زانكۆى سەلاحەدين-هەولێر

Salahaddin University-Erbil

S_s *open set in topological spaces*

Submitted to the department of (mathematic) in partial fulfilment of the Requirements for the degree of BSc. In mathematic

By:

Muhammed Taha Hamad

Supervised by:

Nehmat K. Ahmad

April-2023

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: **Assist.Prof.Dr.Nehmat K. Ahmad**

Scientific grade: Assist. Professor

Date: 10 / 4 / 2022

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: **Dr. Rashad Rasheed Haje**

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: 10 / 4 / 2022

Aknowledgment

I would like to thanks Allah for giving me the power to complete this work And I would like to present my profound thanks to supervisor and lecturer **Assist.Prof.Dr.Nehmat K. Ahmad** for his kind valuable suggestions that assisted me to accomplish this work I would also like to extend my gratitude to the head of mathematic department **Assist.Prof.Dr.Rashad Rashid Haji,** and especially thanks for my family to support me and make me what am I today ,and thanks all my friends

Abstract

In this report we have of set which contain three and four elements X={a,b,c) And x={a,b,c,d}

Which the 3 element has 9 comparable topology elements and four element has 33 comparable topology and we try to obtain the S_s *open set in topological space*

Table of Contents

Introduction

General Topology or Point Set Topology. General topology normally considers local properties of spaces, and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered. Sometimes distances can be defined in these spaces, in which case they are called metric spaces; sometimes no concept of distance makes sense.

In 1963, Levine introduced the concept of a semi-open set. The initiation of the study of generalized closed sets was done by Aull in 1968 as he considered sets whose closure belongs to every open superset. The notion of generalized semi-closed sets was introduced by Arya and Nour . In 1987, Bhattacharyya and Lahiri defined and studied the concept of semi- generalized closed sets via the notion of a semi-closed set.

Chapter one

Definition1.1:Topology((KURONYA January 24, 2010)

A topological space is ordered pair (X,π),where X is a set, π a collection of subsets of X satisfying the following properties

∅1) X ∈ π,

2)U, V ∈ π implies U ∩V,

3)$\{U_\alpha | \propto \in I\}$ implies $U_{\propto \in I}\, U_\propto \in \pi$.

The collection π is called topology on X, the pair (X, π) a topological space, the elements of π are called open sets.

Definition1.2: Semi-open (A.A.Nasef Volume 2,2009)

A subset A of a (T.S) is said to be Semi-open set if $A \subseteq clintA$

Definition1.3:interior (أ. م. د. يوسف يعكوب يوسف و ٢٠١٨-٢٠١٩)

Int A is the largest open set contained A

Definition1.4:closure (أ. م. د. يوسف يعكوب يوسف و ٢٠١٨-٢٠١٩)

ClA is the smallest closed set containing A

Definition1.5: S_s-open (K.B. Alias 2(1) 2014)

A subset A of semi-open set is said to be $\boldsymbol{S_s}$**-open** set if $\forall X \in A, \ni a\ semi - closed\ set\ F\ s.t. X \in F \subseteq A$.

Definition1.6: gS_s-closed (A.A.Nasef Volume 2,2009)

A subset A of X, is said to be $\boldsymbol{gS_s}$-closed set if $S_s\, cl\, A \subseteq U, A \subseteq U$ whenever U∈ $S_s o(x)$, U is S_s-open

