{b},{ a,b},{b,c}}

PO(x) = {∅, X, {a},{b},{ a,b},{b,c}}

PC(x) = {∅, X,{ b,c}{a,c},{c},{a}}

SpO(x) = {∅, X,{a},{b,c}}

SpC(x) = {∅, X,{b,c},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c},{b,c}} =gSpO = p(x)

$\boldsymbol{\pi_9}$=$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = p(x)

PO(x) = p(x)

PC(x) = p(x)

SpO(x) = p(x)

SpC(x) = p(x)

gSpC(x) = {∅, X,{a},{b},{c},{a,b}{a,c},{b,c}} = gSpO =p(x)

Let X={a,b,c,d}

π_1={∅,X}

$\pi_2 = \{\emptyset, X, \{a\}\}$

π_3={$\emptyset, X, \{a, b\}$}

π_4={$\emptyset, X, \{a, b, c\}$}

$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}$

π_6={$\emptyset, X, \{a, b\}, \{c, d\}$}

π_7={$\emptyset, X, \{a\}, \{a, b\}$}

π_8={$\emptyset, X, \{a\}, \{a, b, c\}$}

π_9={$\emptyset, X, \{a, b\}, \{a, b, c\}$}

π_{10}={$\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}$}

π_{11}={$\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}$

π_{12}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}$}

π_{13}={$\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}$}

π_{14}={$\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}$}

π_{15}={$\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}$}

π_{16}={$\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$}

π_{17}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}$}

π_{18}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}$}

π_{19}={$\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$}

π_{20}={$\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}$}

π_{21}={$\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}$}

π_{22}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}$}

π_{23}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$}

π_{24}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$}

π_{25}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}$}

π_{26}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}$}

π_{27}={$\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$}

π_{28}={$\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}$}

π_{29}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}$}

π_{30}={$\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}$}

π_{31}={$\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$}

π_{32}={$\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}$,

$\{a, b, d\}, \{a, c, d\}$}

π_{33}={$\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}$,

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$}

π_1={∅,X}

SO(x) = {∅, X}

PO(x) = p(x) PC(x) = p(x)

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO= p(x)

$\pi_2 = \{\emptyset, X, \{a\}\}$

SO(x) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(x) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{c,d},{b,d},{b,c},{d},{c}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSp(x) = gSpO= p(x)

π_3={$\emptyset, X, \{a, b\}$}

SO(x) = {∅, X, {a,b},{a,b,c},{a,b,d}}

PO(x) = P(x) / {{c},{d},{c,d}}

PC(x) = P(x) / {a,b},{a,b,c},{a,b,d}}

SpO(x) = {∅, X,{a,b},{a,b,c},{a,b,d}}

SpC(x) = {∅, X,{c},{d},{c,d}}

gSpC(x) = {∅, X,{c},{d},{c,d},{a,b,d},{b,c,d}}

gSpO = {∅, X,{ a,b,d },{ a,b,c },{ a,b },{ c },{a }}

$\boldsymbol{\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}}$

SO(x) = {∅, X, {a,b,c}}

PO(x) = P(x) / {{d}}

PC(x) = P(x) / {{a,b,c}}

SpO(x) = {∅, X,{a,b,c}}

SpC(x) = {∅, X,{d}}

gSpC(x) = {∅, X,{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}

gSpO (x) = {∅, X,{ a,c,b },{b,c},{ a,c },{ a,b },{c},{b}.{a }}

$\boldsymbol{\pi_5}$**=**$\boldsymbol{\{\emptyset, X, \{a\}, \{a, b\}\}}$

SO(x) = {∅, X, {a},{b,c,d}}

PO(x) = P(x)

PC(x) = P(x)

SpO(x) = {∅, X,{b,c,d}{a}}

SpC(x) = {∅, X,{a } ,{b,c,d}}

gSpC(x) = gSpO(x) = P(x)

$\boldsymbol{\pi_6}$**=**$\boldsymbol{\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}}$

SO(x) = {∅, X, {a.b},{c,d}}

PO(x) = P(x)

PC(x) = P(x)

SpO(x) = {∅, X,{c,d}{a,b}}

SpC(x) = {∅, X,{a , b} ,{c,d}}

gSpC(x) = gSpO(x) = P(x)

$\boldsymbol{\pi_7}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}\}$

SO(x) = {∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(x) = {∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = { ∅, X, {b,c,d},{c,d},{b,d},{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO(x) = P(x)

