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الأتحاد الدولي للمبدعين في العراق
المؤتمر الدولي السادس للدراسات العلمية والأنسانية

On Permutation G-sets with their Basic Characteristics

Abstract. The behavior of permutation G-set, permutation G-equivariant map, permutation retraction, permutation sub-object, permutation quotient object, and permutation co-retractions in the category of permutation G-sets is investigated in this research. In addition, various unique objects are described, resulting in the classification of permutation G-sets as permutation for balanced / well-powered / co-well-powered.

Keywords: Symmetric groups, permutation, cycles, monomorphisms
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1. Introduction

In 2014, the permutations in symmetric groups are studied by Mahmood [1] to consider non-classical set. Next, the permutations in symmetric groups are studied as non-calsical sets [2-5], where in the last years the non-classical sets are became so important because they have many applications in our life like, nano sets [6], fuzzy sets [7], soft sets [8], neutrosophic sets [9], and others. Eilenberg and MacLane [10] proposed categories, functors, and natural transformations in 1945. However, it was not evident in this work that the principles of category theory would be more than a convenient language, so it stayed unpublished for about fifteen years. In 1969, Lawvere proposed an axiomatization of the category of categories [11]. In 1972, the concept was developed and applied in a variety of ways [12]. Finally, category theory has occupied a significant role not only in contemporary mathematics but also in theoretical computer science, where it has deep roots and contributes to the development of programming semantics and new logical systems, among other things ([13], [14]). G-set is, in reality, a permutation on a set extended to a group action on a set. G-sets are derived from the concept of groups with operators, which was explored in [15] and has been used to define permutation orbits and prove Sylow theorems for groups. We investigate various features of a structural category, i.e. a category of permutation G-sets, in this paper. We will merely cover the fundamental ideas of permutation G-sets here to avoid unnecessary length.

2. Preliminaries

Here, we will recall basic ideas and results that are necessary in this research.

Definition 2.1: [1]

For any permutation $\beta=\prod_{i=1}^{c(\beta)}\delta_i$ in a symmetric group S_n, where $\{\delta_i\}_{i=1}^{c(\beta)}$ is a composite of pairwise disjoint cycles $\{\delta_i\}_{i=1}^{c(\beta)}$ where $\delta_i=(t_1^i,t_2^i,...,t_{\alpha_i}^i)$,$1\le i\le c(\beta)$, for some $1\le \alpha_i, c(\beta)\le n$. If $\delta=(t_1,t_2,...,t_k)$ is $k-$cycle in S_n, we define $\beta-$set as $\delta^\beta=\{t_1,t_2,...,t_k\}$ and is called $\beta-$set of cycle λ. So the $\beta-$sets of $\{\delta_i\}_{i=1}^{c(\beta)}$ are defined by $\{\delta_i^\beta=\{t_1^i,t_2^i,...,t_{\alpha_i}^i\}\,|\,1\le i\le c(\beta)\}$.

Definition 2.2:[1]

Let β be permutation in S_n with $\beta=(t_1,t_2,...,t_n)$. We say β has single $\beta-$set $\{a\}$, where $a=\delta^\beta$ and $|\delta^\beta|=n$.

Definition 2.3[1]

Let $\delta_i^\beta,\delta_j^\beta\subseteq\Omega=\{1,2,3,...,n\}$ be two $\beta-$sets . Define $\wedge$ and $\vee$ by:

$$\delta_i^\beta \wedge \delta_j^\beta = \begin{cases} \delta_i^\beta, \text{ if } \sum_{k=1}^{\sigma} t_k^i < \sum_{k=1}^{\upsilon} t_k^j \\ \delta_j^\beta, \text{ if } \sum_{k=1}^{\sigma} t_k^i > \sum_{k=1}^{\upsilon} t_k^j \\ \delta^\beta, \text{ if } \delta_i^\beta = \delta_j^\beta = \delta^\beta \\ \phi, \text{if } \delta_i^\beta \,\&\, \delta_j^\beta \text{ are disjoint} \end{cases} \quad \text{and} \quad \delta_i^\beta \vee \delta_j^\beta = \begin{cases} \delta_i^\beta, \text{ if } \sum_{k=1}^{\sigma} t_k^i > \sum_{k=1}^{\upsilon} t_k^j \\ \delta_j^\beta, \text{ if } \sum_{k=1}^{\sigma} t_k^i < \sum_{k=1}^{\upsilon} t_k^j \\ \delta^\beta, \text{ if } \delta_i^\beta = \delta_j^\beta = \delta^\beta \\ \Omega, \text{if } \delta_i^\beta \,\&\, \delta_j^\beta \text{ are disjoint} \end{cases}$$

