



زانكۆی سه‌لاحه‌دین - هه‌ولێر
Salahaddin University-Erbil

Ps-open set in topological space

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc. in mathematics

By:

Sumaia Abdullah Haji

Supervised by:

Nehmet Ahmed

April– 2023

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics/ College of Education /Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics

Signature:

Supervisor : **Dr.Nehmet Ahmed**

Scientific grade: Assist. Professor

Date: 5 /4 /2023

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature : 

Name: **Dr. Rashad Rasheed Haje .**

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: 5/4 /2023

ACKNOWLEDGMENT

I would like to thank my supervisor, Dr. Nehmet Ahmed , for bringing the weight of his considerable experience and knowledge to this project. His high standards have made me better at what I do.

I sincerely thank Dr. Rashad Rashid, the head of the Department of Mathematics, College of Education, for all the facilities provided to us in the pursuit of this project.

I acknowledge our deep sense of gratitude to my loving parents for being a constant source of inspiration and motivation. I also thank my friends who have played significant roles throughout my life.

Abstract

In this report we have of set which contain three and four elements $X = \{a, b, c\}$ and $X = \{a, b, c, d\}$

Which the 3 element has 9 comparable topology elements and four element has 33 comparable topology and we try to obtain the Ps open set in topological space

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INTRODUCTIONS

Throughout this paper, a space X means a topological space with out any separation axioms. We recall the following

definitions, notions and characterizations. The closure (resp. interior) of a subset A of X is denoted by ClA (resp. $IntA$). A subset A of X is said to be preopen [4] (resp. semi-open [5], regular open [6] and regular semi-open [7]) if $A \subset IntClA$ (resp. $A \subset ClIntA$, $A \subset IntClIntA$, $A \subset ClIntClA$, $A = IntClA$ and $A = sIntsClA$). The complement of a preopen (resp. semi-open, regular open and regular semi-open) set is said to be preclosed (resp. semi-closed). The intersection of all preclosed (resp. semi-closed) set of X containing a subset A is called the preclosure (resp. semi-closure) of A and is denoted by $pClA$ (resp. $sClA$). The union of all preopen (resp. semi-open) sets of X contained in A is called the preinterior (resp. semi-interior) of A and is denoted by $pIntA$ (resp. $sIntA$). The family of all preopen (resp. semi-open, regular semi-open, preclosed, semi-closed and regular closed) subsets of a topological space X is denoted by $PO(X)$ (resp. $SO(X)$, $RSO(X)$, $PC(X)$, $SC(X)$).

CHAPTER ONE

Definition 1.1:[3] A subset A of a topological space X is said to be Semi-open iff $A \subseteq \text{cl int } A$.

Definition 1.2: [A] A subset A of a topological space X is said to be Pre-open iff $A \subseteq \text{int cl } A$.

Definition 1.3:[1] A subset A of pre-open set is said to be p_s -open set of X if for each $x \in A$ there exist a semi-closed set F such that $x \in F \subseteq A$.

Definition 1.4:[2] A subset A of X is said to be $g p_s$ -closed set iff $p_s \text{ cl } A \subseteq u$, when ever $A \subseteq u$, u is p_s open set.

Example: Let $X = \{a, b, c\}$

$$\pi_1 = \{\emptyset, X\}$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$\pi_5 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\pi_6 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\pi_9 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\pi_1 = \{\emptyset, X\}$$

$$PO(x) = P(x)$$

$$SO(x) = \{\emptyset, X\}$$

$$SC(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$$

$$SC(x) = \{\emptyset, X, \{b,c\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \}$$

$$PsC(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_3 = \{\emptyset, X, \{a,b\}\}$$

$$SO(x) = \{\emptyset, X, \{a,b\}\}$$

$$SO(x) = \{\emptyset, X, \{c\}\}$$

$$PO(x) = P(x)$$

$$PsC(x) = \{\emptyset, X\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_4 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$PsC(x) = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_5 = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$SC(x) = \{\emptyset, X, \{c, b\}, \{a\}\}$$

$$PO(x) = P(x)$$

$$PsC(x) = \{\emptyset, X\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_6 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$SC(x) = \{\emptyset, X, \{a\}, \{b\}, \{b, c\}, \{a, c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}, Ps \text{ g-open} = \{\emptyset, X, \{b\}, \{a\}, \{a, c\}\}$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{a\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$PsO(x) = \{\emptyset, X, \{b\}, \{a, c\}\}$$

$$PsC(x) = \{\emptyset, X, \{a, c\}, \{b\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_9 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$SO(x) = P(x)$$

$$SC(x) = P(x)$$

$$PO(x) = P(x)$$

$$PsO(x) = P(x)$$

$$PsC(x) = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

let X={a,b,c,d}

let X={a,b,c,d}

$\pi_1=\{\emptyset, X\}$ $\pi_2=\{\emptyset, X, \{a\}\}$

$\pi_3=\{\emptyset, X, \{a, b\}\}$

$\pi_4=\{\emptyset, X, \{a, b, c\}\}$

$\pi_5=\{\emptyset, X, \{a\}, \{b, c, d\}\}$

$\pi_6=\{\emptyset, X, \{a, b\}, \{c, d\}\}$

$\pi_7=\{\emptyset, X, \{a\}, \{a, b\}\}$

$\pi_8=\{\emptyset, X, \{a\}, \{a, b, c\}\}$

$\pi_9=\{\emptyset, X, \{a, b\}, \{a, b, c\}\}$

$\pi_{10}=\{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$

$\pi_{11}=\{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\pi_{12}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$