Three element topology

let X={a,b,c}

π_1={∅,X}

$\pi_2=\{\emptyset, X, \{a\}, \{a,b\}\}$

$\pi_3 = \{\emptyset, X, \{a\}, \{a,c\}\}$

$\pi_4 = \{\emptyset, X, \{a\}, \{b,c\}\}$

$\pi_5 = \{\emptyset, X, \{b\}, \{a,b\}\}$

$\pi_6 = \{\emptyset, X, \{b\}, \{a,c\}\}$

$\pi_7 = \{\emptyset, X, \{b\}, \{b,c\}\}$

$\pi_8 = \{\emptyset, X, \{c\}, \{a,b\}\}$

$\pi_9 = \{\emptyset, X, \{c\}, \{a,c\}\}$

$\pi_{10} = \{\emptyset, X, \{c\}, \{b,c\}\}$

$\pi_{11} = \{\emptyset, X, \{a\}\}$

$\pi_{12} = \{\emptyset, X, \{b\}\}$

$\pi_{13} = \{\emptyset, X, \{c\}\}$

π_{14}= {∅, X, {a}, {b}, {a, b}}

$\pi_{15}=\{\emptyset, X, \{a\}, \{c\}, \{a,c\}\}$

$\pi_{16}=\{\emptyset, X, \{b\}, \{c\}, \{b,c\}\}$

$\pi_{17}=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\pi_{18}=\{\emptyset, X, \{a,b\}\}$

π_{19}=$\{\emptyset, X, \{a, c\}\}$

π_{20}=$\{\emptyset, X, \{b, c\}\}$

π_{21}=$\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

π_{22}=$\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$

π_{23}=$\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$

π_{24}=$\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

π_{25}=$\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$

π_{26}=$\{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$

π_{27}=$\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$

π_{28}=$\{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$

π_{29}=$\{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$

The comparable sets

π_1={∅,X}

$\pi_2 = \{\emptyset, X, \{a\}\}$

π_3=$\{\emptyset, X, \{a, b\}\}$

$\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}$

π_5=$\{\emptyset, X, \{a\}, \{a, b\}\}$

π_6=$\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$

π_7=$\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

π_8=$\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$

π_9=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

let X={a,b,c}

π_1={∅,X}

$\pi^C = \{X, \emptyset\}$

$SO(X) = \{\emptyset, X\}$

$SC(X) = \{\emptyset, X\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

$\pi_2 = \{\emptyset, X, \{a\}\}$

$\pi^C = \{X, \emptyset, \{b, c\}\}$

$SO(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$

$SC(X) = \{X, \emptyset, \{b, c\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_3=$\{\emptyset, X, \{a, b\}\}$

$\pi^C = \{X, \emptyset, \{c\}\}$

$SO(X) = \{\emptyset, X, \{a, b\}\}$

$SC(X) = \{\emptyset, X, \{c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

$\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}$

$\pi^C = \{X, \emptyset, \{b, c\}, \{a\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$

$SC(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$

$S_S O(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$

$S_s C(X) = \{\emptyset, X, \{b, c\}, \{a\}\}$

$gS_S - Closed = P(x)$

π_5=$\{\emptyset, X, \{a\}, \{a,b\}\}$

$\pi^C = \{X, \emptyset, \{c\}, \{b,c\}\}$

$SO(X) = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$

$SC(X) = \{\emptyset, X, \{c\}, \{b\}, \{b,c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_6=$\{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$

$\pi^C = \{X, \emptyset, \{b,c\}, \{a,c\}, \{c\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$SC(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}\{a,c\}, \{b,c\}\}$

$S_S O(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$S_s C(X) = \{\emptyset, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$

$gS_S - Closed = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$

π_7=$\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\pi^C = \{X, \emptyset, \{b\}, \{c\}, \{b,c\}\}$

$SO(X) = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$

$SC(X) = \{\emptyset, X, \{c\}, \{b\}, \{b,c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_8=$\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$

$\pi^C = \{X, \emptyset, \{b,c\}, \{a,c\}, \{c\}\{a\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$

$SC(X) = \{\emptyset, X, \{a\}, \{c\}\{a,c\}, \{b,c\}\}$

$S_S O(X) = \{\emptyset, X, \{a\}, \{b,c\}\}$

$S_s C(X) = P(x)$

$gS_S - Closed P(x)$

π_9=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

π^C =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$SO(X)$ =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$SC(X)$ =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$S_s O(X)$ =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$S_s O(X) = S_s C(X) = gS_S - Closed$