3. Permutation G-set

In this section, we will give new notion is called a permutation G-sets and study some of their basic properties.

Definition 3.1: Let $X = \{\delta_i^\beta\}_{i=1}^{c(\beta)}$ be a collection of $\beta-$sets, where β is a permutation in symmetric group $G = S_n$. Then X is said to be a permutation G-set if there exists a mapping $* : X \times G \longrightarrow G$ such that the following conditions:

a) $\delta_i^\beta * ab = \left(\delta_i^\beta * a\right) * b,$

b) $\delta_i^\beta * e = \delta_i$, for all $a, b \in G$ and $\delta_i^\beta \in X$, where e is the identity of G and it is denoted by X_G^*.

Example 3.2:

Let (S_{12}, o) be a symmetric group and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10 \end{pmatrix}$ be a permutation in S_{12}. Since $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10 \end{pmatrix} =$

$(1\ 3)\ (2)\ (4)(5)\ (6)\ (7)\ (8)(9)(11)(10\ 12)$. Therefore, we have $X = \{\delta_i^\beta\}_{i=1}^{10} = \{\{1,3\},\{2\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{11\},\{10,12\}\}$. Define $*: X \times S_{12} \longrightarrow S_{12}$ by $*\left(\delta_i^\beta, a\right) = \delta_i\ o\ a$, where δ_i is a cycle for δ_i^β and $a \in S_{12}$. Then $X_{S_{12}}^o$ is a permutation G-set.

Definition 3.3: Let X_G^* be a permutation G-set. Then a subset A of X_G^* is called a permutation G-subset of X if A_G^* is also a permutation G-set.

Definition 3.4: Define $\# : X \times G \longrightarrow X$ by $\#\left(\delta_i^\beta, a\right) = \delta_i^\beta \vee \{\bigwedge_{i=1}^{c(a)} \lambda_i^a\}$, where $a = \prod_{i=1}^{c(a)} \lambda_i$ is a permutation in group G and each λ_i^a is $a-$set of cycle λ_i, for all $1 \le i \le c(a)$.

Definition 3.5: Let X_G^* and Y_G^* be two permutation G-sets. Then a mapping $f : X_G^* \longrightarrow Y_G^*$ is called a permutation G-equivarient map if $f(x\#a) = f(x)\#a$ for all $a \in G$ and $x \in X_G^*$.

Remark 3.6:

a) Let $\alpha = \{\delta^{\beta}\}$ be a single set, we define a mapping ϕ: $\{\delta^{\beta}\} \times G \longrightarrow G$ by $\phi\left(\delta^{\beta}, m\right) = \delta$ for each $m \in G$. So, α_G^* is a permutation G-set.
b) The empty set is a permutation G-set and dented by $\emptyset_G^*$.
c) The image of a permutation G-equivariant is a permutation G-set.
d) The composition of two permutation G-equivariants is a permutation G-equivariant, i.e. if $f: X_G^* \longrightarrow Y_G^*$, and $g: Y_G^* \longrightarrow Z_G^*$, then $(g \circ f)(x\#a) = ((g \circ f)(x))\#a$.
e) The identity map $I_{X_G^*}: X_G^* \longrightarrow X_G^*$ is a permutation G-equivariant.

