1- Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.

2- Let X be a topological space, and let A be a subset of X then A is closed if and only if ClA=A.

3- Consider the usual topological space, A is subset of R find the closure of

A=$\{\frac{1}{n}, n \in N\}$.

4- Let τ and $\dot{\tau}$ be topologies for X which have a common base. Show that $\tau = \dot{\tau}$.

5-Define hereditary property. Let U be the usual topology for R. Describe the relativization of U to

the set N of natural numbers.

6-Let X be any set and let $\tau = \{\phi, X, A, B\}$ where A and B are non-empty distinct proper subsets of

X. Find what conditions A and B must satisfy in order that τ may be a topology for X.

7-Show that the discrete space (X,D) is a first countable.

8-Let (X,τ) be a topological space. A subcollection $\Im$ of τ is a base for τ if and only if every

open set can be expressed as the union of member $\Im$.

9- Definition: Let (X,τ) space be a topological , and $A \subset X$, then A is said to be β -open if and

only if $A \subseteq cl \operatorname{int} clA$. Using this definition to fined the β -open set of a topological space

(X,τ) where X={a,b,c,d} and $\tau = \{\phi, X, \{a\}, \{b,c\}, \{a,d\}, \{a,b,c\}\}$.

10: - State and prove lower limit topology.

11 - Show that the usual space $(\boldsymbol{R}, \omega)$ is a second countable.

12 - Let (x,π) be a topological space, a collection β of π is abase for π if and only if every π-open set can be expressed as the union of members of β.

13-Let X be a topological space, and let A be a subset of X. Then A is closed if and only if

$D(A) \subseteq A$.

14 - LetA be any subset of a topological space X. then show that IntA, extA and FrA are disjoint and $X = IntA \cup extA \cup FrA$ Further FrA is a closed set.

15- Let (X, τ) be any topological space and (Y,υ) be the space for which Y={a,b,c} and υ={Φ,Y,{a},{a,c}}, if the function $f: X \longrightarrow Y$ define by $f(x) = a$ for all $x \in X$, then

Discus the continuity, openness, and closeness of f.

16- Show that T_0 is a hereditary property.

17- Show that each singleton subset of a Hausdorff space is a closed.

18-Prove or disprove;

i. The union of an infinite collection of closed sets in a topological space is closed.

ii. The intersection of an arbitrary collection of topologies for X is also a topology.

iii. Every T1-space is a T2-space.

19- Let π and π* be topologies for X, which have a common base β then show that π=π*.

20- Let (X.D) be descried topological space, and let A □□X, find the limit point of A if exist?

21-Let (X, τ) be any topological space, and A be any subset of X .

Prove that ClA=adh(A)={x; each nbd of x intersect A}.

22- Find the topology π for R generated by ρ, where ρ is the collection, of all closed interval

of the form [a,a+1] of length 1.

23- A function f from a space X in the another space Y is continuous, if and only if f(clA) □□clf(A), for every A □□X.

24 - Let $f: \boldsymbol{R} \rightarrow \boldsymbol{R}$ be a function defined as follows: $f(x) = \begin{cases} -1 & if\ x < 0 \\ 1 & if\ \ x \geq 0 \end{cases}$

find whether

f is

$U - U$ *continuous*, $I - U$ *continuous*, $S - U$ *continuous*, $D - U$ *continuous*

25-Prove that the space (X, τ) is T_1-space if and only if every singleton subset of X is closed.

26- Let X be any set and let $\tau = \{\phi, X, A, B\}$ where A and B are non-empty distinct proper subsets of

X. Find what conditions A and B must satisfy in order that τ may be a topology for X.

27- Let (X, τ) be any topological space and (Y,υ) be the space for which Y={a,b,c} and υ={Φ,Y,{a},{a,c}}, if

the function $f: X \longrightarrow Y$ define by $f(x) = a$ for all $x \in X$, then Discus the continuity, openness, and closeness of f.

28-Let τ be the finite-closed topology on a set X. If X has at least 3 distinct clopen subsets, prove that X is a finite set.

29-Let (X, τ) be a topological space and X a non-empty set. Further, let f be a function from X into Y. Put $\tau = \{ f^{-1}(S) \colon S \in \tau\}$. Prove that τ is a topology on X.

30-The set Q of all rational numbers is neither a closed subset of R nor an open subset of R.

31-Let Q be the subset of R consisting of all rational numbers. Prove that $clQ = R$.

33-Let $(X,\tau), (Y,\vartheta)$ *and* (Z,μ) be topological spaces. If $(X,\tau) \cong (Y,\vartheta)$ and $(Y,\vartheta) \cong (Z,\mu)$. Prove that $(X,\tau) \cong (Z,\mu)$.

34- Show that every path-connected space is connected

35-*A subset S of R is open if and if it is a union of open intervals*

36-Show that the discrete metric on a set X induces the discrete topology for X.

37-Let X=R (the set of real numbers) and let τ consist of the empty set and all those subset G of R having the property that $x \in G$ *implies* $-x \in G$, then (R, τ) is a topology for R.

38-Let $X = \{a, b, c\}$ and let $\beta = \{\{a,b\},\{b,c\}, X\}$ Show that if id abase for same topology or not?

