



On para α - Open Sets

Research Project

Submitted to the department of (Mathematic) in partial fulfillment of the
requirement for the degree of BSc. (Mathematics)

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Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: Dr. Nehmat K. Ahmed

Scientific grade: Assist. Professor

Date: 4 / 2024

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: Dr. Rashad Rasheed Haje

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: 4 / 2024

Acknowledgment

Primarily, I would like to thank my god for helping me to complete this research with success.

Then I would like to express special of my supervisor Assist. Prof Dr. Nehmat K. Ahmed whose valuable to guidance has been the once helped me to completing my research.

Words can only inadequately express my gratitude to my supervisor for patiently helping me to think clearly and consistently by discussing every point of this dissertation with me.

I would like to thank my family, friend and library staff whose support has helped me to conceive this research.

Abstract

In this report we have a set which contain three and four elements $X=\{a,b,c\}$ And $x=\{a,b,c,d\}$. The set which containing 3 element has 9 non comparable topology and the set which contains four element has 33 non comparable topology and we try to obtain the α -*open sets* to each such topology. Which we have applied several definitions to obtain on it such maximal, minimal and para- open set., also we studied some separation axioms .

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INTRODUCTION

Topology define on a non-empty set X , by a collection of subsets in X , which satisfy the implemented some cohesive properties enabling one to define it. The order pair of this sets with X is called topological spaces .

The notation of α -open set (originally called α -sets) in topological spaces was introduced by Njastad in 1965. Since then, it has been widely investigated in the articles and research's.

Throughout this report , (X, π) always mean topological spaces. where α -open set define as a subset A of (X, π) such that if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$.

CHAPTER ONE

Definition1.1: (STEVEN A. GAAL, 2009) Topological space

Let X be a non empty set and let π be a collection of subsets of X satisfying the following three conditions:

(i): $\phi \in \pi$ and $X \in \pi$.

(ii): If $O_1 \in \pi$ and $O_2 \in \pi$ then $O_1 \cap O_2 \in \pi$.

(iii): If $O_i \in \pi$ for every $i \in I$, then $\cup \{ O_i : i \in I \} \in \pi$.

The collection π is called topology on X , the pair (X, π) a topological space, the elements of π are called open sets.

Definition1.2: (M. P. Chaudhary and vinesh Kumar, May-2011) A subset A of X is said to be α -open set if $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$.

Definition1.3: (أ. م. د. يوسف يعكوب يوسف و , أ. م. د. رشا ناصر مجيد, 2019-2018) Interior Let (X, π) be a topological spaces subset A of X , $\text{Int } A$ is the largest open set contained in A .

Definition1.4: closure (أ. م. د. يوسف يعكوب يوسف و , أ. م. د. رشا ناصر مجيد, 2019-2018)

Let (X, π) be a topological spaces subset A of X , $\text{Cl } A$ is the smallest closed set containing A .

Definition1.5: (Dr. Haji M. Hasan , February 2015,) A proper non-empty α – open subset U at a topological space X is said to be minimal α – open set if any α – open set which is contained in U is ϕ or U , The family of all minimal α - open sets in a topological space X is denoted by $\text{Min}\alpha o(x)$.

Definition1.6: (K. M. A. AL. Hamadi and E. Q. A. Mohammed, Dec. 2020)

A proper non empty α - open set U of a topological space X is said to be maximal α -open set if any α - open set which contains U is X or U. The family of all maximal α - open sets in a topological space X is denoted by $Max\alpha o(x)$

Definition1.7: (Basaraj M. Ittanagi and S. S. Benchalli, 2016) Any open subset U of a topological space X is said to be a para α -open set if it is neither minimal α -open nor maximal α -open set The family of all para α -open sets in a topological space X is denoted by $Par\alpha o(X)$.

The topologies which define on the set $X = \{a, b, c\}$ which are different in element and property

$$\pi_1 = \{ \phi, X \}$$

$$\pi_2 = \{ \phi, X, \{a\} \}$$

$$\pi_3 = \{ \phi, X, \{a, b\} \}$$

$$\pi_4 = \{ \phi, X, \{a\}, \{b, c\} \}$$

$$\pi_5 = \{ \phi, X, \{a\}, \{a, b\} \}$$

$$\pi_6 = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$$

$$\pi_7 = \{ \phi, X, \{a\}, \{a, b\}, \{a, b\}, \{a, c\} \}$$

$$\pi_8 = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b\} \}$$

$$\pi_9 = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$$

$$\pi_1 = \{ \phi, X \}$$

$$\pi_1^c = \{ X, \phi \}$$

$$\alpha o(x) = \{ \phi, X \}$$

$$\text{Min}\alpha o(x) = \{ \phi, X \}$$

$$\text{Max}\alpha o(x) = \{ \phi, X \}$$

$$\pi_2 = \{ \phi, X, \{a\} \}$$

$$\pi_2^c = \{ X, \phi, \{b, c\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b\}, \{a, c\} \}$$