Chapter Two

Four element topology

let X={a,b,c,d}

π_1={∅,X}

$\pi_2 = \{\emptyset, X, \{a\}\}$

π_3=$\{\emptyset, X, \{a, b\}\}$

π_4=$\{\emptyset, X, \{a, b, c\}\}$

$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}$

π_6=$\{\emptyset, X, \{a, b\}, \{\mathrm{c}, \mathrm{d}\}\}$

π_7=$\{\emptyset, X, \{\mathrm{a}\}, \{a, b\}\}$

π_8=$\{\emptyset, X, \{\mathrm{a}\}, \{a, b, c\}\}$

π_9=$\{\emptyset, X, \{\mathrm{a}, b\}, \{a, b, c\}\}$

π_{10}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{a, b, c\}\}$

π_{11}=$\{\emptyset, X, \{\mathrm{a}, b\}, \{a, b, c\}, \{\mathrm{a}, \mathrm{b}, \mathrm{d}\}$

π_{12}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{a, b\}\}$

π_{13}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}, \mathrm{c}\}, \{a, b, c\}\}$

π_{14}=$\{\emptyset, X, \{\mathrm{a}\}, \{a, b\}, \{\mathrm{a}, \mathrm{c}, \mathrm{d}\}\}$

π_{15}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{\mathrm{a}, \mathrm{c}\}, \{a, b, c\}\}$

π_{16}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{a, b, c\}, \{\mathrm{a}, \mathrm{b}, \mathrm{d}\}\}$

π_{17}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{a, b\}, \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}\}$

π_{18}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{a, b\}, \{\mathrm{a, c, d}\}\}$

π_{19}=$\{\emptyset, X, \{\mathrm{c}\}, \{\mathrm{a}, b\}, \{a, b, c\}, \{\mathrm{a, b, d}\}\}$

π_{20}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{c, d\}, \{\mathrm{a, c, d}\}\}$

π_{21}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{a, c\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

π_{22}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{\mathrm{a}, b\}, \{b, c\}, \{\mathrm{a, b, c}\}\}$

π_{23}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

π_{24}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{c, d}\}, \{\mathrm{a, c, d}\}, \{\mathrm{b, c, d}\}\}$

π_{25}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{b, d}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

π_{26}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{a, c}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, c, d}\}\}$

π_{27}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{a}, b\}, \{\mathrm{a, c}\}, \{\mathrm{a, d}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}, \{\mathrm{a, c, d}\}\}$

π_{28}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{c\}, \{\mathrm{a}, b\}, \{\mathrm{b, c}\}, \{\mathrm{a, b, c}\}\}$

π_{29}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{\mathrm{a, b}\}, \{\mathrm{a, c}\}, \{\mathrm{b, d}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

π_{30}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{c\}, \{\mathrm{a}, b\}, \{\mathrm{a, c}\}, \{\mathrm{b, c}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

π_{31}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{\mathrm{a}, b\}, \{\mathrm{a, c}\}, \{\mathrm{a, d}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}, \{\mathrm{a, c, d}\}\}$

π_{32}=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\},$

$\{a, b, d\}, \{a, c, d\}\}$

π_{33}=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

let X={a,b,c,d}

π_1={∅,X}

$SO(X) = \{\emptyset, X\}$

$SC(X) = \{\emptyset, X\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

$\pi_2 = \left\{\emptyset, X, \{a\}\right\}$

$SO(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$

$SC(X) = \{X, \emptyset, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_3=$\{\emptyset, X, \{a, b\}\}$

$SO(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}$

$SC(X) = \{\emptyset, X, \{c, d\}, \{d\}, \{c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_4=$\{\emptyset, X, \{a, b, c\}\}$

$SO(X) = \{\emptyset, X, \{a, b, c\}\}$

$SC(X) = \{\emptyset, X, \{d\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}$

$\mathrm{SO}(X) = \left\{\emptyset, X, \{a\}, \{b, c, d\}\right\}$

$\mathrm{SC}(X) = \{\emptyset, X, \{a\}, \{b, c, d\}\}$

$S_S O(X) = \left\{\emptyset, X, \{a\}, \{b, c, d\}\right\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_6=$\{\emptyset, X, \{a, b\}, \{\mathrm{c}, \mathrm{d}\}\}$