$\pi_{13}=\{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\pi_{14}=\{\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}\}$

$\pi_{15}=\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$

$\pi_{16}=\{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\pi_{17}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$\pi_{18}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

$\pi_{19}=\{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\pi_{20}=\{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$\pi_{21}=\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\pi_{22}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$\pi_{23}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\pi_{24}=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

$$\pi_{25}=\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\pi_{26}=\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$$

$$\pi_{27}=\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\pi_{28}=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$$

$$\pi_{29}=\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\pi_{30}=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\pi_{31}=\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\pi_{32}=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\pi_{33}=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\},$$

$$\pi_1=\{\emptyset, X\}$$

$$SO(x)=\{\emptyset, X\}$$

$$SC(x)=\{\emptyset, X\}$$

$$PO(x)=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$$

$$PsC(x)=\{\emptyset, X\}$$

$$PsO(x)=\{\emptyset, X\}$$

$$Ps \text{ g-open} =P(x)$$

$$Ps \text{ g-closed} =P(x)$$

$$\pi_2=\{\emptyset, X, \{a\}\}$$

$$SO(x)=\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$$

$$SC(x)=\{\emptyset, X, \{b,c\}, \{c\}, \{b\}\}$$

$$PO(x)=\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$$

$$PsO(x)=\{\emptyset, X\}$$

$$PsC(x)=\{\emptyset, X\}$$

$$Ps \text{ g-open} =P(x), Ps \text{ g-closed} =P(x)$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$SO(x) = \{\emptyset, X, \{a, b\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_4 = \{\emptyset, X, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a, b, c\}\}$$

$$SC(x) = \{\emptyset, X, \{d\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$PO(x) = P(x)$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a\}\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_6 = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$PO(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{c, d\}, \{a, b\}\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{b\}, \{c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_9 = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{c, d\}, \{d\}, \{c\}\}$$

$$PO(x) = P(x) \setminus \{\{c\}, \{d\}, \{c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_{10} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi_{11} = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{c, d\}, \{d\}, \{c\}\}$$

$$PO(x) = P(x) \setminus \{\{c\}, \{d\}, \{c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_{12} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$SO(x) = P(x) \setminus \{\{c\}, \{d\}, \{c, d\}\}$$

$$SC(x) = P(x) \setminus \{\{a, b, d\}, \{a, b, c\}, \{a, b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{13} = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{b, c, d\}, \{b, c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{d\}\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi_{14} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{b, c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi 15 = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,c,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}, \{b\}, \{d\}, \{b,d\}, \{c,d\}, \{b,c,d\}, \{b,c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,c,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi 16 = \{\emptyset, X, \{a\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,c,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}, \{b\}, \{d\}, \{b,d\}, \{c,d\}, \{b,c,d\}, \{b,c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,c,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-open} = P(x)$$

$$Ps \text{ g-closed} = P(x)$$

$$\pi 17 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$$

$$SO(x) = P(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$$

$$SC(x) = P(x) \setminus \{\{a,b,d\}, \{a,b,c\} \setminus \{a,b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{d\}, \{c\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{c\}, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\pi 18 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}, \{b\}, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{b, c, d\}, \{b, c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{b\}, \{a, c, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b\}, \{a, c, d\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi 19 = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(x) = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, b, d\}\}$$

$$PO(x) = P(x) \setminus \{\{d\}, \{c, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{c\}, \{a, b, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi 20 = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{b, c\}\}$$

$$PsO(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$Ps \text{ g-closed} = P(x), Ps \text{ g-open} = P(x)$$

$$\pi 21 = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, d\}\}$$

$$PsO(x) = \{\emptyset, X\}, PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x), Ps \text{ g-open} = P(x)$$

$$\pi 22 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{a\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a, d\}, \{d\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a, b, c\}, \{b, c\}, \{a, c\}, \{a, b\}, \{a\}, \{b\}, \{c\}\}$$

$$\pi 23 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(x) = P(x) \setminus \{\{d\}, \{c\}, \{c, d\}\}$$

$$SC(x) = P(x) \setminus \{\{a, b, c\}, \{a, b, d\}, \{a, b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{d\}, \{c\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a, b, c\}, \{a, b, d\}, \{a, b\}, \{a\}, \{b\}\}$$

$$\pi_{24} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$Ps \text{ g-closed} = P(x), Ps \text{ g-open} = P(x)$$

$$\pi_{25} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b, d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{d\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a, b, c\}, \{a, b, d\}, \{b, d\}, \{a, d\}, \{a, b\}, \{b, c\}\}$$

$$\pi_{26} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SC(x) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$$

