



زانکوی سهالحمدین-ھەولێر

Salahaddin University-Erbil

# On Para semi\_open sets

Research Project

Submitted to the department of (mathematic) in partial fulfilment of the Requirements for the degree of BSc. In mathematic

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## **Certification of the Supervisors**

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: **Assist.Prof.Dr.Nehmat K. Ahmad**

Scientific grade: Assist. Professor

Date: 4 / 4 / 2024

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: **Dr. Rashad Rasheed Haje**

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: 4 / 4 / 2024

## Acknowledgment

I would like to thanks Allah for giving me the power to complete this work And I would like to present my profound thanks to supervisor and lecturer **Assist. Prof. Dr. Nehmat K. Ahmed** for his kind valuable suggestions that assisted me to accomplish this work I would also like to extend my gratitude to the head of mathematic department **Assist. Prof. Dr. Rashad Rashid Haji**, and especially thanks for my family to support me and make me what am I today ,and thanks all my friends

## **Abstract**

In this report we have a set which contain three and four elements  $X=\{a,b,c\}$  And  $x=\{a,b,c,d\}$ . The set which containing 3 element has 9 non comparable topology and the set which contains four element has 33 non comparable topology and we try to obtain some type of *semi-open sets* to each such topology.

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## **Introduction**

***General Topology or Point Set Topology.*** General topology normally considers local properties of spaces, and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered. Sometimes distances can be defined in these spaces, in which case they are called metric spaces; sometimes no concept of distance makes sense.

In 1963, Levine introduced the concept of a semi-open set. The initiation of the study of generalized closed sets was done by Aull in 1968 as he considered sets whose closure belongs to every open superset. The notion of generalized semi-closed sets was introduced by Arya and Nour . In 1987, Bhattacharyya and Lahiri defined and studied the concept of semi- generalized closed sets via the notion of a semi-closed set.

# Chapter one

## Definition1.1: Topology(STEVEN A.GAAL,2009)

A topological space is ordered pair  $(X, \pi)$ , where  $X$  is a non empty set, a collection  $\pi$  of subsets of  $X$  satisfying the following properties

- 1)  $\emptyset, X \in \pi,$
- 2)  $U, V \in \pi$  implies  $U \cap V \in \pi$
- 3)  $\{U_\alpha | \alpha \in I\} \in \pi$  implies  $U_{\alpha \in I} U_\alpha \in \pi.$

is called topology on  $X$ , the pair  $(X, \pi)$  is called a topological space, the elements of  $\pi$  are called open sets.

## Definition1.2: Semi-open (A.A.Nasef volume 2,2009)

Let  $(X, \pi)$ , be a topological spaces subset  $A$  of  $X$  is said to be Semi-open set if  $A \subseteq clint A$

## Definition1.3: interior (أ.م.د. يوسف يعقوب يوسف ٢٠١٩\_٢٠١٨)

Let  $(X, \pi)$ , be a topological spaces and  $A$  be a subset of  $X$  then  $Int A$  is the largest open set contained  $A$

## Definition1.4: closure(أ.م.د. يوسف يعقوب يوسف ٢٠١٩\_٢٠١٨)

Let  $(X, \pi)$ , be a topological spaces and  $A$  be a subset of  $X$  then  $CIA$  is the smallest closed set containing  $A$

## Definition1.5: Minimal semi open (Khalaf,A,B.,&Hasan,H.M.(2012)

A proper non-empty semi – open subset  $U$  at a topological space  $X$  is said to be minimal semi – open set if any semi– open set which is contained in  $U$  is  $\emptyset$  or  $U$ , The family of all minimal semi- open sets in a topological space  $X$  is denoted by  $Minso(x)$ .

**Definition1.6: Maximal semi open**( Khalaf ,B.,&Hasan,H.M.(2012)

A proper non-empty semi-open set  $U$  of a topological space  $X$  is said to be maximal semi-open set if any semi open set which contains  $U$  is  $X$  or  $U$ . The family of all maximal semi- open sets in a topological space  $X$  is denoted by  $\text{Maxso}(X)$

**Definition1.7:Para S-open set**(Basaraj M.Ittangi and S.S. Benchalli,2016)

Any open sunset  $U$  of at topolgical space  $X$  is said to be para s-open set if it is neither minimal s-open nor maximal s-open set the family of all para s-open sets in a topological space  $X$  is denoted  $\text{paraso}(X)$ .

