Questions bank for Network analysis I

The two-phase balanced ac generator of Fig. 11-22 feeds two identical loads. The two voltage sources are 180° out of phase. Find (*a*) the line currents, voltages, and their phase angles, and (*b*) the instantaneous and average powers delivered by the generator.

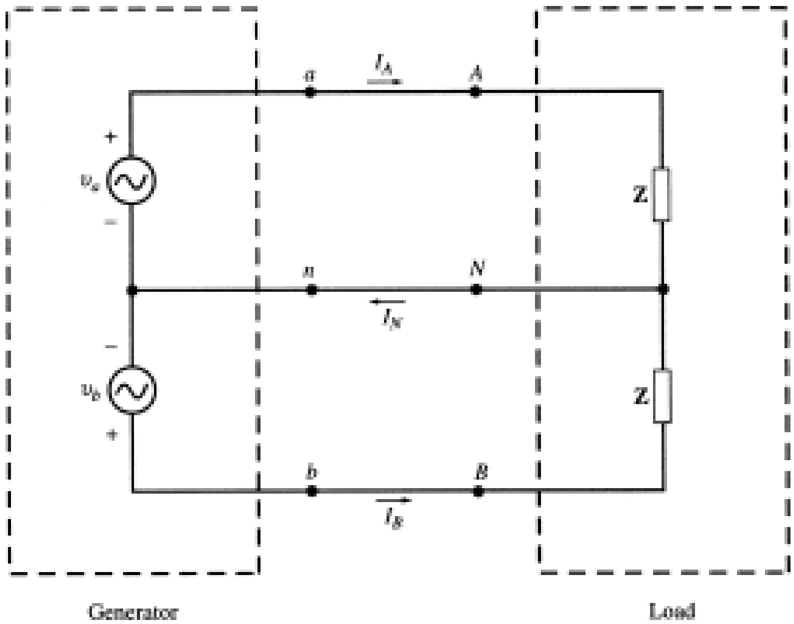

Fig. 11-22

Let $\mathbf{Z} = |Z| \angle\theta$ and $I_p = V_p/|Z|$.

(a) The voltages and currents in the phasor domain are

$$\mathbf{V}_{AN} = V_p \angle 0 \qquad V_{BN} = V_p \angle -180^\circ = -V_p \angle 0 \qquad \mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = 2V_p \angle 0$$

Now, from I_p and $\mathbf{Z}$ given above, we have

$$\mathbf{I}_A = I_p \angle -\theta \qquad \mathbf{I}_B = I_p \angle -180^\circ - \theta = -I_p \angle -\theta \qquad \mathbf{I}_N = \mathbf{I}_A + \mathbf{I}_B = 0$$

(*b*) The instantaneous powers delivered are

$$p_a(t) = v_a(t) i_a(t) = V_p I_p \cos\theta + V_p I_p \cos(2\omega t - \theta)$$

$$p_b(t) = v_b(t) i_b(t) = V_p I_p \cos\theta + V_p I_p \cos(2\omega t - \theta)$$

The total instantaneous power $p_T(t)$ is

$$p_T(t) = p_a(t) + p_b(t) = 2V_p I_p \cos\theta + 2V_p I_p \cos(2\omega t - \theta)$$

The average power is $P_{avg} = 2V_p I_p \cos\theta$.

11.4. Show that the line-to-line voltage V_L in a three-phase system is $\sqrt{3}$ times the line-to-neutral voltage V_{Ph}.

See the voltage phasor diagram (for the ABC sequence), Fig. 11-23.

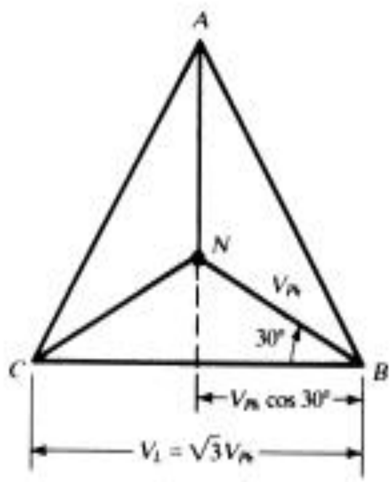

Fig. 11-23

Fig. 11-23

11.5. A three-phase, ABC system, with an effective voltage 70.7 V, has a balanced Δ-connected load with impedances $20\angle 45^\circ$ Ω. Obtain the line currents and draw the voltage-current phasor diagram.