$$\pi_3 = \{ \phi, X, \{a, b\} \}$$

$$\pi_3^c = \{ X, \phi, \{c\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a, b\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a, b\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b\} \}$$

$$\pi_4 = \{ \phi, X, \{a\} \}$$

$$\pi_4^c = \{ X, \phi, \{b, c\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b, c\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a\}, \{b, c\} \}$$

$$\pi_5 = \{ \phi, X, \{a\}, \{a, b\} \}$$

$$\pi_5^c = \{ X, \phi, \{b, c\}, \{c\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha(x) = \{ X, \{a, b\}, \{a, c\} \}$$

$$\pi_6 = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$$

$$\pi_6^c = \{ X, \phi, \{b, c\}, \{a, c\}, \{c\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b\} \}$$

$$\pi_7 = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\pi_7^c = \{ \phi, X, \{b, c\}, \{c\}, \{b\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b\}, \{a, c\} \}$$

$$\pi_8 = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\} \}$$

$$\pi_8^c = \{ X, \phi, \{b, c\}, \{a, c\}, \{c\}, \{b\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b\}, \{a, c\} \}$$

$$\pi_9 = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$$

$$\pi_9^c = \{ X, \phi, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\}, \{c\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b\}, \{a, c\}, \{b, c\} \}$$

The topologies which define on the set $X = \{a, b, c, d\}$ which are different in element and property

$$\pi_1 = \{ \emptyset, X \}$$

$$\pi_2 = \{ \emptyset, X, \{a\} \}$$

$$\pi_3 = \{ \emptyset, X, \{a, b\} \}$$

$$\pi_4 = \{ \emptyset, X, \{a, b, c\} \}$$

$$\pi_5 = \{ \emptyset, X, \{a\}, \{b, c, d\} \}$$

$$\pi_6 = \{ \emptyset, X, \{a, b\}, \{c, d\} \}$$

$$\pi_7 = \{ \emptyset, X, \{a\}, \{a, b\} \}$$

$$\pi_8 = \{ \emptyset, X, \{a\}, \{a, b, c\} \}$$

$$\pi_9 = \{ \emptyset, X, \{a, b\}, \{a, b, c\} \}$$

$$\pi_{10} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, b, c\} \}$$

$$\pi_{11} = \{ \emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{12} = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\} \}$$

$$\pi_{13} = \{ \emptyset, X, \{a\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{14} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c, d\} \}$$

$$\pi_{15} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\} \}$$

$$\pi_{16} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{17} = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\} \}$$

$$\pi_{18} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\} \}$$

$$\pi_{19} = \{ \phi, X, \{c\}, \{a, b\}, \{c, d\}, \{a, c, d\} \}$$

$$\pi_{20} = \{ \phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\} \}$$

$$\pi_{21} = \{ \phi, X, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{22} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{23} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{24} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\} \}$$

$$\pi_{25} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{26} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\} \}$$

$$\pi_{27} = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\pi_{28} = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{29} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{30} = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{31} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\pi_{32} = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\pi_{33} = P(X)$$

$$\pi_1 = \{ \phi, X \}$$

$$\pi_1^c = \{ X, \phi \}$$

$$\alpha\omega(x) = \{ \phi, X \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, X \}$$

$$\text{Max}\alpha\omega(x) = \{ \phi, X \}$$

$$\pi_2 = \{ \phi, X, \{a\} \}$$

$$\pi_2^c = \{ X, \phi, \{b, c, d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, d\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Para}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_3 = \{ \phi, X, \{a, b\} \}$$

$$\pi_3^c = \{ X, \phi, \{c, d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a, b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_4 = \{ \phi, X, \{a, b, c\} \}$$

$$\pi_4^c = \{ X, \phi, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a, b, c\} \}(x) = \{ X, \{a\}, \{b, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a, b, c\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\} \}$$

$$\pi_5 = \{ \phi, X, \{a\}, \{b, c, d\} \}$$

$$\pi_5^c = \{ X, \phi, \{b, c, d\}, \{a\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b, c, d\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a\}, \{b, c, d\} \}$$

