زانکۆی سەلاحەدین-هەولێر
Salahaddin University-Erbil

On Para β-Open Set in Topological Space

Research Project

Submitted to the department of (Mathematic) in partial fulfillment of the requirement for the degree of BSc. (Mathematics)

By:

Yerivan osman youns

Supervised by:

Assist. Prof. Dr. Nehmat K. Ahmad

April-2024

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: **Assist.Prof.Dr.Nehmat K. Ahmad**

Scientific grade: **Assist. professor**

Data: 8/4/2024

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: **Dr. Rashad Rasheed Haje**

Scientific grade: **Assist. Professor**

Chairman of the Mathematics Department

Date: 8 /4 / 2024

Aknowledgment

I would like to thanks Allah for giving me the power to complete this work And I would like to present my profound thanks to supervisor and lecturer **Assist.Prof.Dr.Nehmat K. Ahmad** for his kind valuable suggestions that assisted me to accomplish this work I would also like to extend my gratitude to the head of mathematic department **Assist.Prof.Dr.Rashad Rashid Haji,** and especially thanks for my family to support me and make me what am I today ,and thanks all my friends

Abstract

In this report we have a set which contain three and four elements X= {a, b, c} And X= {a, b, c, d}.

The set Which containing 3 element has 9 non comparable topology and the set which contains four element has 33 non comparable topology and we try to obtain some type of β-open sets to each such topology.

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Introduction

Throughout this paper, a space means a topological space on which no separation axioms are assumed unless explicitly stated.
Abd-El-Moonsef in 1983 defined the class of β-open set. We recall the following definitions and characterizations. The closure (resp., interior) of a subset A of X is denoted by clA (resp., intA). A subset A of X is said to be semi-open, β-open, A subset A of a topological spaces is said to be β-open set if $A \subseteq$ cl int cl A, In general we applied the following definitions on the set which contains three and four elements to obtain some other concept in General topology.

Chapter one

Definition1.1: Topology (Abd El-Monsef, M.E., Kozae, A.M. and Abu-Gdairi, R.A., 2011)

A topological space is ordered pair (X , π),where X is a set, π a collection of subsets of X satisfying the following properties

1) $X \in \pi$ and $\emptyset \in \pi$

2)U, V$\in$ π implies U∩V$\in$ π

3) {Uα| $\propto \in I$}$\in$ π implies U$\propto$ ϵI U$\propto \in \pi$

The collection π is called topology on X, the pair (X, π) a topological space, the elements of π are said to be open sets with this topology.

Definition 1.2: β−open set (Abd El-Monsef, M.E., Kozae, A.M. and Abu-Gdairi, R.A., 2011)

A subset A of a topological space X is said to be β−open set if A ⊆ cl int cl A.

Definition 1.3: interior (2019-2018 أ.م.د.يوسف يعكوب يوسف و)

Let (X, π), be a topological spaces subset A of X, Int A is the largest open set contained in A.

Definition 1.4: closure (2019-2018 أ.م.د.يوسف يعكوب يوسف و)

Let (X, π), be a topological spaces subset A of X, Cl A is the smallest closed set containing A.

Definition 1.5: minimal β−open set (Shakir, Q.R., 2014)

A proper non−empty β−open subset U at a topological space X is said to be minimal β−open set if any β−open set which is contained in U is ϕ or U, the family of all minimal β−open sets in a topological space X is denoted by M¡nβo(x).

Definition 1.6: maximal β−open set (Shakir, Q.R., 2014)

A proper non−empty β−open set U of a topological space X is said to be maximal β−open set if any β−open set which contains U is X or U. The family of all maximal β−open sets in a topological space X is denoted by Maxβo(x).

Definition 1.7: para β-open sets (Basaraj M. lttanagi and S. S. Benchalli, 2016)

Any open subset U of a topological space X is said to be a para β-open set if it is neither minimal β-open nor maximal β-open set the family of all para β-open sets in a topological space X is denoted by parβo(x).

The topologies which define on the set X= {*a, b, c*} which are different in element and property

P(x) = {ϕ, {*a*}, {*b*}, {*c*}, {*a, b*}, {*a, c*}, {*b, c*}, X}

τ_1 = {ϕ, X}

τ_2 = {ϕ, X, {*a*}}

τ_3 = {ϕ, X, {*a, b*}}

τ_4 = {ϕ, X, {*a*}, {*b, c*}}

τ_5 = {ϕ, X, {*a*}, {*a, b*}}

τ_6 = {ϕ, X, {*a*}, {*b*}, {*a, b*}}

τ_7 = {ϕ, X, {*a*}, {*a, b*}, {a, c}}

τ_8 = {ϕ, X, {*a*}, {*b*}, {*a, b*}, {*a, c*}}

τ_9 = {ϕ, X, {*a*}, {*b*}, {*c*}, {*a, b*}, {*a, c*}, {*b, c*}}

$\tau_1 = \{\phi, X\}$

$\tau_1^c = \{X, \phi\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, X\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_2^c = \{X, \phi, \{b, c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_3^c = \{X, \phi, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, X\}$