The circuit is shown in Fig. 11-24. The phasor voltages have magnitudes $V_{max} = \sqrt{2}V_{eff} = 100$ V. Phase angles are obtained from Fig. 11-7(*a*). Then,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}} = \frac{100\angle 120^\circ}{20\angle 45^\circ} = 5.0\angle 75^\circ \text{ A}$$

Similarly, $\mathbf{I}_{BC} = 5.0\angle -45^\circ$ A and $\mathbf{I}_{CA} = 5.0\angle 195^\circ$ A. The line currents are

$$\mathbf{I}_A = \mathbf{I}_{AB} + \mathbf{I}_{AC} = 5\angle 75^\circ - 5\angle 195^\circ = 8.65\angle 45^\circ \text{ A}$$

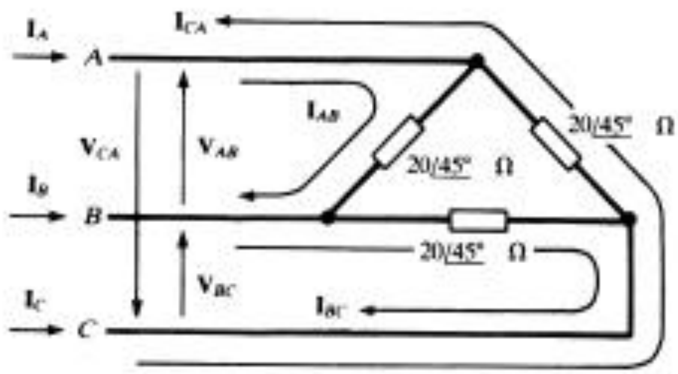

Fig. 11-24

Similarly, $\mathbf{I}_B = 8.65\angle -75^\circ$ A, $\mathbf{I}_C = 8.65\angle 165^\circ$ A.

The voltage-current phasor diagram is shown in Fig. 11-25.

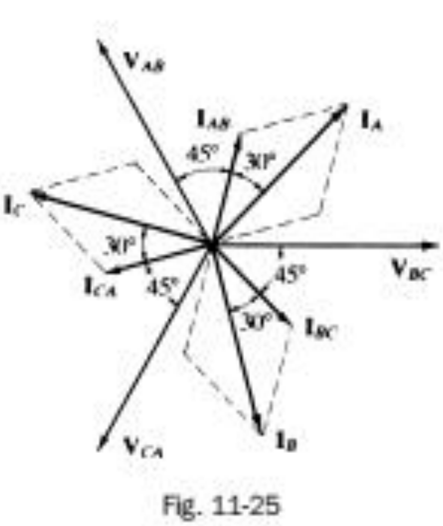

Fig. 11-25

11.6. A three-phase, three-wire *CBA* system, with an effective line voltage 106.1 V, has a balanced Y-connected load with impedances $5\,\angle{-30^\circ}\ \Omega$ (Fig. 11-26). Obtain the currents and draw the voltage-current phasor diagram.

With balanced Y-loads the neutral conductor carries no current. Even though this system is three-wire, the neutral may be added to simplify computation of the line currents. The magnitude of the line voltage is $V_L = \sqrt{2}(106.1) = 150$ V. Then the line-to-neutral magnitude is $V_{LN} = 150/\sqrt{3} = 86.6$ V.