$$\pi_6 = \{ \phi, X, \{a, b\}, \{c, d\} \}$$

$$\pi_6^c = \{ X, \phi, \{c, d\}, \{a, b\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a, b\}, \{c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a, b\}, \{c, d\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b\}, \{c, d\} \}$$

$$\pi_7 = \{ \phi, X, \{a\}, \{a, b\} \}$$

$$\pi_7^c = \{ X, \phi, \{b, c, d\}, \{c, d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_8 = \{ \phi, X, \{a\}, \{a, b, c\} \}$$

$$\pi_8^c = \{ X, \phi, \{b, c, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_9 = \{ \phi, X, \{a, b\}, \{a, b, c\} \}$$

$$\pi_9^c = \{ X, \phi, \{c, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a, b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{10} = \{ \phi, X, \{a\}, \{a, b\}, \{a, b, c\} \}$$

$$\pi_{10}^c = \{ X, \phi, \{b, c, d\}, \{c, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, d\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_{11} = \{ \phi, X, \{a, b\}, \{a, b, c\}, \{a, c, d\} \}$$

$$\pi_{11}^c = \{ X, \phi, \{c, d\}, \{d\}, \{b\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a, b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{12} = \{ \phi, X, \{a\}, \{b\}, \{a, b\} \}$$

$$\pi_{12}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, b\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\} \}$$

$$\pi_{13} = \{ \emptyset, X, \{a\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{13}^c = \{ X, \emptyset, \{b, c, d\}, \{a, d\}, \{d\} \}$$

$$\alpha_o(x) = \{ \emptyset, X, \{a\}, \{b, c\}, \{a, b, c\} \}$$

$$\text{Min}\alpha_o(x) = \{ \emptyset, \{a\}, \{b, c\} \}$$

$$\text{Max}\alpha_o(x) = \{ X, \{a, b, c\} \}$$

$$\pi_{14} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c, d\} \}$$

$$\pi_{14}^c = \{ X, \emptyset, \{b, c, d\}, \{c, d\}, \{b\} \}$$

$$\alpha_o(x) = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha_o(x) = \{ \emptyset, \{a\} \}$$

$$\text{Max}\alpha_o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha_o(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_{15} = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\} \}$$

$$\pi_{15}^c = \{ X, \emptyset, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\} \}$$

$$\alpha_o(x) = \{ \emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha_o(x) = \{ \emptyset, \{a\} \}$$

$$\text{Max}\alpha_o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha_o(x) = \{ \{a, b\}, \{a, d\} \}$$

$$\pi_{16} = \{ \phi, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{16}^c = \{ X, \phi, \{b, c, d\}, \{c, d\}, \{d\}, \{c\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{a, d\} \}$$

$$\pi_{17} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{17}^c = \{ X, \phi, \{a, b, c\}, \{a, c, d\}, \{d\}, \{c\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\} \}$$

$$\pi_{18} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, c, d\} \}$$

$$\pi_{18}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, c\}, \{b\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_{19} = \{ \Phi, X, \{c\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{19}^c = \{ X, \Phi, \{a, b, d\}, \{d\}, \{c\} \}$$

$$\alpha o(x) = \{ \Phi, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \Phi, \{c\} \}$$

$$\text{Max}\alpha o(x) = \{ \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\} \}$$

$$\pi_{20} = \{ \Phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\} \}$$

$$\pi_{20}^c = \{ X, \Phi, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\} \}$$

$$\alpha o(x) = \{ \Phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \Phi, \{a\}, \{c, d\} \}$$

$$\text{Max}\alpha o(X) = \{ X, \{a, b\}, \{a, c, d\} \}$$

$$\pi_{21} = \{ \Phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{21}^c = \{ X, \Phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{c\} \}$$

$$\alpha o(x) = \{ \Phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \Phi, \{a\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_{22} = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{22}^c = \{ X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \emptyset, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{b, c\} \}$$

$$\pi_{23} = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{23}^c = \{ X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\} \}$$

$$\alpha\omega(x) = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \emptyset, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\} \}$$

$$\pi_{24} = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\} \}$$

$$\pi_{24}^c = \{ X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, b\}, \{b\}, \{a\} \}$$

$$\alpha\omega(x) = \{ \emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \emptyset, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, c, d\}, \{b, c, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{c, d\} \}$$

$$\pi_{25} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{25}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{d\}, \{c\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{b, d\} \}$$

$$\pi_{26} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\} \}$$

$$\pi_{26}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{b\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{a, c\} \}$$

