S_β -Open Set in Topological Space

Research Project

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Abstract:

In this report we introduced a subclass of semi open sets called S_β -Open Sets in topological spaces. This class of sets used to defined and study the concept of gs_β-spaces.

Keywords: $\beta - open\ sets$,semi-open sets, S_β -Open Sets.

Introduction:

Throughout this paper, a space means a topological space on which no separation axioms are assumed unless explicitly stated. In 1963 [1] Levine was initiated semi open sets and their properties, Mathematicians gives in several papers in- teresting and different new types of sets. In [2], Abd-El-Moonsef in 1983 defined the class of β-open set. In 2013, Nehmat [3] introduced a new class of semi-open sets called S_β -open sets. We recall the following definitions and characterizations. The closure (resp., interior) of a subset A of X is denoted by clA (resp., intA). A subset A of X is said to be semi-open [1] β-open [3] set if A ⊆ clintclA, In general we applied the following definitions we use which contains three and four elements.

Definition 1.1: A subset A of a topological space X is said to be semi– open iff A⊆cl int A.

Definition 1.2: A subset A of a topological space X is said to be β– open iff A⊆cl int cl A

Definition 1.3: A subset A of semi – open set is said to be S_β -Open set of X if for each x∈A there exist a β-closed set F such that x∈F⊆A.

Definition 1.4: A subset A of X is said to be gs_β closed set iff $S_\beta$cl A ⊆ u , when ever A⊆ u , u is $S_\beta$open set.

Let $x = \{a,b,c\}$

$\tau_1 = \{\Phi, X\}$

$\tau_2 = \{\Phi, X, \{a\}\}$

$\tau_3 = \{\Phi, X, \{a,b\}\}$

$\tau_4 = \{\Phi, X, \{a\}, \{a,b\}\}$

$\tau_5 = \{\Phi, X, \{a\}, \{b,c\}\}$

$\tau_6 = \{\Phi, X, \{a\}, \{b\}, \{a,b\}\}$

$\tau_7 = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\tau_8 = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$

$\tau_9 = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\tau_1 = \{\Phi, X\}$

$SO(x) = \{\Phi, X\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta C(x) = \{\Phi, X, \{b,c\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_2 = \{\Phi, X, \{a\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\beta C(x) = \{\Phi, X, \{b,c\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_3 = \{\Phi, X, \{a,b\}\}$

$SO(x) = \{\Phi, X, \{a,b\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta C(x) = \{\Phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_4 = \{\Phi, X, \{a\}, \{b,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b,c\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b\}, \{b,c\}\}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b,c\}\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_5 = \{\Phi, X, \{a\}, \{a,b\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\beta C(x) = \{X, \Phi, \{b,c\}, \{c\}, \{b\}\}$

$S_\beta O(x)=\{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_6=\{\Phi, X, \{a\}, \{b\}, \{a,b\}\}$

$SO(x)=\{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta O(x)=\{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta C(x)=\{X, \Phi, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x)=\{\Phi, X, \{a,c\}, \{b,c\}, \{a\}, \{b\}, \{a,b\}\}$

$gs_\beta C(X)$=P(X)\{{a,b}}

$gs_\beta O(X)$=P(X)\{c}

$\tau_7=\{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$SO(x)=\{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\beta O(x)=\{\Phi, X, \{a\}, \{a,b\}, \{a,c\}\}$

$\beta C(x)=\{X, \Phi, \{b,c\}, \{c\}, \{b\}\}$

$S_\beta O(x)=\{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_8 = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$

$\beta C(x) = \{\Phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X, \{b\}, \{a,c\}\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_9 = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b\}, \{b,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$\beta C(x) = \{\Phi, X, \{b,c\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

Let $x = \{a,b,c,d\}$

$\tau_1 = \{\Phi, X\}$

$\tau_2 = \{\Phi, X, \{a\}\}$

$\tau_3 = \{\Phi, X, \{a,b\}\}$

$\tau_4 = \{\Phi, X, \{a,b,c\}\}$

$\tau_5 = \{\Phi, X, \{a\}, \{b,c,d\}\}$

$\tau_6 = \{\Phi, X, \{a,b\}, \{c,d\}\}$

$\tau_7 = \{\Phi, X, \{a\}, \{a,b\}\}$

$\tau_8 = \{\Phi, X, \{a\}, \{a,b,c\}\}$

$\tau_9 = \{\Phi, X, \{a,b\}, \{a,b,c\}\}$

$\tau_{10} = \{\Phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$

$\tau_{11} = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\tau_{12} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}\}$