$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{\mathbf{Z}} = \frac{86.6\,\angle{-90^\circ}}{5\,\angle{-30^\circ}} = 17.32\,\angle{-60^\circ}\ \text{A}$$

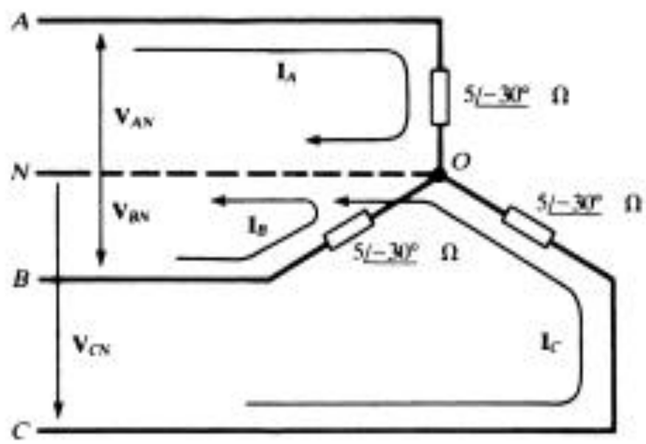

Fig. 11-26

Similarly, $\mathbf{I}_B = 17.32\,\angle{60^\circ}$ A, $\mathbf{I}_C = 17.32\,\angle{180^\circ}$ A. See the phasor diagram, Fig. 11-27, in which the balanced set of line currents leads the set of line-to-neutral voltages by 30°, the negative of the angle of the impedances.

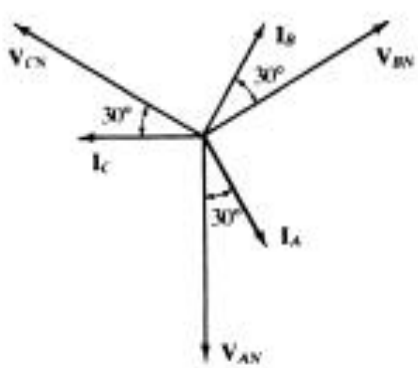

Fig. 11-27

11.7. A three-phase, three-wire *CBA* system, with an effective line voltage 106.1 V, has a balanced Δ-connected load with impedances $\mathbf{Z} = 15\,\underline{/30^\circ}\ \Omega$. Obtain the line and phase currents by the single-line equivalent method.

Referring to Fig. 11-28, $V_{LN} = (141.4\sqrt{2})/\sqrt{3} = 115.5$ V, and so

$$\mathbf{I}_L = \frac{115.5\,\underline{/0^\circ}}{(15/3)\,\underline{/30^\circ}} = 23.1\,\underline{/-30^\circ}\ \text{A}$$

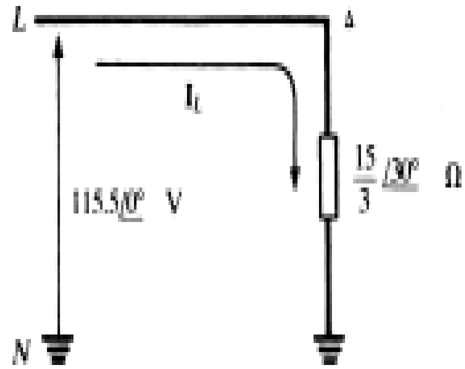

Fig. 11-28

The line currents lag the *ABC*-sequence, line-to-neutral voltages by 30°:

$$\mathbf{I}_A = 23.1\,\underline{/60^\circ}\ \text{A} \qquad \mathbf{I}_B = 23.1\,\underline{/-60^\circ}\ \text{A} \qquad \mathbf{I}_C = 23.1\,\underline{/180^\circ}\ \text{A}$$

The phase currents, of magnitude $I_{Ph} = I_L/\sqrt{3} = 13.3$ A, lag the corresponding line-to-line voltages by 30°:

$$\mathbf{I}_{AB} = 13.3\,\underline{/90^\circ}\ \text{A} \qquad \mathbf{I}_{BC} = 13.3\,\underline{/-30^\circ}\ \text{A} \qquad \mathbf{I}_{CA} = 13.3\,\underline{/210^\circ}\ \text{A}$$

A sketch of the phasor diagram will make all of the foregoing angles evident.

11.8. A three-phase, three-wire system, with an effective line voltage 176.8 V, supplies two balanced loads, one in delta configuration with $\mathbf{Z}_\Delta = 15\angle 0^\circ\ \Omega$ and the other in wye form with $\mathbf{Z}_Y = 10\angle 30^\circ\ \Omega$. Obtain the total power.

First convert the Δ-load to Y, and then use the single-line equivalent circuit, Fig. 11-29, to obtain the line current.