Example 3.7: Let $X = \{\delta_1^{\beta}, \delta_2^{\beta}\}$ be a family of β–sets and $g: X \times G \longrightarrow G$ be a mapping, where X is a subset of group G and h defined by

$$g(x,m) = \begin{cases} \delta_1 & if \quad x = \delta_1^{\beta} \\ \delta_2 & if \quad x = \delta_2^{\beta} \end{cases}$$

for all $m \in G$, $x \in X$. Then X is a permutation G-set.

Also, define $h: X \times G \longrightarrow X$ by

$$h(x,m) = \begin{cases} \delta_1^{\beta} & if \quad x = \delta_1^{\beta} \\ \delta_2^{\beta} & if \quad x = \delta_2^{\beta} \end{cases}$$

for all $m \in G$, $x \in X$. Then h is a permutation G-equivariant.

Lemma 3.8: Assume that X_G^* and Y_G^* are two permutation G-sets. Then,

a) $X_G^* \wedge Y_G^*$ is a permutation G-set,
b) $X_G^* \times Y_G^*$permutation G-set,
c) Disjoint union of X_G^* and Y_G^* is a permutation G-set.

Note: Since the composition of permutation G-equivariant is again a permutation G-equivariant and every identity mapping is a permutation G-equivariant, therefore we can construct a category by taking the class of all permutation G-sets as the class of objects of such category and the class of all permutation G-equivariant as the class of permutation equivariant of the category. It is called the category of permutation G-sets.

Proposition 3.9: A permutation G-equivariant $p: X_G^* \longrightarrow Y_G^*$ of permutation G-sets is injective if and only if it is left cancellable.

Proof: Suppose that $p: X_G^* \longrightarrow Y_G^*$ is an injective permutation equivariant in permutation $\mathbb{G}$-sets. For any Z_G^* permutation G-set, suppose there are two permutation G-equivariants $h, k: Z_G^* \longrightarrow X_G^*$ such that $p \circ h = p \circ k$. Then for any $z \in Z_G^*$, we have

$$(p \circ h)(z) = (p \circ k)(z)$$

Then, $p\big(h(z)\big) = p\big(k(z)\big)$

Therefore, $h(z) = k(z)$ (as p is injective)

Hence, $h = k$

Implies that p is left cancellable.

Conversely, let p be left cancellable and $p(x_1) = p(x_2)$ for $x_1, x_2 \in X_G^*$. Consider the set $C_G^* = \{c\}$ and define two permutation equivariants $h, k: C_G^* \longrightarrow X_G^*$ with $h(c) = x_1$ and $h(c) = x_2$. So, h and k are permutation G-equivariants. Hence we get

$$C_G^* \xrightarrow{h} X_G^* \xrightarrow{p} Y_G^* = C_G^* \xrightarrow{k} X_G^* \xrightarrow{p} Y_G^*$$

Then $p \circ h = p \circ k$

Therefore, $h = k$ (as p is left cancellable)

Hence, $h(c) = k(c), \forall c \in C_G^*$

Which implies that $x_1 = x_2$ and hence p is injective.

Corollary 3.10: A permutation equivariant in the category $\mathbb{G}$-sets is a monomorphism if and only if it is an injective.

Proof: By proposition (3.9), the proof is obvious.

Proposition 3.11: A permutation equivariant $p: X_G^* \longrightarrow Y_G^*$ in permutation $\mathbb{G}$-sets is surjective if and only if it is right cancellable.

Proof: Assume $p: X_G^* \longrightarrow Y_G^*$ is a surjective permutation equivariant in $\mathbb{G}$-sets. For any Z_G^* permutation G-set, suppose there are two permutation equivariants $h, k: Y_G^* \longrightarrow Z_G^*$ in permutation $\mathbb{G}$-sets with $h \circ p = k \circ p$. Since p is surjective, for every $y \in Y_G^*$ there exists $x \in X_G^*$ satisfies $y = p(x)$. Thus we get

$$\begin{aligned} h(y) &= h\big(p(x)\big) \\ &= (h \circ p)(x) \\ &= (k \circ p)(x) \\ &= k\big(p(x)\big) \end{aligned}$$

$= k(y), \forall y \in Y_G^*$

implying that $h = k$, therefore p is right cancellable.