$\tau_{13} = \{\Phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}$

$\tau_{14} = \{\Phi, X, \{a\}, \{a,b\}, \{a,c,d\}\}$

$\tau_{15} = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$

$\tau_{16} = \{\Phi, X, \{a\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\tau_{17} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$

$\tau_{18} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c,d\}\}$

$\tau_{19} = \{\Phi, X, \{c\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\tau_{20} = \{\Phi, X, \{a\}, \{a,b\}, \{c,d\}, \{a,c,d\}\}$

$\tau_{21} = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}\}$

$\tau_{22} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}, \{a,b,c\}\}$

$\tau_{23} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\tau_{24} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$

$\tau_{25} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$

$$\tau_{26} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$$

$$\tau_{27} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

$$\tau_{28} = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\tau_{29} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$$

$$\tau_{30} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,b,d\}\}$$

$$\tau_{31} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\tau_{32} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$$

$$\tau_{33} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$\tau_1 = \{\Phi, X\}$$

$$SO(x) = \{\Phi, X\}$$

$$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{b,d\}, \{b,c\}, \{a,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$$

$$\beta C(x) = \{\Phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,b,c\}, \{c,d\}, \{b,d\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b\}, \{d\}, \{c\}, \{b\}, \{a\}\}$$

$$S_\beta O(x) = \{\Phi, X\}$$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_2 = \{\Phi, X, \{a\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{\Phi, X, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_3 = \{\Phi, X, \{a,b\}\}$

$SO(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

$\beta C(x) = \{\Phi, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{a,c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$gs_\beta C(X)$=$\{\Phi, X, \{c\}, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$

$gs_\beta O(X)$= $\{\Phi, X, \{a,b,d\}, \{a,b,c\}, \{a,b\}, \{b\}, \{a\}\}$

$\tau_4 = \{\Phi, X, \{a,b,c\}\}$

$SO(x) = \{\Phi, X, \{a,b,c\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,c\}, \{a,b\}, \{a,d\}, \{a,c\}, \{b,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{c,b,d\}\}$

$\beta C(x) = \{\Phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,c\}, \{b,d\}, \{a,c\}, \{a,d\}, \{a,b\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x) = \{\Phi, X, \{a,b,c\}\}$

$gs_\beta C(X)=\{\Phi, X, \{d\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

$gs_\beta O(X)= \{\Phi, X, \{a,b,c\}, \{c,b\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$

$\tau_5 = \{\Phi, X, \{a\}, \{b,c,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b,c,d\}\}$

$\beta O(x) = \mathrm{P(X)}$

$\beta C(x) = \mathrm{P(X)}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b,c,d\}\}$

$gs_\beta C(X)=\mathrm{P(X)}$

$gs_\beta O(X)=\mathrm{P(X)}$

$\tau_6 = \{\Phi, X, \{a,b\}, \{c,d\}\}$

$SO(x) = \{\Phi, X, \{a,b\}, \{c,d\}\}$

$\beta O(x) = p(x)$

$\beta C(x) = p(x)$

$S_\beta O(x) = \{\Phi, X, \{a,b\}, \{c,d\}\}$

$gs_\beta C(X)=\mathrm{P(X)}$

$gs_\beta O(X)=\mathrm{P(X)}$

$\tau_7 = \{\Phi, X, \{a\}, \{a,b\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_8 = \{\Phi, X, \{a\}, \{a,b,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{\Phi, X, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_9 = \{\Phi, X, \{a,b\}, \{a,b,c\}\}$

$SO(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\beta O(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta C(x) = p(x) \setminus \{\{a,b,d\}, \{a,b,c\}, \{a,b\}\}$

$S_\beta O(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$gs_\beta C(X) = \{\Phi, X, \{c,d\}, \{c\}, \{d\}, \{a,c,d\}, \{b,c,d\}\}$

$gs_\beta O(X) = \{\Phi, X, \{a,b\}, \{a,b,d\}, \{a,b,c\}, \{b\}, \{a\}\}$

$\tau_{10} = \{\Phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X\}$

$gs_\beta C(X)$=P(X)

$gs_\beta O(X)$=P(X)

$\tau_{11} = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$SO(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\beta O(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta C(x) = p(x) \setminus \{\{a,b,d\}, \{a,b,c\}, \{a,b\}\}$

$S_\beta O(x) = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$gs_\beta C(X) = \{\Phi, X, \{c,d\}, \{c\}, \{d\}, \{a,c,d\}, \{b,c,d\}\}$

$gs_\beta O(X) = \{\Phi, X, \{a,b\}, \{a,b,d\}, \{a,b,c\}, \{b\}, \{a\}\}$

$\tau_{12} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}\}$

$SO(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta O(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta C(x) = p(x) \setminus \{\{a,b,d\}, \{a,b,c\}, \{a,b\}\}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,c\}, \{a,d\}, \{b,c\}\{b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,d\}, \{a,b,c\}, \{a,b\}\}$