$\tau_4 = \{\phi, X, \{a\}, \{b, c\}\}$

$\tau_4^c = \{X, \phi, \{b, c\}, \{a\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, X\}$

$\tau_5 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_5^c = \{X, \phi, \{b, c\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_6 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_6^c = \{X, \phi, \{b, c\}, \{a, c\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$\tau_7^c = \{X, \phi, \{b, c\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$\tau_8^c = \{X, \phi, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x\} = \{X, \{a, b\}, \{a, c\}\}$

$\tau_9 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\tau_9^c = \{X, \phi, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}, \{b, c\}\}$

The topologies which define on the set X = {*a, b, c, d*} which are different in element and property

P(x) = {ϕ, {*a*}, {*b*}, {*c*}, {*d*}, {*a, b*}, {*a, c*}, {*a, d*}, {*b, c*}, {*b, d*}, {*c, d*}, {*a, b, c*}, {*a, b, d*}, {*a, c, d*}, {*b, c, d*}, X}

τ_1 = {ϕ, X}

τ_2 = {ϕ, X, {*a*}}

τ_3 = {ϕ, X, {*a, b*}}

τ_4 = {ϕ, X, {*a, b, c*}}

τ_5 = {ϕ, X, {*a*}, {*b, c, d*}}

τ_6 = {ϕ, X, {*a, b*}, {*c, d*}}

τ_7 = {ϕ, X, {*a*}, {*a, b*}}

τ_8 = {ϕ, X, {*a*}, {*a, b, c*}}

τ_9 = {ϕ, X, {*a, b*}, {*a, b, c*}}

τ_{10} = {ϕ, X, {*a*}, {*a, b*}, {*a, b, c*}}

τ_{11} = {ϕ, X, {*a, b*}, {*a, b, c*}, {*a, b, d*}}

τ_{12} = {ϕ, X, {*a*}, {*b*}, {*a, b*}}

τ_{13} = {ϕ, X, {*a*}, {*b, c*}, {*a, b, c*}}

τ_{14} = {ϕ, X, {*a*}, {*a, b*}, {*a, c, d*}}

τ_{15} = {ϕ, X, {*a*}, {*a, b*}, {*a, c*}, {*a, b, c*}}

τ_{16} = {ϕ, X, {*a*}, {*a, b*}, {*a, b, c*}, {*a, b, d*}}

$\tau_{17} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{18} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{19} = \{\phi, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{20} = \{\phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$\tau_{21} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{22} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{23} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{24} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c. d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_{25} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{26} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$

$\tau_{27} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{28} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{29} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{30} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{31} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{32} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{33} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_1 = \{\phi, X\}$

$\tau_1^c = \{X, \phi\}$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_2^c = \{X, \phi, \{b, c, d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_3^c = \{X, \phi, \{c, d\}\}$

$\beta o(x) = P(X)/\{\{c\}, \{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_4 = \{\phi, X, \{a, b, c\}\}$

$\tau_4^c = \{X, \phi, \{d\}\}$

$\beta o(x) = P(X)/\{d\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$

$\tau_5 = \{\phi, X, \{a\}, \{b, c, d\}\}$

$\tau_5^c = \{X, \phi, \{b, c, d\}, \{a\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\},\{b\},\{c\},\{d\}\}$

$Max\beta o(x) = \{X, \{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$Par\beta o(x) = \{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$

$\tau_6 = \{\phi, X, \{a, b\}, \{c, d\}\}$

$\tau_6^c = \{X, \phi, \{c,d\},\{a,b\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\},\{b\},\{c\},\{d\}\}$

$Max\beta o(x) = \{X, \{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

$Par\beta o(x) = \{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_7^c = \{X, \phi, \{b, c, d\}, \{c, d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{a, b, c\}\}$

$\tau_8^c = \{X, \phi, \{b, c, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_9 = \{\phi, X, \{a, b\}, \{a, b, c\}\}$

$\tau_9^c = \{X, \phi, \{c, d\}, \{d\}\}$

$\beta o(x) = P(X)/\{\{c\}, \{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{10} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{10}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{11} = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{11}^{c} = \{X, \phi, \{c, d\}, \{d\}, \{b\}\}$

$\beta o(x) = P(X)/\{\{c\}, \{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{12} = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_{12}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, b\}\}$

$\beta o(x) = P(X)/\{\{c\}, \{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{13} = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{13}^{c} = \{X, \phi, \{b, c, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = P(X)/\{d\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{14} = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{14}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{15} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$