$\boldsymbol{\pi_8}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = {∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(x) = {∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = { ∅, X, {b,c,d},{c,d},{b,d},{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO(x) = P(x)

$\boldsymbol{\pi_9}$**=**$\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = {∅, X,{a,b},{a,b,c},{a,b,d}}

PO(X) = p(x) / {c},{d},{c,d}

PC(x) = P(x) / {a,b,d},{a,b,c},{a,b}

SpO(x) = {∅, X,{a,b},{a,b,c},{a,b,d}}

SpC(x) = {∅, X,{c},{d},{c,d}}

gSpC(x) = {∅, X,{c},{d},{c,d},{a,c,d},{b,c,d}}

gSpO(x) = {∅, X,{ a,b,d },{ a,b,c },{ a,b },{b},{a}}

$\boldsymbol{\pi_{10}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = {∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(X) ={∅, X, {a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = { ∅, X, {b,c,d},{c,d},{b,d},{b,c},{d},{c},{b}}

Spo(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO(x)=p(x)

$\boldsymbol{\pi_{11}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$

SO(x) = {∅, X, {a,b},{a,b,c},{a,b,d}}

PO(X) = p(x) / {c},{d},{c,d}

PC(x) = P(x) / {a,b,d},{a,b,c},{a,b}

SpO(x) = {∅, X,{a,b},{a,b,c},{a,b,d}}

SpC(x) = {∅, X,{c,d},{c},{d}}

gSpC(x) = {∅, X,{c,d},{c},{d},{a,c,d},{b,c,d}}

gSpO(x) **=** {∅, X,{ a,b },{ a,b,d },{ a,b,c },{b},{a}}

$\boldsymbol{\pi_{12}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$

SO(x) = p(x) /{c},{d},{c,d}

PO(X) = {∅, X,{a},{b},{a,b}{a,b,c}{a,b,d}}

PC(x) = {∅, X,{a,c,d},{b,c,d},{d},{c},{c,d}}

SpO(x) = {∅, X,{a,c,d},{b,c,d}}

SpC(x) = {∅, X,{b},{a}}

gSpC(x) = {∅, X,{a},{b},{a,b}{a,b,c}{a,b,d}}

gSpO(x) **=** {∅, X,{b,c,d}**,**{ a,c,d }{c,d}{d},{c}}

$\boldsymbol{\pi_{13}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}, \mathbf{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) = {∅, X,{a},{b,c},{a,d},{a,b,c},{b,c,d}

PO(X) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d}{a,d},{d},{c},{b}}

SpO(x) = {∅, X,{a,d},{b,c,d}{b,c}}

SpC(x) = {∅, X,{a},{b,c},{a,d}}

gSpC(x) =
{∅, X,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{a,b,c}{a,b,d},{a,c,d}}

gSpO(x) =
{∅, X,{b,c,d},{a,c,d},{a,b,d},{a,b,c},{c,d},{b,d},{b,c},{a,d},{d},{c},{b}}

$\boldsymbol{\pi_{14}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) ={∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PO(X) ={∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{b,d}{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{15}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$

SO(x) ={∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PO(X) ={∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{c,d},{b,d}{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpc(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{16}}$**={**$\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) ={∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PO(X) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c}{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{c,d},{b,d}{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{17}}$**={**$\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$

SO(x) =p(x) / {d},{c}{c,d}}

PO(X) = {∅, X,{a},{b},{a,b},{a,b,c}{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{d},{c}}

SpO(x) = {∅, X,{b,c,d},{a,c,d}}

SpC(x) = {∅, X,{b},{a}}

gSpC(x) = {∅, X,{a},{b},{a,b},{a,b,c}{a,b,d}}

gSpO(x) ={∅, X, {b, c, d}, {a, c, d}, {c, d}, {d}, {c}}

$\boldsymbol{\pi_{18}}$**={**$\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(X) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{b,d},{b,c},{d},{c},{b}}

SpO(x) = {∅, X,{b},{a,c,d}}

SpC(x) = {∅, X,{a,c,d},{b}}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{19}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{c}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = {∅, X,{c},{a,b},{a,b,c},{a,b,d}}