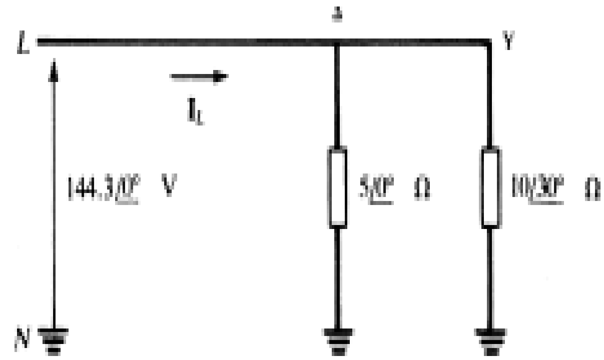

Fig. 11-29

$$\mathbf{I}_L = \frac{144.3\angle 0^\circ}{5\angle 0^\circ} + \frac{144.3\angle 0^\circ}{10\angle 30^\circ} = 42.0\angle -9.9^\circ \text{ A}$$

Then
$$P = \sqrt{3}\, V_{Leff} I_{Leff} \cos\theta = \sqrt{3}(176.8)(29.7)\cos 9.9^\circ = 8959 \text{ W}$$

11.9. Obtain the readings when the two-wattmeter method is applied to the circuit of Problem 11.8.

The angle on $\mathbf{I}_L$, $-9.9°$, is the negative of the angle on the equivalent impedance of the parallel combination of $5\ \underline{/0°}\ \Omega$ and $10\ \underline{/30°}\ \Omega$. Therefore, $\theta = 9.9°$ in the formulas of Section 11.13.

$$W_1 = V_{Leff} I_{Leff} \cos(\theta + 30°) = (176.8)(29.7)\cos 39.9° = 4028\ \text{W}$$

$$W_2 = V_{Leff} I_{Leff} \cos(\theta - 30°) = (176.8)(29.7)\cos(-20.1°) = 4931\ \text{W}$$

As a check, $W_1 + W_2 = 8959$ W, which is in agreement with Problem 11.8.

11.10. A three-phase supply, with an effective line voltage 240 V, has an unbalanced Δ-connected load shown in Fig. 11-30. Obtain the line currents and the total power.

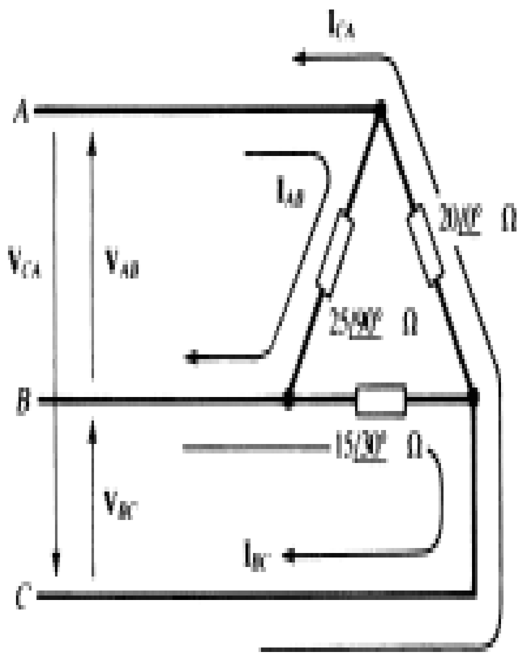

Fig. 11-30

The power calculations can be performed without knowledge of the sequence of the system. The effective values of the phase currents are

$$I_{AB\,\text{eff}} = \frac{240}{25} = 9.6\ \text{A} \qquad I_{BC\,\text{eff}} = \frac{240}{15} = 16\ \text{A} \qquad I_{CA\,\text{eff}} = \frac{240}{20} = 12\ \text{A}$$

Hence, the complex powers in the three phases are

$$\mathbf{S}_{AB} = (9.6)^2(25\angle 90^\circ) = 2304\angle 90^\circ = 0 + j2304$$

$$\mathbf{S}_{BC} = (16)^2(15\angle 30^\circ) = 3840\angle 30^\circ = 3325 + j1920$$

$$\mathbf{S}_{CA} = (12)^2(20\angle 0^\circ) = 2880\angle 0^\circ = 2880 + j0$$

and the total complex power is their sum,

$$\mathbf{S}_T = 6205 + j4224$$

That is, $P_T = 6205$ W and $Q_T = 4224$ var (inductive).