Conversely, assume that p is right cancellable. Define two permutation G-equivariants $h, k: Y_G^* \longrightarrow \{\delta_1^\beta, \delta_2^\beta\} \vee Im(p)$, by

$h(y) = \begin{cases} y & if\ y \in Im(p) \\ \delta_1^\beta & if\ y \notin Im(p) \end{cases}$ and $k(y) = \begin{cases} y & if\ y \in Im(p) \\ \delta_2^\beta & if\ y \notin Im(p) \end{cases}$, where β is a member in G.

By Example 3.7, and Remark 3.6-(c), we get $\{\delta_1^{\beta}, \delta_2^{\beta}\}$ and $Im(p)$are permutation G-sets respectively.

In other side to show that h and k are permutation G-equivariant, we show that of $y \in Im(p)$, then $y\#a \in Im(p)$, $\forall a \in G$. To substantiate this, let $y \in Im(p)$, then

$p(x) = y$ for some $x \in X_G^*$

Thus, $$p(x)\#a = y\#a$$

Hence, $$p(x\#a) = y\#a$$

Which yields $y\#a \in Im(p)$. Thus we have

$$h(y\#a) = y\#a$$
$$= h(y)\#a$$

Next, we show that if $y \notin Im(p)$, then $y\#a \notin Im(p)$ for all $a \in G$. Suppose, on contrary that $y \notin Im(p)$ that is to say $y\#a \in Im(p)$, then

$$p(x) = y\#a\ , \quad \text{for some } x \in X_G^*$$

Thus, $$\big(p(x)\big)\#a^{-1} = y$$

So, $$p(x\#a^{-1}) = y$$

Implying that $y \in Im(p)$ which is a contradiction. Thus,

$$h(y\#a) = \delta_1^{\beta}$$

$$= \delta_1^{\beta}\ \#a \qquad \text{(from Example 3.7)}$$

$$= \big(h(y)\big)\#a.$$

Therefore, h is a permutation G-equivariant. Similarly, we can show that k is also a permutation G-equivariant. Hence h, k are elements of permutation G-sets.

Now, for any $x \in X_G^*$, we have

$$(h \circ p)(x) = h\big(p(x)\big)$$

$$= p(x) \qquad \text{(from definition of } h\text{)}$$

$$= k\big(p(x)\big) \qquad \text{(from definition of } k\text{)}$$

Implies $h \circ p = k \circ p$ which gives $h = k$ (since p is right cancellable). Suppose $p: X_G^* \longrightarrow Y_G^*$ is not surjective, then there exists some $y \in Y_G^*$ such that $y \notin Im(\alpha)$. Thus, we have $\delta_1^{\beta} = h(y) = k(y) = \delta_1^{\beta}$ which is a contradiction and the results follows.

Corollary 3.12: A permutation G-equivariant in the category G-sets is an epimorphism if and only if it is surjective.

Definition 3.13: A permutation equivariant $p: X_G^* \longrightarrow Y_G^*$ in permutation G-sets is called permutation co-retraction (permutation section) if and only if there exists a permutation equivariants $\vartheta: Y_G^* \longrightarrow X_G^*$ in permutation $\mathbb{G}$-sets such that $\vartheta \circ \alpha = I_{X_G^*}$.

Definition 3.14: A permutation equivariant $p: X_G^* \longrightarrow Y_G^*$ in permutation G-sets is called permutation retraction if and only if there exists a permutation equivariants $\vartheta: Y_G^* \longrightarrow X_G^*$ in permutation $\mathbb{G}$-sets such that $\alpha \circ \vartheta = I_{Y_G^*}$.

Theorem 3.15: A permutation equivariant $p: X_G^* \longrightarrow Y_G^*$ in permutation G-sets is a monomorphism if and only if it is a permutation section(permutation co-retraction).

