On Para Pre-Open Set in Topological Space

Research Project

Submitted to the department of (Mathematic) in partial fulfillment of therequirement for the degree of BSc. (Mathematics)

By:

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Supervised By: Nehmat K. Ahmed

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Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor: **Assist. Prof. Dr. Nehmat K. Ahmed**

Scientific grade: Assist. professor

Data: 4/4/2024

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: **Dr. Rashad Rasheed Haje .**

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: 4 /4 / 2024

Acknowledgment

I would like to thanks Allah for giving me the power to complete this work. And I would like to present my profound thanks to supervisor and lecturer **Assist. Prof. Dr. Nehmat K. Ahmad** for his kind valuable suggestions that assisted me to accomplish this work I would also like to extend my gratitude to the head of mathematic department **Assist. Prof. Dr. Rashad Rashid Haji,** and especially thanks for my family to support me and make me what am I today ,and thanks all my friends

Abstract:

In this report we have a set which contain three and four elements X={a,b,c) And x={a,b,c,d}. The set which containing 3 element has 9 non comparable topology and the set which contains four element has 33 non comparable topology and we try to obtain some type of β-*open set*s to each such topology.

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Introduction:

Throughout this paper, a space means a topological space on which no separation axioms are assumed unless explicitly stated. In [2], Abd-El-Moonsef in 1983 defined the class of β-open set. We recall the following definitions and characterizations. The closure (resp., interior) of a subset A of X is denoted by clA (resp., intA). A subset A of X is said to be semi-open [1] β-open [2] A subset A of a topological spaces is said to be β-open if $A \subseteq int\ clintA$, In general we applied the following definitions on the set which contains three and four elements to obtain some other concept in General topology.

Chapter one

Definition1.1:Topology

A topological space is ordered pair (X , π),where X is a set, π a collection of subsets of X satisfying the following properties

1) $X \in \pi$ and $\emptyset \in \pi$

2) U, V $\in \pi$ implies $U \cap V \in \pi$,

3) $\{ U\alpha | \propto \in I\} \in \pi$ implies $\cup_{\propto \in I} U\propto \in \pi$

The collection π is called topology on X, the pair (X, π) a topological space, the elements of π are said to be open sets with this topology.

Definition 1.2: A subset A of a topological space X is said to be β–open set if $A \subseteq cl\ int\ cl\ A$.

Definition 1.3: A proper non–empty β–open subset U at a topological space X is said to be minimal β–open set if any β–open set which is contained in U is ϕ or U, the family of all minimal β–open sets in a topological space X is denoted by Minβo(x).

Definition 1.4: A proper non–empty β–open set U of a topological space X is said to be maximal β–open set if any β–open set which contains U is X or U. The family of all maximal β–open sets in a topological space X is denoted by Maxβo(x).

Definition1.5: (Basaraj M. Ittanagi and S. S. Benchalli, 2016) Any open subset U of a topological space X is said to be a para β -open set if it is neither minimal β -open nor maximal β -open set The family of all para β -open sets in a topological space X is denoted by Par β o(X).

The topologies which define on the set X = {a , b , c}which are different in its element and property

$\tau_1 = \{\phi, X\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_4 = \{\phi, X, \{a\}, \{b, c\}\}$

$\tau_5 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_6 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$\tau_9 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\tau_1 = \{\phi, X\}$

$\tau_1^c = \{X, \phi\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, X\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_2^c = \{X, \phi, \{b, c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_3^c = \{X, \phi, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, X\}$

$\tau_4 = \{\phi, X, \{a\}, \{b, c\}\}$

$\tau_4^c = \{X, \phi, \{b, c\}, \{a\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b\}, \{a, c\}, X\}$

$\tau_5 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_5^c = \{X, \phi, \{b, c\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_6 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_6^c = \{X, \phi, \{b, c\}, \{a, c\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{X, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$\tau_7^c = \{X, \phi, \{b, c\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$Mi\beta o(x) = \{\phi, \{a\}\}$

$Ma\beta o(x) = \{X, \{a, b\}, \{a, c\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$\tau_8^c = \{X, \phi, \{b, c\}, \{a, c\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

$Mi\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Ma\beta o(x\} = \{X, \{a, b\}, \{a, c\}\}$

$\tau_9 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$\tau_9^c = \{X, \phi, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

$Mi\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Ma\beta o(x) = \{X, \{a, b\}, \{a, c\}, \{b, c\}\}$

The topologies which define on the set X= {a,b,c,d}which are different in element and property

$\tau_1 = \{\phi, X\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_4 = \{\phi, X, \{a, b, c\}\}$

$\tau_5 = \{\phi, X, \{a\}, \{b, c, d\}\}$

$\tau_6 = \{\phi, X, \{a, b\}, \{c, d\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{a, b, c\}\}$

$\tau_9 = \{\phi, X, \{a, b\}, \{a, b, c\}\}$

$\tau_{10} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{11} = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{12} = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_{13} = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{14} = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{15} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$

$\tau_{16} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{17} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{18} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{19} = \{\phi, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{20} = \{\phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$\tau_{21} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{22} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{23} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{24} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c. d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_{25} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{26} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$

$\tau_{27} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{28} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{29} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{30} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{31} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{32} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{33} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_1 = \{\phi, X\}$

$\tau_1^c = \{X, \phi\}$

$\beta o(x) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$\text{Max}\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_2 = \{\phi, X, \{a\}\}$

$\tau_2^c = \{X, \phi, \{b, c, d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}\}$