$\mathrm{SO}(X) = \left\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\right\}$

$\mathrm{SC}(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, d\}\}$

$S_S O(X) = \left\{\emptyset, X, \{a, c\}, \{b, c, \}\right\}$

$S_s C(X) = \{\emptyset, X, \{c, d\}, \{a, b\}\}$

$gS_S - Closed = P(x)$

π_7=$\{\emptyset, X, \{\text{a}\}, \{a, b\}\}$

$\text{SO}(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{SC}(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_8=$\{\emptyset, X, \{\text{a}\}, \{a, b, c\}\}$

$\text{SO}(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{SC}(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_9=$\{\emptyset, X, \{\text{a}, b\}, \{a, b, c\}\}$

$SO(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$SC(X) = \{\emptyset, X, , \{c, d\}, \{d\}, \{c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{10}={∅, X, {a}, {a, b}, {a, b, c}}

$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{11}={∅, X, {a, b}, {a, b, c}, {a, b, d}

$SO(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$SC(X) = \{\emptyset, X, , \{c, d\}, \{d\}, \{c\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{12}={∅, X, {a}, {b}, {a, b}}

$SO(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$

$SC(X) = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$S_S O(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$

$S_s C(X) = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$gS_S - Closed = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

π_{13}={∅, X, {a}, {b, c}, $\{a,b,c\}$}

$SO(X)=\{\emptyset,X,\{a\},\{b,c\},\{a,d\},\{a,b,c\},\{b,c,d\}\}$

$SC(X)=\{\emptyset,X,\{b,c,d\},\{a,d\},\{b,c\},\{d\},\{a\}\}$

$S_SO(X)=\{\emptyset,X,\{a\},\{b,c\},\{a,d\},\{a,b,c\},\{b,c,d\}\}$

$S_sC(X)=\{\emptyset,X,\{b,c,d\},\{a,d\},\{b,c\},\{d\},\{a\}\}$

$gS_S-Closed=\{P(x)/\{a,b\},\{a,c\},\{a,b,c\}\}$

π_{14}={∅, X, {a}, $\{a,b\}$, {a, c, d}}

$SO(X)=\{\emptyset,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$SC(X)=\{\emptyset,X,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$

$S_SO(X)=\{\emptyset,X\}$

$S_sC(X)=\{\emptyset,X\}$

$gS_S-Closed=P(x)$

π_{15}={∅, X, {a}, {a, b}, {a, c}, $\{a,b,c\}$}

$SO(X)=\{\emptyset,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$SC(X)=\{\emptyset,X,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$

$S_SO(X)=\{\emptyset,X\}$

$S_sC(X)=\{\emptyset,X\}$

$gS_S-Closed=P(x)$

π_{16}={∅, X, {a}, {a, b}, {a, b, c}, {a, b, d}}

$SO(X) = \left\{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\right\}$

$SC(X) = \{\emptyset, X, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{17}={∅, X, {a}, {b}, {a, b}, {a, b, c}}

$SO(X) = \left\{P(x)/\{c,d\}, \{d\}, \{c\}\right\}$

$SC(X) = \{P(x)/\{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$S_S O(X) = \left\{P(x)/\{c,d\}, \{d\}, \{c\}\right\}$

$S_s C(X) = \{P(x)/\{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$gS_S - Closed = \{P(x)/\{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

π_{18}={∅, X, {a}, {b}, {a, b}, {a, c, d}}

$SO(X) = \left\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\right\}$

$SC(X) = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}\{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X, \{b\}, \{a,c,d\}\}$

$S_s C(X) = \{\emptyset, X, \{a,c,d\}, \{b\}\}$

$gS_S - Closed = P(x)$

π_{19}={∅, X, {c}, {a, b}, {a, b, c}, {a, b, d}}

$SO(X) = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \}$

$SC(X) = \{\emptyset, X, \{a, b, d\}, \{c, d\}, \{d\}, \{c\}\}$

$S_S O(X) = \{\emptyset, X, \{c\}, \{a, b, d\}\}$

$S_s C(X) = \{\emptyset, X, \{a, b, d\}, \{c\}\}$

$gS_S - Closed = P(x)$

π_{20}={∅, X, {a}, {a, b}, {c, d}, {a, c, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{c, d\}, \{a, b\}\}$