Proof: Let X be a permutation G-set with a fixed element $w \in X_G^*$ such that $aw = w$ for all $a \in G$ and let $p: X_G^* \longrightarrow Y_G^*$ be a monomorphism in permutation $\mathbb{G}$-sets. For any $y \in Y_G^*$, define a mapping $\vartheta: Y_G^* \longrightarrow X_G^*$ by

$$\vartheta(y) = \begin{cases} x & \text{if } \; y \in Im(p) \text{ and } p(x) = y \text{ for some } x \in X_G^* \\ w & \qquad\qquad\qquad otherwise. \end{cases}$$

To show that ϑ is well defined, suppose $y = y'$ for all $y, y' \in Y_G^*$. Then either both $y, y' \in Im(p)$ or both $y, y' \notin Im(p)$. If $y, y' \notin Im(p)$, then $\vartheta(y) = w = \vartheta(y')$.

Suppose, $y, y' \in Im(p)$, then there exist unique $x, x' \in X_G^*$ such that $p(x) = y$ and $p(x') = y'$ implying thereby $\vartheta(y) = x$ and $\vartheta(y') = x'$. Then ϑ is well defined, for if

$$y = y'$$

Therefore, $$p(x) = p(x')$$

Then, $$x = x' \qquad \text{(since } p \text{ is injective)}$$

Thus, $$\vartheta(y) = \vartheta(y').$$

In order to prove that ϑ is a permutation G-equivariant, we show that if $y \notin Im(p)$, then $y'\#a \notin Im(p)$ for all $a \in G$. Suppose on a contrary note that $y' \notin Im(p)$ implying thereby $y \in Im(p)$ which in turn yields

$$p(x) = y\#a, \text{ for some } x \in X_G^*$$

Thus, $$\big(p(x)\big)\#a^{-1} = y$$

Thus, $$p(x\#a^{-1}) = y$$

Which implies that $y \in Im(p)$ a contradiction.

Therefore, we have

$$\begin{aligned} \vartheta(y\#a\ ) &= w \\ &= w\#a \\ &= \big(\vartheta(y)\big)\#a, \qquad \text{for all } y' \notin Im(p) \end{aligned}$$

Again we show that if $y \in Im(p)$, then $y\#a \in Im(p)$ for all $a \in G$. If $y \in Im(p)$, then

$$p(x) = y, \text{ for some } x \in X_G^*$$

Then, $\big(p(x)\big)\#a = y\#a$

Thus, $p(x\#a) = y\#a, \text{ for all } y\#a \in Im(p)$

Which implies $y\#a \in Im(p)$.

Now, if $y \in Im(p)$, then we have

$$p(x) = y\ , \text{ for some } x \in X_G^*$$

Then, $\big(p(x)\big)\#a = y\#a$

Thus, $p(x\#a) = y\#a, \text{ for all } y\#a \in Im(p)$

Implying thereby $\vartheta(y\#a) = x\#a = \big(\vartheta(y)\big)\#a$ which means that ϑ is a permutation G-equivariant.

Lastly, we show that $\vartheta \circ p = I_{X_G^*}$.

Let $x' \in X_G^*$ and $p(x') = y'$ for some $y' \in Y_G^*$. Then $\vartheta(y') = x'$ by definition of ϑ. Thus, we have

$$\begin{aligned}(\vartheta \circ p)(x') &= \vartheta\big(p(x')\big)\\ &= \vartheta(y')\\ &= x'\\ &= I_{X_G^*}(x'), \qquad \text{for all } x' \in X_G^*\end{aligned}$$

Which gives $\vartheta \circ p = I_{X_G^*}$ and henceforth α is a permutation section.

Conversely, suppose that $p: X_G^* \longrightarrow Y_G^*$is a permutation section, then there exists a permutation equivariant $\vartheta: Y_G^* \longrightarrow X_G^*$ such that $\vartheta \circ p = I_{X_G^*}$ which means that p is an injective and by Corollary 3.12, it is a monomorphism.

Proposition 3.16: A permutation equivariant $\alpha: X_G^* \longrightarrow Y_G^*$ in permutation $\mathbb{G}$-sets is an epimorphism if and only if it is a permutation retraction.