$$\pi_{27} = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\pi_{27}^c = \{ X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\} \}$$

$$\alpha o(x) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Min}\alpha o(x) = \{ \phi, \{a\} \}$$

$$\text{Max}\alpha o(x) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\} \}$$

$$\text{Par}\alpha o(x) = \{ \{a, b\}, \{a, c\}, \{a, d\} \}$$

$$\pi_{28} = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{28}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{a, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\}, \{c\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{b, c\} \}$$

$$\pi_{29} = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\pi_{29}^c = \{ X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{a, c\}, \{d\}, \{c\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{b, d\} \}$$

$$\pi_{30} = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$$

$$\pi_{30}^c = \{ X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\} \}$$

$$\alpha\omega(x) = \{ \phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Min}\alpha\omega(x) = \{ \phi, \{a\}, \{b\}, \{c\} \}$$

$$\text{Max}\alpha\omega(x) = \{ X, \{a, b, c\}, \{a, b, d\} \}$$

$$\text{Par}\alpha\omega(x) = \{ \{a, b\}, \{a, c\}, \{b, d\} \}$$

$$\pi_{31} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{31}^c = \{X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$$

$$\alpha\omega(x) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Min}\alpha\omega(x) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{Max}\alpha\omega(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}\alpha\omega(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_{32} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{32}^c = \{X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$$

$$\alpha\omega(x) = \pi$$

$$\text{Min}\alpha\omega(x) = \{\emptyset, \{a\}, \{b\}, \{c\}\}$$

$$\text{Max}\alpha\omega(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}\alpha\omega(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}\}$$

$$\pi_{33} = P(X)$$

$$\pi_{33}^c = P(X)$$

$$\alpha\omega(x) = P(X)$$

$$\text{Min}\alpha\omega(x) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}\}$$

$$\text{Max}\alpha\omega(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\text{Par}\alpha\omega(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$$

CHAPTER TWO

Definition2.1: (STEVEN A. GAAL, 2009) α - T_0 -Space

A topological space (X, π) is said to be para α - T_0 -space if for each pair at distinct points $x, y \in P(x)$, there exist para α -open sets $\{G \& H$ such that $x \in G$ but $y \notin G$ or $y \in H$ but $x \notin H\}$.

Definition2.2: (STEVEN A. GAAL, 2009) α - T_1 - Space

A topological space (X, π) is said to be para α - T_1 -space if for each pair of distinct points $x, y \in X$, there exist two para α -open set G, H such that $x \in G, y \notin G$ and $x \notin H, y \in H$.

Definition2.3: (STEVEN A. GAAL, 2009) α - T_2 -space

A topological space (X, π) is said to be α - T_2 -space if for each pair at distinct point $x, y \in X$, there exist two distinct para α -open set G, H such that $x \in G, y \in H$ and $G \cap H = \emptyset$

$X = \{a, b, c, d\}$	Para- α - T_0 -space	Para- α - T_1 -space	Para- α - T_2 -space
π_2	1	0	0
π_7	1	0	0
π_8	1	0	0
π_9	1	0	0
π_{10}	1	0	0
π_{12}	0	0	0

π_{14}	1	0	0
π_{15}	1	0	0
π_{16}	1	0	0
π_{17}	0	0	0
π_{18}	1	0	0
π_{19}	1	0	0
π_{21}	1	0	0
π_{22}	0	0	0
π_{23}	0	0	0
π_{25}	0	0	0
π_{26}	0	0	0
π_{27}	1	0	0
π_{28}	0	0	0
π_{29}	1	0	0
π_{30}	1	0	0
π_{31}	1	0	0
π_{32}	1	0	0
π_{33}	1	1	1

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پوخته

لهم راپورتهدا دوو كۆمهلهمان و مرگرتوه يهكئيكيان ۳ توخم و ۹ توپۆلۆجى بهراوردكراوى هميه وه دووهميان ۴ توخم و ۳۳ توپۆلۆجى بهراوردكراوى هميه كه چند پئناسهيهكمان لهسهر جئيهجئيكردون بۆ بهدهست هئينانى (Para- α -open set)

خلاصة

في هذا التقدير أخذنا مجموعتين واحدة تحتوي على ۳ عناصر و ۹ طوبولوجيا متشابهة والأخرى تحتوي على ۴ عناصر و ۳۳ طوبولوجيا قابلة للمقارنة والتي طبقنا عليها عدة تعريفات للحصول على مجموعة (Para- α -open set)