To obtain the currents, a sequence must be assumed; let it be *ABC*. Then, using Fig. 11-7(*a*),

$$\mathbf{I}_{AB} = \frac{339.4\angle 120^\circ}{25\angle 90^\circ} = 13.6\angle 30^\circ\ \text{A}$$

$$\mathbf{I}_{BC} = \frac{339.4\angle 0^\circ}{15\angle 30^\circ} = 22.6\angle -30^\circ\ \text{A}$$

$$\mathbf{I}_{CA} = \frac{339.4\angle 240^\circ}{20\angle 0^\circ} = 17.0\angle 240^\circ\ \text{A}$$

The line currents are obtained by applying KCL at the junctions.

$$\mathbf{I}_A = \mathbf{I}_{AB} + \mathbf{I}_{AC} = 13.6\angle 30^\circ - 17.0\angle 240^\circ = 29.6\angle 46.7^\circ\ \text{A}$$

$$\mathbf{I}_B = \mathbf{I}_{BC} + \mathbf{I}_{BA} = 22.6\angle -30^\circ - 13.6\angle 30^\circ = 19.7\angle -66.7^\circ\ \text{A}$$

$$\mathbf{I}_C = \mathbf{I}_{CA} + \mathbf{I}_{CB} = 17.0\angle 240^\circ - 22.6\angle -30^\circ = 28.3\angle -173.1^\circ\ \text{A}$$

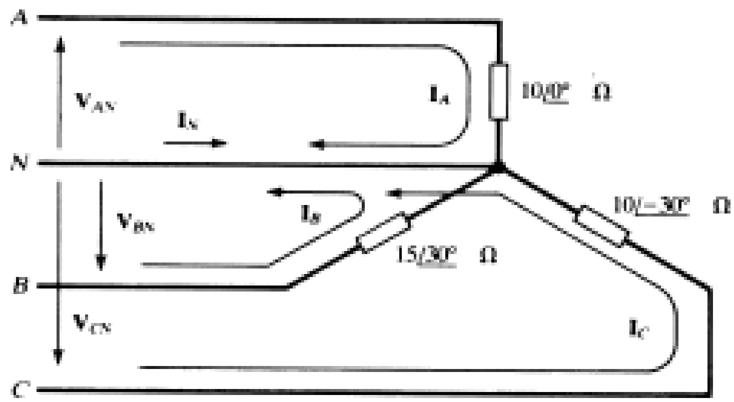

Fig. 11-31

$$\mathbf{I}_A = \frac{169.9\underline{/90^\circ}}{10\underline{/0^\circ}} = 16.99\underline{/90^\circ}\text{ A}$$

$$\mathbf{I}_B = \frac{169.9\underline{/-30^\circ}}{15\underline{/30^\circ}} = 11.33\underline{/-60^\circ}\text{ A}$$

$$\mathbf{I}_C = \frac{169.9\underline{/-150^\circ}}{10\underline{/-30^\circ}} = 16.99\underline{/-120^\circ}\text{ A}$$

$$\mathbf{I}_N = -(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C) = 8.04\underline{/69.5^\circ}\text{ A}$$

11.11. Obtain the readings of wattmeters placed in lines *A* and *B* of the circuit of Problem 11.10. (Line *C* is the potential reference for both meters.)

$$W_A = \text{Re}\,(\mathbf{V}_{AC\,\text{eff}}\mathbf{I}^*_{A\,\text{eff}}) = \text{Re}\left[(240\underline{/60^\circ})\left(\frac{29.6}{\sqrt{2}}\underline{/-46.7^\circ}\right)\right]$$

$$= \text{Re}\,(5023\underline{/13.3^\circ}) = 4888\text{ W}$$

$$W_B = \text{Re}\,(\mathbf{V}_{BC\,\text{eff}}\mathbf{I}^*_{B\,\text{eff}}) = \text{Re}\left[(240\underline{/0^\circ})\left(\frac{19.7}{\sqrt{2}}\underline{/66.7^\circ}\right)\right]$$

$$= \text{Re}\,(3343\underline{/66.7^\circ}) = 1322\text{ W}$$

Note that $W_A + W_B = 6210$ W, which is very close to P_T as found in Problem 11.10.

