University of Salahadden-Erbil
School of Engineering
Electrical Department
2022-2023

Circuit analysis I (2113)

Lecture 3

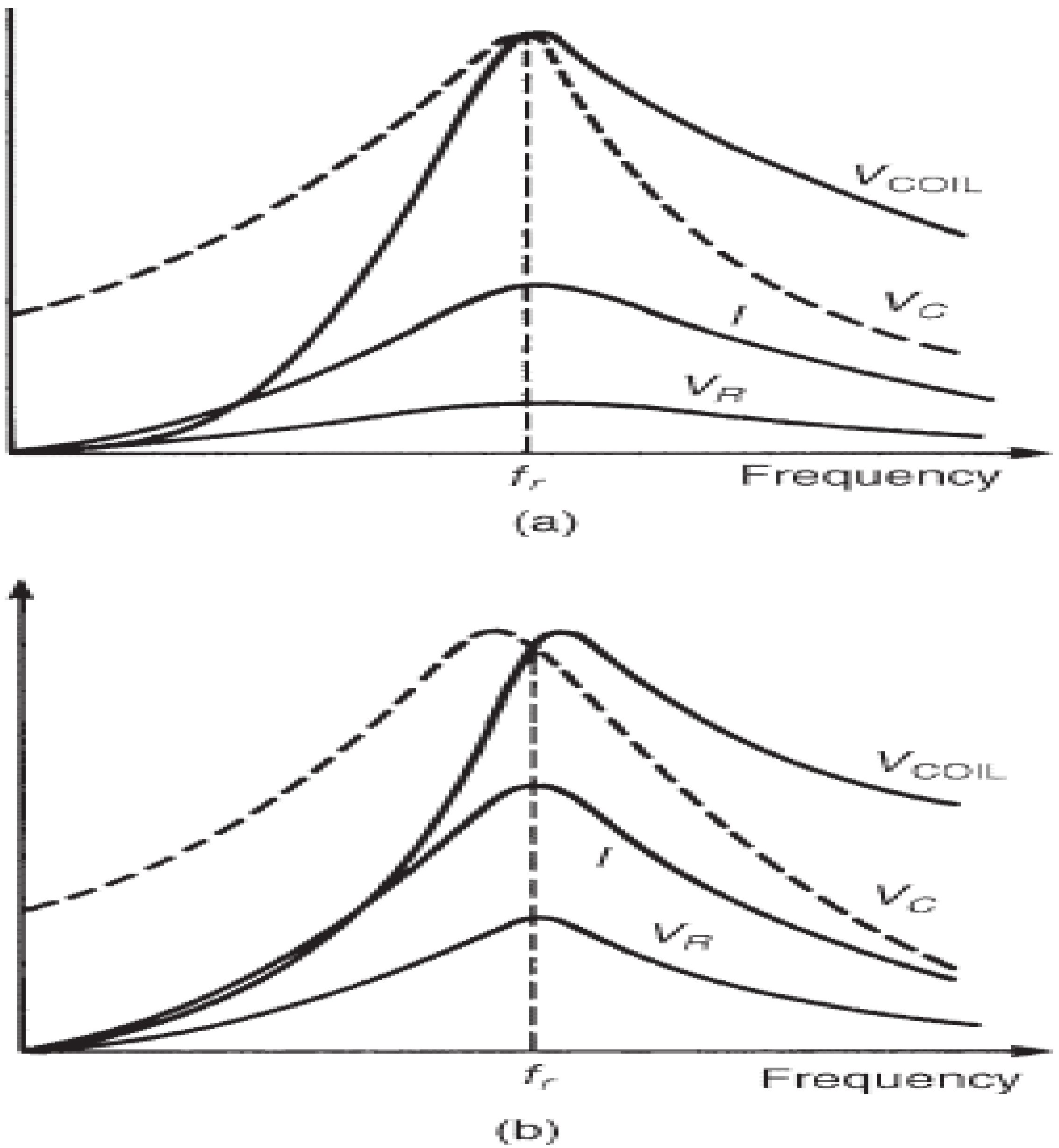
By

Asst. Lect. Niyaz O. Ali

Voltage magnification

- $V_{Cm} = \frac{QV_S}{\sqrt{[1-(\frac{1}{2Q})^2}}$
- $f_{rc} = \sqrt{(1 - \frac{1}{2Q^2})}$