Proof: Assume $p: X_G^* \longrightarrow Y_G^*$ is an epimorphism in permutation category G-sets. Then for every $y \in Y_G^*$ there exists $x \in X_G^*$ such that $p(x) = y$. For each $y \in Y_G^*$ choose by the axiom of choice and fix such an element x, say x_y, where $x_y \in p^{-1}(y)$. Therefore, we define a mapping $q: Y_G^* \longrightarrow X_G^*$ by $q(y) = x_y$ for all $y \in Y_G^*$.

We show that q is a permutation G-equivariant.

Since, $x_y \in p^{-1}(y)$

Hence, $p(x_y) = y$

Thus, $p(x_y)\#a = y\#a,\quad$ for all $a \in G$

So, $p(x_y\#a) = y\#a$

Then, $x_y\#a \in p^{-1}(y\#a)$

Thus, $q(y\#a) = x_y\#a$

Therefore, $q(y\#a) = \big(q(y)\big)\#a$

Which show that q is a permutation equivariant in G-sets.

Next, for any $y \in Y_G^*$, we have

$$\begin{aligned}(p \circ q)(y) &= p\big(q(y)\big)\\ &= p(x_y)\\ &= y\end{aligned}$$

Which implies that $p \circ q = I_{Y_G^*}$ and so p is a permutation retraction.

Conversely, suppose that $p: X_G^* \longrightarrow Y_G^*$ is a permutation retraction, then there exists a permutation equivariant $\vartheta: Y_G^* \longrightarrow X_G^*$ such that $p \circ \vartheta = I_{Y_G^*}$ which means that p is surjective and from Corollary 3.12, it is a epimorphism.

Proposition 3.17: The permutation category G-sets is balanced.

Proof: Since in the permutation category G-sets, every bimorphism is an isomorphism, therefore the permutation G-sets is balanced.

Definition 3.18: Let X_G^* be a permutation G-set. Then, a permutation G-subset Y_G^* of X_G^* together with inclusion equivariant $i: Y_G^* \longrightarrow X_G^*$ is called the permutation sub-object of X_G^* in permutation G-sets.

Definition 3.19: Let X_G^* be a permutation G-set and let $\backsim_G$ be a permutation G-equivalence relation on X_G^*. Then, the quotient set $X_G^*/\backsim_G$ together with natural projection $p: X_G^* \longrightarrow X_G^*/\backsim_G$ is called a permutation quotient object of X_G^* in permutation $\mathbb{G}$-sets.

Trivially, $\emptyset_G^*$ forms a permutation G-set, that is., $\emptyset_G^*$ in permutation G-sets and for any other object $X_G^* \in G$-sets, there is only one equivariant from $\emptyset_G^*$ to X_G^* with no assignment, that is, $Hom(\emptyset_G^*, X_G^*)$ is singleton. Thus, we have the following proposition:

Proposition 3.20: The permutation category $\mathbb{G}$-sets has initial object.

Also, from Remark 3.6, every singleton set $\{w\}$ forms a permutation G-set, that is $\{w\} \in$ $\mathbb{G}$-sets and for any other object $X_G^* \in G$-sets, there is only one equivariant from X_G^* to $\{w\}$, that is, $Hom(X_G^*, \{w\})$ is singleton. Thus, we have the following proposition:

Proposition 3.21: The permutation category G-sets has terminal object.

Remark 3.22: the permutation category G-sets has no zero object.

Proposition 3.23: The permutation category 𝔾-sets is well powered.

Proof: Since for any permutation G-set X_G^*, the collection of all sub-objects of X_G^* is permutation equivariant to the collection of all subsets $P(X_G^*)$, the power set of X. But $P(X_G^*)$ is a set. Hence, G-sets is well powered.

Proposition 3.24: The permutation category 𝔾-sets is co-well powered.

4. Conclusion

Some new notions of non-classical sets by using permutation sets in symmetric group are given and discussed their basic properties. In future work, we will use other classes of non-classical sets to investigate and study new classes of G-sets. Moreover, we will study permutation sets in other groups like Alternating, Dihedral, Mathieu, and others.

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