## **Chapter Two**

The topologies which define on the set  $X = \{a, b, c\}$  which are different in element and property

$$\pi_1 = \{\emptyset, X\}$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$, \quad \pi_5 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\pi_6 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\pi_9 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\text{let } X = \{a, b, c\}$$

$$\pi_1 = \{\emptyset, X\}$$

$$\pi^c = \{X, \emptyset\}$$

$$SO(X) = \{\emptyset, X\}$$

$$\text{MinSO}(x) = \{\emptyset\}$$

$$\text{MaxSO}X) = \{X\}$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$\pi^c = \{X, \emptyset, \{b, c\}\}$$

$$SO(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a,b\}, \{a,c\}\}$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$\pi^c = \{X, \emptyset, \{c\}\}$$

$$SO(X) = \{\emptyset, X, \{a, b\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a,b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a,b\}\}$$

$$\pi_4 = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$\pi^c = \{X, \emptyset, \{b, c\}, \{a\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b, c\}\}$$

$$\text{MaxSO}(X) = \{X, \{a\}, \{b, c\}\}$$

$$\pi_5=\{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\pi^c = \{X, \emptyset, \{c\}, \{b, c\}\}$$

$$SO(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b\}, \{a, c\}\}$$

$$\pi_6=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$\pi^c = \{X, \emptyset, \{b, c\}, \{a, c\}, \{c\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b\}, \{a, c\}\}$$

$$\pi_7=\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\pi^c = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$$

$$SO(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{ManSO}(X) = \{\emptyset, \{a, b\}, \{a, c\}\}$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$$

$$\pi^c = \{X, \emptyset, \{b, c\}, \{a, c\}, \{c\}, \{a\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$MaxSO(X) = \{X, \{a, b\}, \{b, c\}\}$$

$$\pi_9 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\pi^c = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}, \{c\}\}$$

$$MaxSO(X) = \{X, \{a, b\}, \{a, c\}, \{b, c\}\}$$

The topologies which define on the set  $X = \{a, b, c, d\}$  which are different in element and property

$$\pi_1 = \{\emptyset, X\}$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$\pi_4 = \{\emptyset, X, \{a, b, c\}\}$$

$$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$\pi_6 = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$$

$$\pi_9 = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$$

$$\pi_{10} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$$

$$\pi_{11} = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{12} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$\pi_{13} = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$$

$$\pi_{14} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}\}$$

$$\pi_{15} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$$

$$\pi_{16} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{17} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$$

$$\pi_{18} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$$

$$\pi_{19} = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{20} = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$$

$$\pi_{21} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{22} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$$

$$\pi_{23} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{24} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\pi_{25} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{26} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$$

$$\pi_{27} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{28} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$$

$$\pi_{29} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{30} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{31} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{32} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{33} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\},$$

$$\{a, c, d\}, \{b, c, d\}\}$$

$$\pi_1 = \{\emptyset, X\}$$

$$\text{SO}(X) = \{\emptyset, X\}$$

$$\text{MinSO}(X) = \{\emptyset\}$$

$$\text{MaxSO}(X) = \{X\}$$

$$\pi_2 = \{\emptyset, X, \{a\}\}$$

$$\text{SO}(X) = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, c, d\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_3 = \{\emptyset, X, \{a, b\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a, b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_4 = \{\emptyset, X, \{a, b, c\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a, b, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a, b, c\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}\}$$

$$\pi_5 = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a\}, \{b, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b, c, d\}\}$$

$$\text{MaxSO}(X) = \{X, \{a\}, \{b, c, d\}\}$$

$$\pi_6 = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a, b\}, \{c, d\}\}$$

$$\text{MaxSO}(X) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$$

$$\pi_7 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_8 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, b, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_9 = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$$

$$\text{SO}(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a, b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{10} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_{11} = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a, b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{12} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$$

$$SO(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{13} = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$$

$$SO(X) = \{\emptyset, \{a\}, \{b, c\}\}$$

$$MinSO(X) = \{\emptyset, X, \{b, c, d\}, \{a, d\}, \{b, c\}, \{d\}, \{a\}\}$$

$$MaxSO(X) = \{X, \{a, d\}, \{a, b, c\}, \{b, c, d\}\}$$

$$\pi_{14} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}\}$$

$$MaxSO(X) = \{\emptyset, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_{15} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{16} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}(x) = \{\{a, b\}, \{a, b\}, \{a, c\}\}$$

$$\pi_{17} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$$

$$SO(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\pi_{18} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, b\}, \{a, d\}\}$$

$$\pi_{19} = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a, b\}, \{c\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{a, b, d\}\}$$

$$\pi_{20} = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{c, d\}\}$$

$$MaxSO(X) = \{X, \{a, b\}, \{a, c, d\}\}$$

$$\pi_{21} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\pi_{22} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{ X, \{a\}, \{a, b, d\}, \{b, c, d\}, \{a, b, d\}\}$$