- $V_{Lm} = \frac{QV_S}{\sqrt{[1-(\frac{1}{2Q})^2}}$
- $f_{rL} = \sqrt{(1 + \frac{1}{2Q^2})}$



EX: A series L−R−C circuit has a sinusoidal input voltage of maximum value 12 V. If inductance, L = 20 mH, resistance, R = 80 and capacitance, C = 400 nF, determine

- a) the resonant frequency,
- b) the value of the p.d. across the capacitor at the

resonant frequency,

- c) the frequency at which the p.d. across the capacitor

is a maximum

- d) the value of the maximum voltage across the capacitor.
- Solution

(a) The resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{(LC)}}$$

$$= \frac{1}{2\pi\sqrt{[(20\times10^{-3})(400\times10^{-9})]}}$$

$$= \mathbf{1779.4\,Hz}$$

(b) $V_C = QV$ and

$$Q = \frac{\omega_r L}{R}\left(\text{or } \frac{1}{\omega_r CR} \text{ or } \frac{1}{R}\sqrt{\frac{L}{C}}\right)$$

Hence $Q = \dfrac{(2\pi\,1779.4)(20\times10^{-3})}{80} = 2.80$

Thus $V_C = QV = (2.80)(12) = \mathbf{33.60\,V}$

(c) From equation (7), the frequency f at which V_C is a maximum value,

$$f = f_r\sqrt{\left(1-\frac{1}{2Q^2}\right)}$$

$$= (1779.4)\sqrt{\left(1-\frac{1}{2(2.80)^2}\right)}$$

$$= \mathbf{1721.7\,Hz}$$

(d) From equation (9), the maximum value of the p.d. across the capacitor is given by:

$$V_{C_m} = \frac{QV}{\sqrt{\left[1-\left(\frac{1}{2Q}\right)^2\right]}}$$

$$= \frac{(2.80)(12)}{\sqrt{\left[1-\left(\frac{1}{2(2.80)}\right)^2\right]}}$$

$$= \mathbf{34.15\,V}$$

- Q – factor in series

The overall Q-factor, $Q_T = \frac{1}{R_T}\sqrt{\frac{L}{C}}$ from Section

where $R_T = R_L + R_C$

Since $Q_L = \frac{\omega_r L}{R_L}$ then $R_L = \frac{\omega_r L}{Q_L}$ and since

$$Q_C = \frac{1}{\omega_r C R_C} \text{ then } R_C = \frac{1}{Q_C \omega_r C}$$

Hence

$$Q_T = \frac{1}{R_L + R_C}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{\left(\frac{\omega_r L}{Q_L} + \frac{1}{Q_C \omega_r C}\right)}\sqrt{\frac{L}{C}}$$

$$Q_T = \frac{Q_L Q_C}{Q_L + Q_C}$$

Problem 6. An inductor of Q-factor 60 is connected in series with a capacitor having a Q-factor of 390. Determine the overall Q-factor of the circuit.

From above, overall Q-factor,

$$Q_T = \frac{Q_L Q_C}{Q_L + Q_C} = \frac{(60)(390)}{60 + 390} = \frac{23\,400}{450} = \mathbf{52}$$

- Bandwidth and selectivity

$$10\lg\left[\frac{I_r^2 R/2}{I_r^2 R}\right] = 10\lg\frac{1}{2} = \mathbf{-3\,dB}$$

It is for this reason that the half-power points are often referred to as **'the −3 dB points'**.
At the half-power frequencies, $I = 0.707 I_r$, thus impedance

$$Z = \frac{V}{I} = \frac{V}{0.707 I_r} = 1.414\left(\frac{V}{I_r}\right) = \sqrt{2}Z_r = \sqrt{2}R$$