$$PO(x) = \boxed{\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}}$$

$$PsO(x) = \{\emptyset, X, \{b\}, \{a, c, d\}\}$$

$$Ps \text{ g-closed} = P(x), Ps \text{ g-open} = P(x)$$

$$\pi_{27} = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$SC(x) = \{\emptyset, X, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$PsO(x) = \{\emptyset, X\}$$

$$PsC(x) = \{\emptyset, X\}$$

$$Ps \text{ g-closed} = P(x), Ps \text{ g-open} = P(x)$$

$$\pi_{28} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$$

$$SO(x) = P(x) \setminus \{\{d\}\}$$

$$SC(x) = P(x) \setminus \{\{a,b,c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

$$PsC(x) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{d\}, \{a,d\}, \{c,d\}, \{b,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a,b,c\}, \{b,c\}, \{a,b\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$$

$$\pi_{29} = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SC(x) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{b,d\}, \{a,c\}, \{d\}, \{c\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsO(x) = \{\emptyset, X, \{a,c\}, \{b,d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b,d\}, \{a,c\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi 30 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

$$SO(x) = P(x) \setminus \{\{d\}, \{c,d\}\}$$

$$SC(x) = P(x) \setminus \{\{a,b,c\}, \{a,b\}\}$$

$$PsO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

$$PsC(x) = \{\emptyset, X, \{b,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}, \{c\}\}$$

$$Ps \text{ g-closed} = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{a,d\}, \{b,d\}, \{d\}, \{c\}\}$$

$$Ps \text{ g-open} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\pi 31 = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$SC(x) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$$

$$PO(x) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$PsO(x) = \{\emptyset, X, \{b\}, \{a,c,d\}\}$$

$$PsC(x) = \{\emptyset, X, \{a,c,d\}, \{b\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi 32 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$SO(x) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\} = PO(x)$$

$$SC(x) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{d\}, \{c\}, \{b\}\}$$

$$Ps \text{ g-closed} = P(x)$$

$$Ps \text{ g-open} = P(x)$$

$$\pi 33 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$SO(x) = SC(x) = PO(x) = PsO(x) = PsC(x) = P(x)$$

$$Ps \text{ g-closed} = Ps \text{ g-open} = P(x)$$

CHAPTER TWO

Definition 2.1:[8](To Space)

A topological space X is said to be a To space if For any two points x, y in X , there is an open set U such that x in U and y not in U or y in U and x not in U .

Definition 2.2[8](T1 Space)

A topological space X is said to be a T1 space if For any two points x, y in X there exists two open sets U and V such that x in U and y not in U , and y in V and x not in V .

Definition 2.3:[8](T2 Space or Hausdorff Space)

A Hausdorff space is a topological space in which each pair of distinct points can be separated by a disjoint open set. In other words, a topological space x is said to be a T2 space or Hausdorff space if For any two points x, y in X there exists two open sets U and V such that x in U , y in V , and $U \cap V = \emptyset$.

$X=\{a,b,c\}$	g Ps-To - space	g Ps-T1 - space	g Ps-T2 - space
t 1	1	1	1
t 2	1	1	1
t 3	1	1	1
t 4	1	1	1
t 5	1	1	1
t 6	1	0	0
t 7	1	1	1
t 8	1	1	1
t 9	1	1	1

X={a,b,c,d}	g Ps To -space	g Ps T1 -space	T g Ps 2 -space	X={a,b,c,d}	g Ps To -space	g Ps T1 -space	g PsT 2 -space
t 1	1	1	1	t 10	1	1	1
t 2	1	1	1	t 11	1	1	1
t 3	1	1	1	t 12	1	0	0
t 4	1	1	1	t 13	1	1	1
t 5	1	1	1	t 14	1	1	1
t 6	1	1	1	t 15	1	1	1
t 7	1	1	1	t 16	1	1	1
t 8	1	1	1	t 17	1	1	1
t 9	1	1	1	t 18	1	1	1

X={a,b,c,d}	g Ps To -space	g Ps T1 -space	g Ps T2 -space	X={a,b,c,d}	g Ps To -space	g Ps T1 -space	g Ps T2 -space
t 19	1	0	0	t 26	1	1	1
t 20	1	1	1	t 27	1	1	1
t 21	1	1	1	t 28	1	0	0
t 22	1	0	0	t 29	1	1	1
t 23	1	0	0	t 30	1	0	0
t 24	1	1	1	t 31	1	1	1
t 25	1	0	0	t 32	1	1	1
				t 33	1	1	1

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پوخته:

لهم راپورتهدا كؤمه ليكمان ههيه كه ۳ توخم و ۴ توخم لهخؤ دهگريٽ كه توخمي ۳, ۹ توپولوجي بهراوردكراوى ههيه وه توخمي ۴, ۳۳ توپولوجي ههيه, نيمه ههول دهدهين كؤمه لهي كراوهي

Ps-open set

به دست بهينين.

خلاصة

حيث يحتوي العنصر 3 على 9 عناصر طوبولوجيا قابلة للمقارنة وأربعة عناصر بها 33 طوبولوجيا

قابلة للمقارنة ونحاول الحصول على Ps -open set in topological space