11.12. A three-phase, four-wire, *ABC* system, with line voltage $\mathbf{V}_{BC} = 294.2\underline{/0^\circ}$ V, has a Y-connected load of $\mathbf{Z}_A = 10\underline{/0^\circ}\ \Omega$, $\mathbf{Z}_B = 15\underline{/30^\circ}\ \Omega$, and $\mathbf{Z}_C = 10\underline{/-30^\circ}\ \Omega$ (Fig. 11-31). Obtain the line and neutral currents.

11.13. The Y-connected load impedances $\mathbf{Z}_A = 10\angle 0^\circ\ \Omega$, $\mathbf{Z}_B = 15\angle 30^\circ\ \Omega$, and $\mathbf{Z}_C = 10\angle -30^\circ\ \Omega$, in Fig. 11-32, are supplied by a three-phase, three-wire, *ABC* system in which $\mathbf{V}_{BC} = 208\angle 0^\circ$ V. Obtain the voltages across the impedances and the displacement neutral voltage $\mathbf{V}_{ON}$.

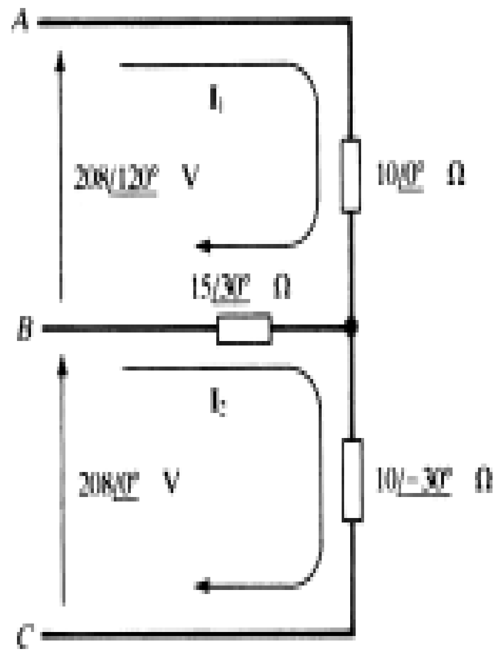

Fig. 11-32

The method of Example 11.7 could be applied here and one node-voltage equation solved. However, the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ suggested in Fig. 11-32 provide another approach.

$$\begin{bmatrix} 10\angle 0^\circ + 15\angle 30^\circ & -15\angle 30^\circ \\ -15\angle 30^\circ & 15\angle 30^\circ + 10\angle -30^\circ \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 208\angle 120^\circ \\ 208\angle 0^\circ \end{bmatrix}$$

Therefore, $\mathbf{I}_1 = 14.16\,\underline{/86.09^\circ}$ A and $\mathbf{I}_2 = 10.21\,\underline{/52.41^\circ}$ A. The line currents are then

$$\mathbf{I}_A = \mathbf{I}_1 = 14.16\,\underline{/86.09^\circ}\ \text{A} \qquad \mathbf{I}_B = \mathbf{I}_2 - \mathbf{I}_1 = 8.01\,\underline{/-48.93^\circ}\ \text{A} \qquad \mathbf{I}_C = -\mathbf{I}_2 = 10.21\,\underline{/-127.59^\circ}\ \text{A}$$

Now the phasor voltages at the load may be computed.

$$\mathbf{V}_{AO} = \mathbf{I}_A\mathbf{Z}_A = 141.6\underline{/86.09^\circ}\ \text{V}$$

$$\mathbf{V}_{BO} = \mathbf{I}_B\mathbf{Z}_B = 120.2\underline{/-18.93^\circ}\ \text{V}$$

$$\mathbf{V}_{CO} = \mathbf{I}_C\mathbf{Z}_C = 102.1\underline{/-157.59^\circ}\ \text{V}$$

$$\mathbf{V}_{ON} = \mathbf{V}_{OA} + \mathbf{V}_{AN} = 141.6\underline{/-93.91^\circ} + 120.1\,\underline{/90^\circ} = 23.3\underline{/-114.53^\circ}\ \text{V}$$

The phasor diagram is given in Fig. 11-33.