$\text{Max}\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Pa}\beta\text{or}(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_3 = \{\phi, X, \{a, b\}\}$

$\tau_3^c = \{X, \phi, \{c, d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$\text{Max}\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$[i]

$\tau_4 = \{\phi, X, \{a, b, c\}\}$

$\tau_4^c = \{X, \phi, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_5 = \{\phi, X, \{a\}, \{b, c, d\}\}$

$\tau_5^c = \{X, \phi, \{b, c, d\}, \{a\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_6 = \{\phi, X, \{a, b\}, \{c, d\}\}$

$\tau_6^c = \{X, \phi, \{c, d\}, \{a, b\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_7 = \{\phi, X, \{a\}, \{a, b\}\}$

$\tau_7^c = \{X, \phi, \{b, c, d\}, \{c, d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o\ (x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_8 = \{\phi, X, \{a\}, \{a, b, c\}\}$

$\tau_8^c = \{X, \phi, \{b, c, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Pa\beta or(x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_9 = \{\phi, X, \{a, b\}, \{a, b, c\}\}$

$\tau_9^c = \{X, \phi, \{c, d\}, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{10} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{10}^c = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o\ (x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{11} = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{11}^c = \{X, \phi, \{c, d\}, \{d\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{12} = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

$\tau_{12}^c = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{13} = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{13}{}^{c} = \{X, \phi, \{b, c, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$\text{Max}\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{14} = \{\phi, X, \{a\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{14}{}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}\}$

$\text{Max}\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Par}\beta o\ (x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{15} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$

$\tau_{15}{}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}\}$

$\text{Max}\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\text{Par}\beta o\ (x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{16} = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{16}^c = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, X, \{a\}, \{a, b,\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{X, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$Par\beta o\ (x) = \{\{a, b,\}, \{a, c\}, \{a, d\}\}$

$\tau_{17} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$\tau_{17}^c = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{18} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$

$\tau_{18}^c = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Pa\beta or(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{19} = \{\phi, X, \{c\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{19}^{c} = \{X, \phi, \{a, b, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{20} = \{\phi, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$

$\tau_{20}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{a, b\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}\}$

$\tau_{21} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{21}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{22} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{22}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{23} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{23}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{24} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c. d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_{24}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, b\}, \{b\}, \{a\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) == \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{25} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{25}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

$Pa\beta or(x) = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}$

$\tau_{26} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$

$\tau_{26}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{d\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{27} = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{27}^{c} = \{X, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o(x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{28} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$\tau_{28}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$\text{Max}\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

$\tau_{29} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{29}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{a, c\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$\text{Max}\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, X\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{b, d\}\}$

$\tau_{30} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$

$\tau_{30}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, \{c\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Min}\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$\text{Max}\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$\text{Par}\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}\}$

$\tau_{31} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{31}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

$\tau_{32} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$\tau_{32}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$

$\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Min\beta o(x) = \{\phi, \{a\}, \{b\}, \{c\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}\}$

$\tau_{33} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$

$\tau_{33}^{c} = \{X, \phi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{d\}, \{c\}, \{b\}, \{a\}\}$

$\beta o(x) = P(X)$

$Min\beta o(x) == \{\phi, \{a\}, \{b\}, \{c\}, \{d\}\}$

$Max\beta o(x) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$

$Par\beta o\ (x) = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$

Chapter two

Definition 2.1: A topological space (X, C) is said to be para β−T_o−space if for each pair at distinct points X,Y ϵ P(x) , there exist para β − open sets { G & H such that XЄG but Yϵ/ G or YЄH but Xϵ/H}.

Definition 2.2: para β−T_1−space if for each pair of distinct points X, Y Є P(x), there exist two para β−open set G, H such that X Є G, Y ϵ/ G and X ϵ/ H, Y Є H.

Definition 2.3: para β−T_2 −space if for each pair at distinct point X, Y Є X, there exist two distinct para β−open set G, H containing X and Y respectively.

X = {a, b, c, d}	Para β−T_o−space	Para β−T_1−space	Para β−T_2−space
τ_1	1	1	1
τ_2	1	0	0
τ_3	1	1	1
τ_4	1	1	1
τ_5	1	1	1
τ_6	1	1	1
τ_7	1	0	0
τ_8	1	0	0
τ_9	1	1	1
τ_{10}	1	0	0
τ_{11}	1	1	1
τ_{12}	1	1	1
τ_{13}	1	1	1
τ_{14}	1	0	0
τ_{15}	1	0	0
τ_{16}	1	0	0
τ_{17}	1	1	1
τ_{18}	1	0	0
τ_{19}	1	1	1

τ_{20}	1	0	0
τ_{21}	1	0	0
τ_{22}	1	0	0
τ_{23}	1	1	1
τ_{24}	1	1	1
τ_{25}	1	0	0
τ_{26}	1	0	0
τ_{27}	1	0	0
τ_{28}	1	1	1
τ_{29}	1	0	0
τ_{30}	1	1	1
τ_{31}	1	0	0
τ_{32}	1	0	0
τ_{33}	1	1	1

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پوخته

لەم راپۆتەدا کۆمەڵێکمان هەیە کە 3 توخم و 4 توخم لە خۆ دەگرێت کە توخمی 3 ,9 تۆپۆلۆجی بەراوردکراوی هەیە وە توخمی 4, 33 تۆپۆلۆجی هەیە, ئێمە هەوڵ دەدەین کۆمەڵەی کراوەی

On para β-open set

بە دەست بهێنین

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