PO(X) = P(X) / {d}, {c,d}

PC(x) = p(x) / {a,b,c},{a,b}

SpO(x) = {∅, X,{c},{a,b,d},{a,b,c},{a,b}}

SpC(x) = {∅, X,{a,b,d},{d},{c},{c,d}}

gSpC(x) =
{∅, X,{a},{b},{c},{d},{a,b},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}}

gSpO(x)
=$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{b, c\}, \{a, c\}, \{a, b\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\pi_{20}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{c}, \boldsymbol{d}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{a,b},{c,d},{a,c,d}}

PO(X) = {∅, X,{a},{c},{d},{a,b},{a,c},{a,d},{c,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{a,b,d},{a,b,c},{b,d},{c,d},{b,c},{a,b},{d},{c},{b}}

SpO(x) = {∅, X,{c,d},{a,b}}

SpC(x) = {∅, X,{a,b},{c,d}}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{21}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(X) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{c,d},{b,d},{b,c},{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) =
{∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

gSpO(x)
=$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$\boldsymbol{\pi_{22}}$=$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{b,c},{a,d},{b,d},{a,b,c},{a,b,d},{b,c,d}}

PO(X) = {∅, X,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{a,d},{d},{c}}

SpO(x) = {∅, X,{b,c,d},{a,d}}

SpC(x) = {∅, X,{b,c},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{b,d},{a,b,c},{a,b,d},
{a,c,d},{b,c,d}}

gSpO(x)
=$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$\boldsymbol{\pi_{23}}$=$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = p(x) / {c},{d},{c,d}

PO(X) = {∅, X,{a},{b},{a,b},{a,b,c},{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{a,d},{d},{c}}

SpO(x) = {∅, X,{b,c,d},{a,c,d}}

SpC(x) = {∅, X,{b},{a}}

gSpC(x) = {∅, X,{a},{b},{a,b},{a,b,c},{a,b,d}}

gSpO(x) ==$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\boldsymbol{\pi_{24}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{c}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}, \{\mathbf{b}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{c,d},{a,c,d},{b,c,d}}

PO(X)=PC(x) = p(x)

SpO(x) = {∅, X,{a},{b},{a,b},{c,d},{a,c,d},{b,c,d}}

SpC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{a,b},{b},{a}}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{25}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{b}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{a,c},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d}}

PO(x) = {∅, X,{a},{b},{a,b},{b,d},{a,b,c},{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{a,b},{d},{c}}

SpO(x) = {∅, X,{b,c,d},{a,c}}

SpC(x) = {∅, X,{b,d},{a}}

gSpC(x) = {∅, X,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,c},{a,b,d},{a,c,d}}

gSpO(x) =$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, c\}, \{c, d\}, \{b, c\}, \{a, c\}, \{d\}, \{c\}, \{b\}\}$

$\boldsymbol{\pi_{26}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PO(x) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{b,d},{b,c},{d},{c},{b}}

SpO(x) = {∅, X,{a,c,d},{b}}

SpC(x) = {∅, X,{b},{a,c,d}}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{27}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{b,c,d}}

PO(x) = {∅, X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{b,c,d}}

PC(x) = {∅, X,{b,c,d},{b,d},{b,c},{d},{c},{b}}

SpO(x) = {∅, X}

SpC(x) = {∅, X}

gSpC(x) =gSpO(x) = p(x)

$\boldsymbol{\pi_{28}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}\}$

SO(x) = p(x) / {d}}

PO(X) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{a,d},{d}}

SpO(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{a,d}}

SpC(x) = {∅, X,{a,c},{a,b},{b,c},{c},{b},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

gSpO(x) =$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$

$\boldsymbol{\pi_{29}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{b}\}, \{\mathbf{a}, \mathbf{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{b}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{a,c},{b,d},{a,b,c},{a,b,d}}

PO(x) = {∅, X,{a},{b},{a,b},{a,c},{b,d},{a,b,c},{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{b,d},{a,c},{d},{c}}

SpO(x) = {∅, X,{b,d},{a,c}}

SpC(x) = {∅, X,{a,c},{b,d}}

gSpC(x) = gSpO(x) = p(x)

$\boldsymbol{\pi_{30}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}\}$

SO(x) = p(x) / {d},{c,d}

P(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,b,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{a,d},{d},{c}}

SpO(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{b,d},{a,d},{c}}

SpC(x) = {∅, X,{a,b,d},{b,c},{a,c},{c},{b},{a}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,b,d}}

gSpO(x) =$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, \{c\}\}$

$\boldsymbol{\pi_{31}}$**=**$\{\emptyset, \boldsymbol{X}, \{\mathbf{a}\}, \{\boldsymbol{b}\}, \{\mathbf{a}, \boldsymbol{b}\}, \{\mathbf{a}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}, \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}, \{\mathbf{a}, \mathbf{c}, \mathbf{d}\}\}$