39/ Prove or disprove:

1-Let (X,τ) be any topological space and (Y,I) be any indiscrete space, every function of X into Y is
τ-I continuous.

2-Let (X,D) be any Discrete space and (Y,υ) be any topological space, every function of X into Y is
D- υ continuous.

3-Every closed subspace of a lindelof space is lindelof.

40 A topological space (X,τ), is normal iff each open nbd G of a closed set F contains as a subset the
closure of another open nbd H of F.

41/-If x and y are two distinct points of a tychonoff space (X,τ), then there exists a real valued
continuous function f of X such that $f(x) \neq f(y)$.

42- Every compact hausdorff space is a T_3-space

43- a-Let (X,τ) be any topological space and (Y,υ) be the space for which Y={a,b,c} and
υ={Φ,Y,{a},{a,c}}, if the function $f: X \to Y \text{ such that } f(x) = a \ \forall x \in X$.
Discus the continuity, openness, and closeness of the function f.

44- Prove that if $f^{-}[A^{\circ}] \subset (f^{-1}[A])^{\circ} \quad \forall A \in Y$, then the function $f: X \to Y$ is continuous

45- Let X be a non empty set, and let $\tau = \{\emptyset, X, A, B\}$, where A and B are non-empty disjoint
proper subsets of X. Find what condition A and B must satisfy in order that τ may be a
topology for X.

46- Prove that a subset of a topological space is open if and only if it is neighborhood of
each of its points.

47- Let (X, τ) be a topological space, and let A be a subset of X. Show that A° equals the set of all those points of A which are not limit points of A^c.

48- Let $X = \{a, b, c, d, e\}$ and let $\rho = \{\{a, b\}, \{b, d\}, \{a, c, e\}\}$, find the topology generated by ρ.

49 -Show that the usual space $(\boldsymbol{R}, \omega)$ is a second countable.

50- Let $X = \{0,1,2\}$ and $\tau = \{\emptyset, X, \{0\}, \{0,1\}\}$ a topology on X. Let f be a function on
(X, τ) into itself such that $f(1) = 0$ and $f(2) = 1$. What Value of $f(0)$ make f
continuous?

51- Let (X, τ) be a topological space , and (Y, ϑ) be a hausdorff space. If f, g are continuous
functions of X into Y. Show that the set $A = \{x \in X : f(x) = g(x)\}$ is a closed subset of
X.

52- Show that a topological space (X, τ) is a T_1- space if and only if every singleton subset
of X is closed.

53-Consider the Co-finite topological space (X, τ), and find the closure of any sub set A of X.

54--Show that the discrete space (X,D) is a first countable.

55-Let (X,τ) be a topological space. A sub-collection $\Im$ of τ is a base for τ if and only if every

open set can be expressed as the union of member $\Im$.

56- Let U be the usual topology for R. Describe the relativization of U to the set N of natural

numbers.

57: a- Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ and let $N(a) = \{\{a\}, \{a, c\}\}$, $N(b) = \{\{b\}, \{b, c\}\}$, $N(c) = \{X\}$

i-If τ is the topology for X induced by $N(a)$, $N(b)$ and $N(c)$, then find all $\tau - open\ set$.

ii- Is (X, τ) a door space ?

58- Let X be a topological space, and let A be a subset of X. Prove that $\bar{A}$ =A$\cup$ $D(A)$

59: - Let A be a subset of a topological space X, then show that a point x in X

is an exterior point of A iff x is not an adherent point of A .

60- Let $X = \{a, b, c, d, e\}$ and let $\rho = \{\{a, b\}, \{b, d\}, \{a, c, e\}\}$, find the topology generated by ρ.

61 – Show that every compact subset A of a Hausdorff space X is closed.

62- show that the function f:R→R defined by

$$F(x)=\begin{cases} X & \text{where } x<1 \\ 1 & \text{where } x\in[1,2] \\ X^2/4 & \text{where } x>2 \end{cases}$$

Discuses the continuity and openness of f .

63- Show that a topological space $(X,\tau\}$ is regular if and only if for each point x in X and

every neighborhood N of x, there exists a neighborhood M of x such that $\overline{M} \subseteq N$

64- Show that if normality is a topological property?

65/ Prove or disprove: (

1-Let (X,τ) be any topological space and (Y,I) be any indiscrete space, every function of X into Y is τ-I continuous.

2-Let (X,D) be any topological space and (Y,υ) be any indiscrete space, every function of X into Y is D- υ continuous.

3-Every closed subspace of a lindelof space is lindelof.

66/ A topological space (X,τ), is normal iff each open nbd G of a closed set F contains as a subset the closure of another
open nbd H of F.

67/ a-If x and y are two distinct points of a tychonoff space (X,τ), then there exists a real valued continuous function f
of X such that $f(x) \neq f(y)$.

68- Every compact hausdorff space is a T_3-space

69\Prove or disprove:

1-Let (X,τ) be any topological space and (Y,I) be any indiscrete space, every function of X into Y
is τ-I continuous.

2-Let (X,D) be any Discrete space and (Y,υ) be any topological space, every function of X into Y
is D- υ continuous.

3-Every closed subspace of a compact space is compact. (30 marks)

70\ A topological space X is disconnected if and only if there exists a non empty proper subset
which is both open and closed. (10 marks)