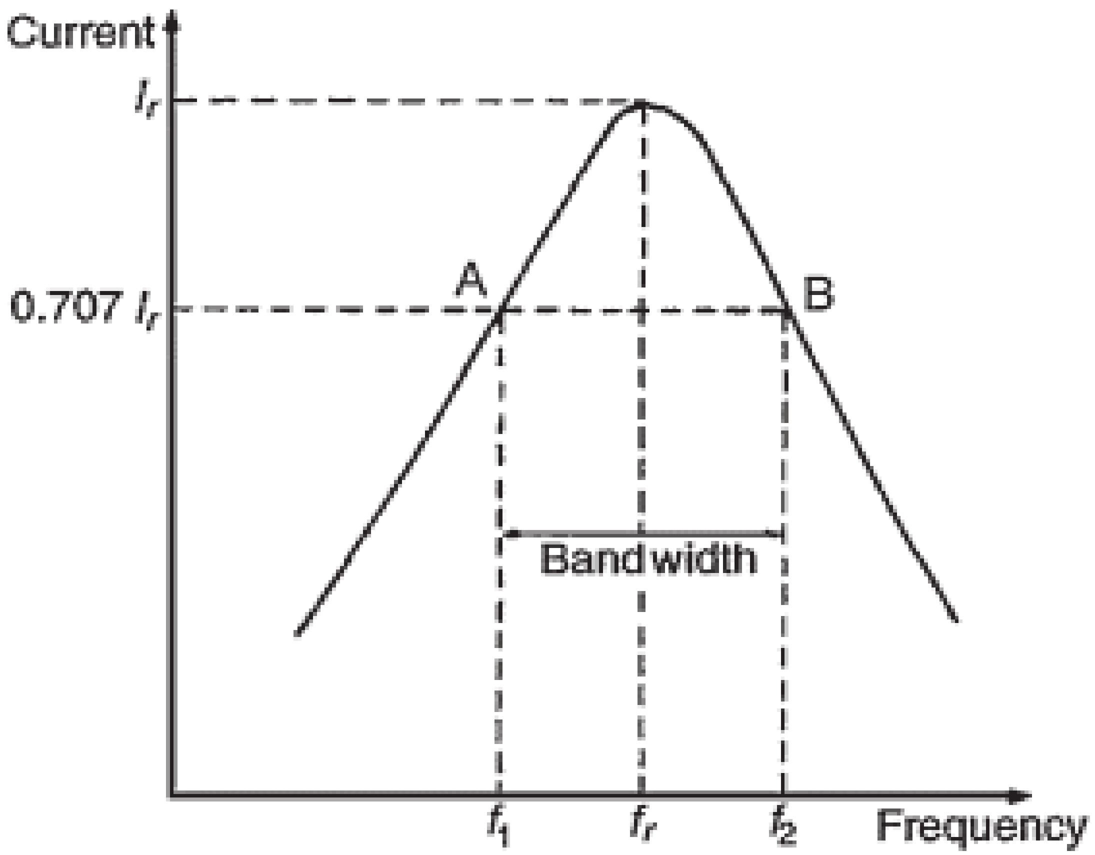
(since at resonance $Z_r = R$)

- At F1 |Xc|>|XL|

Thus $\dfrac{1}{2\pi f_1 C} - 2\pi f_1 L = R$

from which, $1 - 4\pi^2 f_1^2 LC = 2\pi f_1 CR$

i.e. $(4\pi^2 LC) f_1^2 + (2\pi CR) f_1 - 1 = 0$



Bandwidth and half-power points f_1, f_2

This is a quadratic equation in f_1. Using the quadratic formula gives:

$$f_1 = \frac{-(2\pi CR) \pm \sqrt{[(2\pi CR)^2 - 4(4\pi^2 LC)(-1)]}}{2(4\pi^2 LC)}$$