$gs_\beta C(X) = P(X)\setminus\{\{a,b\},\{a,b,c\},\{a,b,d\}\}$

$gs_\beta O(X) = P(X)\setminus\{\{c,d\},\{c\},\{d\}\}$

$\tau_{13} = \{\Phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,d\}\}$

$\beta O(x) = p(x) \setminus \{d\}$

$\beta C(x) = p(x) \setminus \{a,b,c\}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,d\}\}$

$gs_\beta C(X)$
$= \{\Phi, X, \{a\}, \{d\}, \{b,c\}, \{b,c,d\}, \{a,d\}, \{b\}, \{c\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}\}$

$gs_\beta O(X)$
$= \{\Phi, X, \{b,c,d\}, \{a,b,c\}, \{a,d\}, \{a\}, \{c,b\}, \{a,c,d\}, \{a,b,d\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}\}$

$\tau_{14} = \{\Phi, X, \{a\}, \{a,b\}, \{a,c,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$$\beta C(x)=\{X,\Phi,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$$

$$S_{\beta}O(x)=\{\Phi,X\}$$

$$gs_{\beta}C(X)=P(X)$$

$$gs_{\beta}O(X)=P(X)$$

$$\tau_{15}=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,b,c\}\}$$

$$SO(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$$

$$\beta O(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$$

$$\beta C(x)=\{X,\Phi,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$$

$$S_{\beta}O(x)=\{\Phi,X\}$$

$$gs_{\beta}C(X)=P(X)$$

$$gs_{\beta}O(X)=P(X)$$

$$\tau_{16}=\{\Phi,X,\{a\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$$

$$SO(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$$

$$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$$

$$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$$

$$S_{\beta}O(x)=\{\Phi,X\}$$

$$gs_{\beta}C(X)=P(X)$$

$$gs_{\beta}O(X)=P(X)$$

$\tau_{17} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$

$SO(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta O(x) = p(x) \setminus \{\{c\}, \{d\}, \{c,d\}\}$

$\beta C(x) = p(x) \setminus \{\{a,b,d\}, \{a,b,c\}, \{a,b\}\}$

$S_\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{18} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X, \{b\}, \{a,c,d\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{19} = \{\Phi, X, \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{c\}\}$

$SO(x) = \{\Phi, X, \{c\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{a,c\}, \{d\}, \{c\}, \{a\}, \{b\}\}$

$S_\beta O(x)=\{\Phi,X,\{c\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{20}=\{\Phi,X,\{a\},\{a,b\},\{c,d\},\{a,c,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{a,b\},\{c,d\},\{a,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,b,d\},\{a,b,c\},\{b,d\},\{c,d\},\{b,d\},\{b,c\},\{a,b\},\{d\},\{c\},\{b\}\}$

$S_\beta O(x)=\{\Phi,X,\{a,b\},\{c,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{21}=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,b,c\},\{a,b,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$

$S_\beta O(x)=\{\Phi,X\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{22}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,c\},\{a,d\},\{a,c\},\{d\},\{c\},\{a\}\}$

$S_\beta O(x)=\{\Phi,X,\{a\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{23}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,d\},\{b,c\},\{a,d\},\{a,c\},\{d\},\{c\},\{b\},\{a\}\}$

$S_\beta O(x)=P(X)\backslash\{\{d\},\{c\},\{c,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{24}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$

$\beta O(x)=p(x)$

$\beta C(x)=p(x)$

$S_\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{25}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{b,d\},\{a,b,c\},\{a,b,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,d\},\{a,c\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,d\},\{a,c\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,d\},\{a,d\},\{a,c\},\{d\},\{c\},\{a\}\}$

$S_\beta O(x)=\{\Phi,X,\{a\},\{a,c\},\{b,d\},\{a,b,d\},\{b,c,d\}\}$

$gs_\beta C(X)=P(X)\setminus\{\{a,d\},\{a,b,d\},\{a,b\}\}$

$gs_\beta O(X)=P(X)\setminus\{\{c,b\},\{c\},\{c,d\}\}$

$\tau_{26}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$

$S_\beta O(x)=\{\Phi,X,\{b\},\{a,c,d\}\}$

$gs_\beta C(X)=P(X)$

$gs_\beta O(X)=P(X)$

$\tau_{27}=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$

$S_\beta O(x)=\{\Phi,X\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{28}=\{\Phi,X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{c\},\{a,b\},\{a,d\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{c\},\{a,b\},\{a,d\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{c,d\},\{b,d\},\{b,c\},\{a,d\},\{a,c\},\{a,b\},\{d\},\{c\},\{b\},\{a\}\}$