$gS_S - Closed = P(x)$/{a}

π_{21}={∅, X, {a}, {a, b}, {a, c}, {a, b, c}, {a, b, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{22}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{\mathrm{a}, b\}, \{b, c\}, \{\mathrm{a, b, c}\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\{, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}\{c, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{a\}\}$

$S_S O(X) = \{\emptyset, X, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{b, c, d\}, \{b, c\}, \{a, d\}, \{d\}, \{a\}\}$

$gS_S - Closed = P(x)$/{a,b},{a,c},{a,b,c}

π_{23}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}\}$

$SO(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$

$SC(X) = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$S_S O(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$

$S_s C(X) = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$gS_S - Closed = \{P(x)/\{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

π_{24}=$\{\emptyset, X, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, b\}, \{\mathrm{c, d}\}, \{\mathrm{a, c, d}\}, \{\mathrm{b, c, d}\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}\{c, d\}, \{a, b\}, \{b\}, \{a\}\}$

$S_S O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\}, \{a\}\}$

$gS_S - Closed = \{P(x)/\{a, c\}, \{a, d\}, \{a, c, d\}\}$

π_{25}={∅, X, {a}, {b}, {a, b}, {b, d}, {a, b, c}, {a, b, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{a\}\}$

$S_S O(X) = \{\emptyset, X, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{b, c, d\}, \{b, d\}, \{a, c\}, \{c\}, \{a\}\}$

$gS_S - Closed = \{P(x)/\{a, b\}, \{a, d\}, \{a, b, d\}\}$

π_{26}={∅, X, {a}, {b}, {a, b}, {a, c}, {a, b, c}, {a, c, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}\{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X, \{b\}, \{a, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{a, c, d\}, \{b\}\}$

$gS_S - Closed = P(x)$

π_{27}={∅, X, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X\}$

$S_s C(X) = \{\emptyset, X\}$

$gS_S - Closed = P(x)$

π_{28}={∅, X, {a}, {b}, {c}, {a, b}, {b, c}, {a, b, c}}

$SO(X) = \{P(x)/\{d\}\}$

$SC(X) = \{P(x)/\{a, b, c\}\}$

$S_SO(X) = \{P(x)/\{d\}\}$

$S_sC(X) = \{P(x)/\{a, b, c\}\}$

$gS_S - Closed = \{P(x)/\{a, b, c\}\}$

π_{29}={∅, X, {a}, {b}, {a, b}, {a, c}, {b, d}, {a, b, c}, {a, b, d}}

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, c\}, \{d\}, \{c\}\}$

$S_SO(X) = \{\emptyset, X, \{a, c\}, \{b, d\}\}$

$S_sC(X) = \{\emptyset, X, \{b, d\}, \{a, c\}\}$

$gS_S - Closed = P(x)$

π_{30}={∅, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, b, d}}

$SO(X) = P(x)/\{d\}$,{c,d}

$SC(X) = \{P(x)/\{a, b, c\}\{a, b\}\}$

$S_SO(X) = \{P(x)/\{d\}, \{c, d\}$

$S_sC(X) = \{P(x)/\{a, b\}, \{a, b, c\}\}$

$gS_S - Closed = \{P(x)/\{a, b\}, \{a, b, c\}\}$

π_{31}=$\{\emptyset, X, \{\mathrm{a}\}, \{b\}, \{\mathrm{a}, b\}, \{\mathrm{a, c}\}, \{\mathrm{a, d}\}, \{\mathrm{a, b, c}\}, \{\mathrm{a, b, d}\}, \{\mathrm{a, c, d}\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}\{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$S_S O(X) = \{\emptyset, X, \{b\}, \{a, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{a, c, d\}, \{b\}\}$