$\tau_{15}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{16} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{16}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{17} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{17}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = P(X)/\{\{c\}, \{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{18} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{18}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{19} = \{\phi, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{19}^{c} = \{X, \phi, \{a, b, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = P(X)/\{\{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{20} = \{\phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$\tau_{20}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}\}$

$\tau_{21} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{21}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{22} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{22}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{23} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{23}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{24} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c. d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_{24}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, b\}, \{b\}, \{a\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) == \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{25} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{25}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}$

$\tau_{26} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$

$\tau_{26}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Man\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{27} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{27}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{28} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{28}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = P(X)/\{d\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{29} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{29}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, c\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, d\}\}$

$\tau_{30} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{30}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, \{c\}\}$

$\beta o(x) = P(X)/\{\{d\}, \{c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{31} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{31}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{32} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{32}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}\}$

$\tau_{33} = P(X)$

$\tau_{33}^{c} = P(X)$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}$

Chapter two

Definition 2.1: To–space (أ.م.د.يوسف يعكوب يوسف و 2019-2018)

A topological space (X, π)is said to be para β–T_o–space if for each pair at distinct points X, Y ϵ P(x), there exist para β–open sets { G & H such that XЄG but Yϵ/ G or YЄH but Xϵ/H}.

.

Definition 2.2: T_1–space (أ.م.د.يوسف يعكوب يوسف و 2019-2018)

A topological space (X, π)is said to be para β–T_1–space if for each pair of distinct points X, Y Є P(x), there exist two para β–open set G, H such that X Є G, Y ϵ/ G and X ϵ/ H, Y Є H.

Definition 2.3: T_2–space (أ.م.د.يوسف يعكوب يوسف و 2019-2018)

A topological space (X, π)is said to be para β–T_2 –space if for each pair at distinct point X, Y Є X, there exist two distinct para β–open set G, H containing X and Y respectively.

X = {*a*, *b*, *c*, *d*}	Para β−T_o−space	Para β−T_1−space	Para β−T_2−space
τ_1	1	1	1
τ_2	1	0	0
τ_3	1	1	1
τ_4	1	1	1
τ_5	1	1	1
τ_6	1	1	1
τ_7	1	0	0
τ_8	1	0	0
τ_9	1	1	1
τ_{10}	1	0	0
τ_{11}	1	1	1
τ_{12}	1	1	1
τ_{13}	1	1	1
τ_{14}	1	0	0
τ_{15}	1	0	0
τ_{16}	1	0	0
τ_{17}	1	1	1
τ_{18}	1	0	0
τ_{19}	1	1	1
τ_{20}	1	0	0
τ_{21}	1	0	0
τ_{22}	1	0	0
τ_{23}	1	1	1
τ_{24}	1	1	1
τ_{25}	1	0	0
τ_{26}	1	0	0
τ_{27}	1	0	0
τ_{28}	1	1	1
τ_{29}	1	0	0
τ_{30}	1	1	1
τ_{31}	1	0	0
τ_{32}	1	0	0
τ_{33}	1	1	1

References

Ittanagi, B.M. and Benchalli, S.S., 2016. On paraopen sets and maps in topological spaces. *Kyungpook Mathematical Journal*, *56*(1), pp.301-310.

Shakir, Q.R., 2014. Minimal and Maximal Beta Open Sets. *Al-Nahrain Journal of Science*, *17*(1), pp.160-166.

Abd El-Monsef, M.E., Kozae, A.M. and Abu-Gdairi, R.A., 2011. Generalized near rough probability in topological spaces. *Int. J. Contemp. Math. Sciences*, *6*(23), pp.1099-1110.

Nakaoka, F. and Oda, N., 2001. Some applications of minimal open sets. *International journal of mathematics and mathematical sciences*, *27*, pp.471-476.

Nakaoka, F. and Oda, N., 2003. Some properties of maximal open sets. *International Journal of Mathematics and Mathematical Sciences*, *2003*, pp.1331-1340.

Velicko, N.V., 1967. H-closed topological spaces. *Amer. Math. Soc. Transl.*, *78*(20), pp.103-118.

أ.م. د. يوسف يعكوب يوسف و , أ. م. د. رشا ناصر مجيد, 2018-2019. General topology. Baghdad:Baghada: University-Department of Mathematic.

پوختە

لەم راپۆتەدا کۆمەڵێکمان هەیە کە 3 توخم و 4 توخم لە خۆ دەگرێت کە توخمی 3 ,9 تۆپۆلۆجی بەراوردکراوی هەیە وە توخمی 4, 33 تۆپۆلۆجی هەیە, ئێمە هەوڵ دەدەین کۆمەڵەی کراوەی

On para β-open set

بە دەست بهێنین