$$\text{Par}(X) = \{\{b, d\}, \{b, c\}, \{a, d\}, \{a, b\}\}$$

$$\pi_{23} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{P(x)/\{c, d\}, \{d\}, \{c\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{24} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}, \{c, d\}\}$$

$$\text{MaxSO}(X) = \{ X, \{a, b\}, \{b, c, d\}, \{a, c, d\}\}$$

$$\pi_{25} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{26} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{27} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}\}$$

$$MaxSO(X) = \{X, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_{28} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$$

$$SO(X) = \{P(x)/\{d\}\}$$

$$\text{MinSO}(X) = \{ \emptyset, \{a\}, \{b\}, \{c\} \}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{29} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{a, b, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{b, d\}\}$$

$$\pi_{30} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$$

$$SO(X) = \{P(x)/\{d\}, \{c, d\}\}$$

$$\text{MinSO}(X) = \{\emptyset, \{a\}, \{b\}, \{c\}\}$$

$$\text{MaxSO}(X) = \{X, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$$

$$\text{Par}(X) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$$

$$\pi_{31} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}\}$$

$$MaxSO(X) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

$$\pi_{32} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$MinSO(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$$

$$MaxSO(X) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}\}$$

$$\pi_{33} = P(X)$$

$$SO(X) = P(X)$$

$$MinSO(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}\}$$

$$MaxSO(X) = \{ X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$Par(X) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$$

## Chapter Three

### **Definition3.1: $T_0$ -Space** (STEVEN A.GAAL,2009)

A topological  $(X, \pi)$  is said to be para  $S_{T_0}$ -space if for each pair at ditanct points  $x, y \in X$ , There exist a Para  $S_{\text{open}}$  sets  $\{G \& H$  such that  $x \in G$  but  $y \notin G$  or  $y \in H$  but  $x \notin H\}$

### **Definition3.2: $T_1$ -Space** (STEVEN A.GAAL,2009)

A topological space  $(X, \pi)$  is said to be Para  $S_{T1}$ -space if for each pair of distinct points  $x, y \in P(X)$ , There exist two Para  $S_{\text{open}}$  set  $G, H$  such that  $x \in G, y \notin G$  and  $x \notin H, y \in H$ .

### **Definition3.3: $T_2$ -Space.** ( STEVEN A.GAAL,2009)

A topological space  $(X, \pi)$  is said to be  $S_{T2}$ -space if for each pair at distancet  $x, y$ ,there exist two distinct para  $S_{\text{open}}$  set  $G, H$  such that  $x \in G, y \in H$  and  $G \cap H = \emptyset$ .

$X = \{a, b, c, d\}$	Para $T_0$ – Space	Para $T_1$ -Space	Para $T_2$ -Space
$\pi_2$	1	0	0
$\pi_7$	1	0	0
$\pi_8$	1	0	0
$\pi_{10}$	1	0	0
$\pi_{12}$	1	1	1
$\pi_{14}$	1	0	0
$\pi_{15}$	1	0	0
$\pi_{18}$	1	0	0
$\pi_{22}$	1	1	1
$\pi_{23}$	1	1	1
$\pi_{25}$	1	0	0
$\pi_{26}$	1	0	0
$\pi_{26}$	1	0	0
$\pi_{27}$	1	0	0
$\pi_{28}$	1	1	1
$\pi_{29}$	1	1	1
$\pi_{30}$	1	1	1
$\pi_{31}$	1	0	0

$\pi_{32}$	1	1	1
$\pi_{33}$	1	1	1

**Note:** In the set which contain 4 elements all 33 topology will be  $T_2$ -Space,  $T_1$ -Space,  $T_0$ -Space

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## پوخته

لەم راپورتىدا كۆمەللىكىمان ھېيە كە سى توخم و ٤ توخم لە خۆ دەگرى ، كە توخمى ٣ ، ٩ توپقۇلوجى بەراوردىكراوى ھېيە وە توخمى ٤ ، ٣٣ توپقۇلوجى ھېيە، ئىمە ھەولۇدەدىن كۆمەلەمى كراوهى

On para semi-open set

بەدەست بەيىنن.

## خلاصە

فى هذا التقرير ، لدينا مجموعة من 3 و 4 عناصر، يحتوي على 9،3 طوبولوجيا قابلة للمقارنة ، وهناك أربعة ، 33 عنصر طوبولوجيا، نحاول الحصول على مجتمع on para semi- sets مفتوح