$S_\beta O(x) = P(X)\backslash\{\{\mathrm{d}\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{29}=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,d\},\{a,b,c\},\{a,b,d\}\}$

$SO(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,d\},\{a,b,c\},\{a,b,d\}\}$

$\beta O(x)=\{\Phi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,d\},\{a,b,c\},\{a,b,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{b,d\},\{a,c\},\{d\},\{c\}\}$

$S_\beta O(x)=\{\Phi,X,\{a,c\},\{b,d\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{30} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,d\}, \{a,c\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{a,c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$S_\beta O(x) = P(X) \setminus \{\{d\}, \{c,d\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{31} = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x) = \{X, \Phi, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$

$S_\beta O(x) = \{\Phi, X, \{b\}, \{a,c,d\}\}$

$gs_\beta C(X) = P(X)$

$gs_\beta O(X) = P(X)$

$\tau_{32} = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$SO(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta O(x) = \{\Phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$\beta C(x)=\{X,\Phi,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{c,d\},\{b,d\},\{b,c\},\{a,d\},\{d\},\{c\},\{b\}\}$

$S_{\beta}O(x)=\{\Phi,X,\{b\},\{c\},\{a,d\},\{b,c\},\{a,b,d\},\{a,c,d\}\}$

$gs_{\beta}C(X)=P(X)$

$gs_{\beta}O(X)=P(X)$

$\tau_{33}=\{\Phi,X,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$SO(x)=\mathrm{P(X)}$

$\beta O(x)=\mathrm{P(X)}$

$\beta C(x)=\mathrm{P(X)}$

$S_{\beta}O(x)=\mathrm{P(X)}$

$gs_{\beta}C(X)=P(X)$

$gs_{\beta}O(X)=P(X)$

Definition2.1: A topological space (X, τ) is said to be gs_{β} T_0-space if and only if for every two distinct points of X there exist a gs_{β} open set G which contains one of them but not the other.

Definition2.2: A topological space (X, τ) is said to be gs_{β} T_1-space if and only if for every two distinct points of X there exist two gs_{β} open sets G&H such that $x\in G$ and $y\notin G$ but $x\notin H$ and $y\in H$.

Definition2.3: A topological space (X, τ) is said to be gs_{β} T_2-space iff for every two distinct points of X there exist two gs_{β} open sets G,H such that $x\in G$, $y\in H$ and $G\cap H=\phi$

Let X={a,b,c}

Topologies	$gs_\beta T_0$	$gs_\beta T_1$	$gs_\beta T_2$
τ_1	1	1	1
τ_2	1	1	1
τ_3	1	0	0
τ_4	1	1	1
τ_5	1	1	1
τ_6	1	1	1
τ_7	1	1	1
τ_8	1	1	1
τ_9	1	1	1

Let X={a,b,c,d}

Topologies	$gs_\beta T_0$	$gs_\beta T_1$	$gs_\beta T_2$
τ_1	1	1	1
τ_2	1	1	1
τ_3	0	0	0
τ_4	0	0	0
τ_5	1	1	1
τ_6	1	1	1
τ_7	1	1	1
τ_8	1	1	1
τ_9	0	0	0
τ_{10}	1	1	1
τ_{11}	0	0	0
τ_{12}	1	1	1
τ_{13}	1	1	1
τ_{14}	1	1	1
τ_{15}	1	1	1
τ_{16}	1	1	1
τ_{17}	1	1	1
τ_{18}	1	1	1
τ_{19}	1	1	1
τ_{20}	1	1	1
τ_{21}	1	1	1
τ_{22}	1	1	1
τ_{23}	1	1	1
τ_{24}	1	1	1
τ_{25}	1	1	1
τ_{26}	1	1	1
τ_{27}	1	1	1
τ_{28}	1	1	1
τ_{29}	1	1	1
τ_{30}	1	1	1
τ_{31}	1	1	1
τ_{32}	1	1	1
τ_{33}	1	1	1

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الملخص:

استخدمنا بعض التعاريف التبولوجية على مجموعتين تحتوي على ثلاثة و اربعة عناصر لايجاد مجموعات شبه مفتوحة من المفاهيم لتبولوجية

ثوختة:

له م ئيش دا دوو كومه له مان وه ركرتوه كه سى دانه وه جوارهدانه ى تيدايه هه نيك بيناسه ي توبولوجيمان له سه ر به كار هيناوه بو دوزينه وهنديك كومه له ى نيمجه كراوه .