$$= \frac{-(2\pi CR) \pm \sqrt{[4\pi^2 C^2 R^2 + 16\pi^2 LC]}}{8\pi^2 LC}$$

$$= \frac{-(2\pi CR) \pm \sqrt{[4\pi^2 C^2 (R^2 + (4L/C))]}}{8\pi^2 LC}$$

$$= \frac{-(2\pi CR) \pm 2\pi C\sqrt{[R^2 + (4L/C)]}}{8\pi^2 LC}$$

Hence $f_1 = \dfrac{-R \pm \sqrt{[R^2 + (4L/C)]}}{4\pi L}$

$$= \frac{-R + \sqrt{[R^2 + (4L/C)]}}{4\pi L}$$

(since $\sqrt{[R^2 + (4L/C)]} > R$ and f_1 cannot be negative).

At f_2, the upper half-power frequency $|X_L| > |X_C|$ (see

Thus $2\pi f_2 L - \dfrac{1}{2\pi f_2 C} = R$

from which, $4\pi^2 f_2^2 LC - 1 = R(2\pi f_2 C)$

i.e. $(4\pi^2 LC) f_2^2 - (2\pi CR) f_2 - 1 = 0$

$$f_2 = \frac{R + \sqrt{[R^2 + (4L/C)]}}{4\pi L}$$

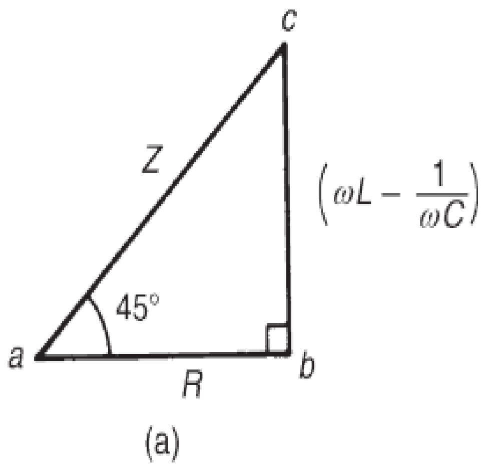
Bandwidth $= (f_2 - f_1)$

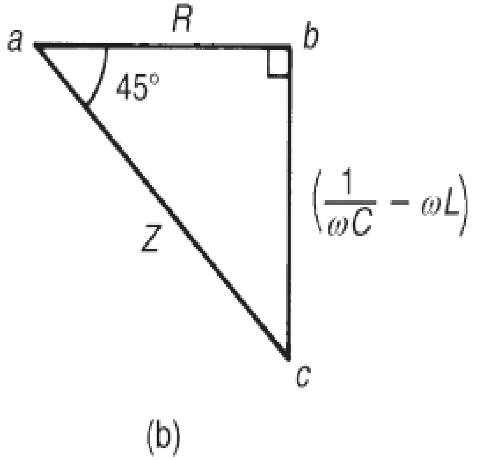
$$= \left\{\frac{R + \sqrt{[R^2 + (4L/C)]}}{4\pi L}\right\} - \left\{\frac{-R + \sqrt{[R^2 + (4L/C)]}}{4\pi L}\right\}$$

$$= \frac{2R}{4\pi L} = \frac{R}{2\pi L} = \frac{1}{2\pi L/R}$$

$$= \frac{f_r}{2\pi f_r L/R} = \frac{f_r}{Q_r}$$

$$Q = \frac{f_r}{f_2 - f_1}\ ,\ f_r = \sqrt{f_1 f_2}$$





(a) Inductive impedance triangle

(b) capacitive impedance triangle

Example An R–L–C series circuit has a resonant frequency of 1.2 kHz and a Q-factor at resonance of 30. If the impedance of the circuit at resonance is 50 Ω determine the values of (a) the inductance and (b) the capacitance. Find also (c) the bandwidth, (d) the lower and upper half-power frequencies and (e) the value of the circuit impedance at the half-power frequencies.

(a) At resonance the circuit impedance, $Z = R$, i.e. $R = 50\,\Omega$.