$gS_S - Closed = P(x)$

π_{32}=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\},$

$\{a, b, d\}, \{a, c, d\}\}$

$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$

$SC(X) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}\}$

$S_S O(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$S_s C(X) = \{\emptyset, X, \{a, c, d\}, \{a, b, d\}, \{b, c\}, \{a, d\}, \{c\}, \{b\}\}$

$gS_S - Closed = P(x)$

π_{33}=$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

π_{33}= $\mathrm{SO}(X) = \mathrm{SC}(X) = S_S O(X)$

$\pi_{33} = P(x) = S_S O(X) = S_s C(X) = gS_S - Closed$

Chapter Three

Definition3.1: T_0**-Space** (أ. م. د. يوسف يعكوب يوسف و ٢٠١٨-٢٠١٩)

Let (X,π) be a topological space. Then the space (X,π) is called $\boldsymbol{T_0}$**-Space** iff for each pair of distinct points x,y∈ X there is $gS_S0(X)$ containing x but not y or an open set containing y but not x. I.e.,

X is $\boldsymbol{T_0}$**-Space** $\leftrightarrow \forall x, y \in X\,; x \neq y\ \exists U \in t\,; (x \in U \wedge y \notin U) \vee (x \notin U \wedge y \in U)$.

Definition3.2: T_1**-Space** (أ. م. د. يوسف يعكوب يوسف و ٢٠١٨-٢٠١٩)

Let (X,π) be a topological space. Then the space (X,π) is called $\boldsymbol{T_1}$**-Space** iff for each pair of distinct points x,y∈ X there is $gS_S0(X)$ containing x but not y And an open set containing y but not x. I.e.,

X is $\boldsymbol{T_1}$**-Space** $\leftrightarrow \forall x, y \in X\,; x \neq y\ \exists U, V \in t\,; (x \in U \wedge y \notin U) \wedge (x \notin V \wedge y \in V)$.

Definition3.3: T_2**-Space** (أ. م. د. يوسف يعكوب يوسف و ٢٠١٨-٢٠١٩)

Let (X,π) be a topological space. Then the space (X,π) is called $\boldsymbol{T_2}$**-Space** or **Hausdorff space** iff for each pair of distinct points x,y∈ X there exist $gS_S0(X)$ sets U And V s.t. $x \in U, y \in V\ And\ U \cap V = \emptyset$ I,e.,

X is $\boldsymbol{T_2}$**-Space** $\leftrightarrow \forall x, y \in X\,; x \neq y\ \exists U, V \in t\,;\ U \cap V = \emptyset\,, (x, y \in U \vee\ x, y \in V)$

X={a,b,c}	$\boldsymbol{T_0} - gS_S0(X)$	$\boldsymbol{T_1}$-$gS_S0(X)$	$\boldsymbol{T_2}$-$gS_S0(X)$
π_1	1	1	1
π_2	1	1	1
π_3	1	1	1
π_4	1	1	1
π_5	1	1	1
π_6	1	0	0
π_7	1	1	1
π_8	1	1	1
π_9	1	1	1

Note: in 4 elements all 33 topology will be $\boldsymbol{T_2}$-$gS_S0(X)$, $\boldsymbol{T_1}$-$gS_S0(X)$, $\boldsymbol{T_0}$-$gS_S0(X)$

References

A.A.Nasef, A.I. El-Maghrabi &. "Between semi-closed and GS- closed sets." *Journal of Taibah University for Science*, Volume 2,2009: 78-87.

K.B. Alias, M.H. Abdulrahman,and J.M. Jamil. "Ss-Open sets and Ss-Continuous Functions." *International Journal of Advanced Mathematical Sciences*, 2(1) 2014: 71-80.

KURONYA, ALEX. *INTRODUCTION TO TOPOLOGY*. Budapest: Alex Küronya, January 24, 2010.

Maurice, Thomas William James. "Semi-Open Sets." Master of Science, Kansas State, August 1965.

أ. م. د. يوسف يعكوب يوسف و، أ. م. د. رشا ناصر مجيد. *General Topology*. Baghdad: Baghdad University-Department of Mathematics, 2018-2019.