SO(x) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{b,c,d}}

PO(x) = {∅, X,{a},{b},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{b,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{c,d},{b,d},{b,c},{d}{c},{b}}

Sp(x) = {∅, X,{b},{a,c,d}}

SpC(x) = {∅, X,{a,c,d},{b}}

gSpC(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,b,d}}

gSpO(x) =$\{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, \{c\}\}$

π_{32}={

$\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{d}\}, \{\boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}\}$

SO(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{b,c,d}}

PO(x) = {∅, X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{b,c,d}}

PC(x) = {∅, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{b,c},{a,d},{d},{c},{b}}

Sp(x) = {∅, X,{b},{c},{a,d},{b,c},{a,b,d},{a,c,d}}

SpC(x) = {∅, X, {a,c,d},{a,b,d}{b,c},{a,d},{c},{b}}

gSpC(x) = gSpO(x) = p(x)

π33 = SO(x) = PO(x)= PC(x) = sp =spc=gsp=p(x)

Chapter TWO

Definition2.1: T_0-Space (أ. م. د. يوسف يعكوب يوسف و 2019-2018)

Let (X,π) be a topological space. Then the space (X,π) is called gSp T_0-Space iff for each pair of distinct points x,y∈ X there is either an gSp- open set containing x but not y or an open set containing y but not x. I.e.,

X is gSp T_0-Space ↔ $\forall x, y \in X ; x \neq y \; \exists U \in \mathrm{gSp}O(X) ; (x \in U \wedge y \notin U \vee x \notin U \wedge y \in U.$

Definition2.2: T_1-Space (أ. م. د. يوسف يعكوب يوسف و 2019-2018)

Let (X,π) be a topological space. Then the space (X,π) is called gSpT_1-Space iff for each pair of distinct points x,y∈ X there is either an gSp- open set containing x but not y And an open set containing y but not x. I.e.,

X is gSp T_1-Space ↔ $\forall x, y \in X ; x \neq y \; \exists U \in \mathrm{gSp}O(X) ; (x \in U \wedge y \notin U \vee x \notin U \wedge y \in U.$

Definition2.3: T_2-Space (أ. م. د. يوسف يعكوب يوسف و 2019-2018)

Let (X,π) be a topological space. Then the space (X,π) is called gSp T_2-Space or **Hausdorff space** iff for each pair of distinct points x,y∈ X there exist gSp -open sets U And V s.t. $x \in U, y \in V \; And \; U \cap V = \emptyset$ I,e.,

X is gSp T_2-Space ↔ $\forall x, y \in X ; x \neq y \; \exists U \in \mathrm{gSp}O(X) ; (x \in U \wedge y \notin U) \vee (x \notin U \wedge y \in U.$

X={a,b,c}	gSpT_0-Space	gSpT_1-Space	gSpT_2-Space
π_1	1	1	1
π_2	1	1	1
π_3	0	0	0
π_4	1	1	1
π_5	1	1	1
π_6	1	1	1
π_7	1	1	1
π_8	1	1	1
π_9	1	1	1

Let X={a,b,c,d}

Let X={a,b,c,d}	gSpT_0-Space	gSpT_1-Space	gSpT_2-Space
π_1	1	1	1
π_2	1	1	1
π_3	0	0	0
π_4	0	0	0
π_5	1	1	1
π_6	1	1	1
π_7	1	1	1
π_8	1	1	1
π_9	0	0	0
π_{10}	1	1	1
π_{11}	0	0	0
π_{12}	0	0	0
π_{13}	1	1	1
π_{14}	1	1	1
π_{15}	1	1	1
π_{16}	1	1	1
π_{17}	0	0	0
π_{18}	1	1	1
π_{19}	1	1	1
π_{20}	1	1	1
π_{21}	1	1	1
π_{22}	1	1	1
π_{23}	0	0	0
π_{24}	1	1	1
π_{25}	0	0	0
π_{26}	1	1	1
π_{27}	1	1	1
π_{28}	0	0	0
π_{29}	1	1	1
π_{30}	0	0	0
π_{31}	1	1	1
π_{32}	1	1	1
π_{33}	1	1	1

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