Q-factor at resonance, $Q_r = \omega_r L/R$

Hence **inductance,** $L = \dfrac{Q_r R}{\omega_r} = \dfrac{(30)(50)}{(2\pi 1200)}$

$= \mathbf{0.199\,H}$ or $\mathbf{199\,mH}$

(b) At resonance $\omega_r L = 1/(\omega_r C)$

Hence **capacitance,** $C = \dfrac{1}{\omega_r^2 L}$

$= \dfrac{1}{(2\pi 1200)^2 (0.199)}$

$= \mathbf{0.088\,\mu F}$ or $\mathbf{88\,nF}$

(c) Q-factor at resonance is also given by $Q_r = f_r/(f_2 - f_1)$, from which,

bandwidth, $(f_2 - f_1) = \dfrac{f_r}{Q_r} = \dfrac{1200}{30} = \mathbf{40\,Hz}$

$f_r = \sqrt{(f_1 f_2)}$, i.e. $1200 = \sqrt{(f_1 f_2)}$

from which, $f_1 f_2 = (1200)^2 = 1.44 \times 10^6$ (12)

From part (c), $f_2 - f_1 = 40$ (13)

From equation (12), $f_1 = (1.44 \times 10^6)/f_2$

Substituting in equation 13 gives:

$$f_2 - \frac{1.44 \times 10^2}{f_2} = 40$$

Multiplying throughout by f_2 gives:

$f_2^2 - 1.44 \times 10^6 = 40 f_2$

i.e. $f_2^2 - 40 f_2 - 1.44 \times 10^6 = 0$

This is a quadratic equation in f_2. Using the quadratic formula gives:

$$f_2 = \frac{40 \pm \sqrt{[(40)^2 - 4(1.44 \times 10^6)]}}{2}$$

$$= \frac{40 \pm 2400}{2}$$

$$= \frac{40 + 2400}{2} \text{ (since } f_2 \text{ cannot be negative)}$$

Hence **the upper half-power frequency,**

$\mathbf{f_2 = 1220\,Hz}$

$f_1 = f_2 - 40 = 1220 - 40 = \mathbf{1180\ Hz}$

Note that the upper and lower half-power frequency values are symmetrically placed about the resonance frequency. This is usually the case when the Q-factor has a high value (say, >10).

(e) At the half-power frequencies, current $I = 0.707\, I_r$

Hence impedance,

$$Z = \frac{V}{I} = \frac{V}{0.707 I_r} = 1.414\left(\frac{V}{I_r}\right) = \sqrt{2}Zr$$
$$= \sqrt{2}R$$

Thus **impedance at the half-power frequencies,**

$Z = \sqrt{2}R = \sqrt{2}(50) = \mathbf{70.71\ \Omega}$

Example. A filter in the form of a series L–R–C circuit is designed to operate at a resonant frequency of 20 kHz. Included within the filter is a 20 mH inductance and 8 Ω resistance. Determine the bandwidth of the filter.

Q-factor at resonance is given by

$$Q_r = \frac{\omega_r L}{R} = \frac{(2\pi 20\,000)(10 \times 10^{-3})}{8} = 157.08$$

Since $Q_r = f_r/(f_2 - f_1)$

$$\textbf{bandwidth, } (f_2 - f_1) = \frac{f_r}{Q_r} = \frac{20\,000}{157.08} = \mathbf{127.3\ Hz}$$

Homework

A series R−L−C circuit is connected to a 0.2 V supply and the current is at its maximum value of 4 mA when the supply frequency is adjusted to 3 kHz. The Q-factor of the circuit under these conditions is 100. Determine the value of (a) the circuit resistance, (b) the circuit inductance, (c) the circuit capacitance and (d) the voltage across the capacitor.

A coil of inductance 351.8 mH and resistance 8.84 is connected in series with a 20μF capacitor. Determine (a) the resonant frequency, (b) the Q-factor at resonance, (c) the bandwidth and (d) the lower and upper −3 dB frequencies.


Thank
you