Question bank of ring

Q1/ Let R be a ring with unity 1. Show that $(-1)a = -a$ for all $a \in R$.

Q2/ Find all units, zero-divisors, and nilpotent elements in the rings $Z \times Z$, $Z_3 \times Z_3$, $Z \times Q$and $Z_4 \times Z_6$.

Q3/ (a) Show that the multiplicative identity in a ring with unity R is unique.

(b) Suppose that R is a ring with unity and that $a \in R$ is a unit of R. Show that the multiplicative inverse of a is unique.

Q4/ Determine the center of the ring $M_2(R)$.

Q5/ Is Z_2 a subring of Z_6? Is $3Z_6$ a subring of Z_6?

Q6/ In Z_{24}, (a) find all nilpotent elements;

(b) An element $a \in R$ is called idempotent if $a^2 = a$. Find all idempotent elements.

Q7/ Find a commutative ring with zero divisors a, b such that $a + b$ is not a zero divisor and $a + b \neq 0$.

Q8/ Consider the equation $x^2 - 5x + 6 = 0$. Find all solutions of this equation in Z_7, Z_8, Z_{12} and Z_{14} .

Q9/ Let R be a commutative ring. Show that $(a - b)^2 = a^2 - 2ab + b^2, for\ all\ a, b \in R$.

Q10/ Let R be an ideal of a ring R. Prove that the quotient ring R/I is a commutative ring if and only if $ab - ba \in I\ for\ all\ a, b \in R$.

Q11/ Let I be an ideal of a commutative ring R. Define the annihilator of I to be the set

$ann(I) = \{r \in R \mid ra = 0 for\ all\ a \in I\}$. Prove that $ann(I)$ is an ideal of R.
In the ring Z_{12}, find $ann(I)$.

Q12/ Let $(R, +, .)$ be a ring which has the property that $a^2 = a\ for\ every\ a \in R$. Prove that $(R, +, .)$ is a commutative ring. [Hint: First show $a + a = 0, for\ any\ a \in R$.].

Q13/ Let $(R, +, .)$ be an arbitrary ring. In R define a new binary operation * by $a * b = a.b + b.a\ for\ all\ a, b \in R$. Show that $(R, +, *)$ is a commutative ring.

Q14/ Let $(I, +, .)$ be an ideal of the $(R, +, .)$. Define $C(I)$ to be the set

$C(I) = \{r \in R \mid ra - a.r \in I, for\ all\ a \in R\}$. Prove that $C(I)$ is a subring of R.

Q15/ If A and B are ideals of a ring, show that the sum of A and B, $A + B = \{a + b : a \in A, b \in B\}$ is an ideal.

Q16/ Describe the quotient rings in $Z/4Z$, $Z_{12}/(3)$, $2Z/8Z$ and $Z \times Z/2Z \times \{0\}$.