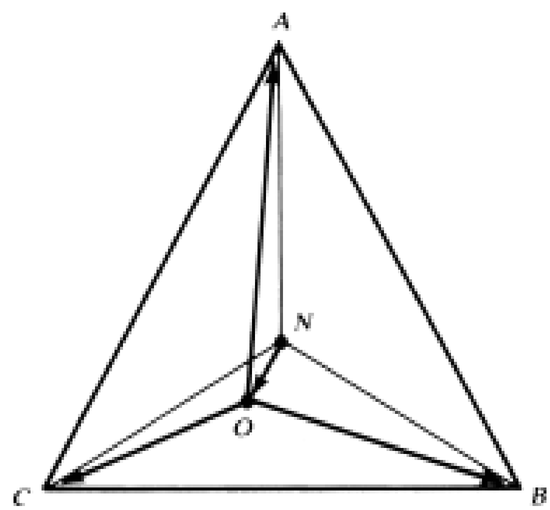

Fig. 11-33

11.14. Obtain the total average power for the unbalanced, Y-connected load in Problem 11.13, and compare with the readings of wattmeters in lines B and C.

The phase powers are

$$P_A = I^2_{A\,\mathrm{eff}} R_A = \left(\frac{14.16}{\sqrt{2}}\right)(10) = 1002.5\ \mathrm{W}$$

$$P_B = I^2_{B\,\mathrm{eff}} R_B = \left(\frac{8.01}{\sqrt{2}}\right)(15\cos 30^\circ) = 417.0\ \mathrm{W}$$

$$P_C = I^2_{C\,\mathrm{eff}} \boldsymbol{R}_C = \left(\frac{10.21}{\sqrt{2}}\right)^2 (10\cos 30^\circ) = 451.4\ \mathrm{W}$$

and so the total average power is 1870.9 W.

From the results of Problem 11.13, the wattmeter readings are:

$$W_B = \mathrm{Re}(\mathbf{V}_{BA\mathrm{eff}}\mathbf{I}^*_{B\mathrm{eff}}) = \mathrm{Re}\left[\left(\frac{208}{\sqrt{2}}\angle -60^\circ\right)\left(\frac{8.01}{\sqrt{2}}\angle 48.93^\circ\right)\right] = 817.1\ \mathrm{W}$$

$$W_C = \mathrm{Re}(\mathbf{V}_{CA\mathrm{eff}}\mathbf{I}^*_{C\mathrm{eff}}) = \mathrm{Re}\left[\left(\frac{208}{\sqrt{2}}\angle 2400^\circ\right)\left(\frac{10.21}{\sqrt{2}}\angle 127.59^\circ\right)\right] = 1052.8\ \mathrm{W}$$

The total power read by the two wattmeters is 1869.9 W.

11.15. A three-phase, three-wire, balanced, Δ-connected load yields wattmeter readings of 1154 W and 557 W. Obtain the load impedance, if the line voltage is 141.4 V.

$$\pm\tan\theta = \sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right) = \sqrt{3}\left(\frac{577}{1731}\right) = 0.577 \qquad \theta = \pm 30.0^\circ$$

and, using $P_T = \sqrt{3}\, V_{L\text{eff}} I_{L\text{eff}} \cos\theta$,

$$Z_\Delta = \frac{V_{L\text{eff}}}{I_{Ph\text{eff}}} = \frac{\sqrt{3}\;V_{L\text{eff}}}{I_{L\text{eff}}} = \frac{3V_{L\text{eff}}^2\cos\theta}{P_T} = \frac{3(100)^2\cos 30.0^\circ}{1154 + 577}\,\Omega = 15.0\,\Omega$$

Thus, $\mathbf{Z}_\Delta = 15.0\,\underline{/\pm 30.0^\circ}\ \Omega$.

11.16. A balanced Δ-connected load, with $\mathbf{Z}_\Delta = 30\underline{/30^\circ}\ \Omega$, is connected to a three-phase, three-wire, 250-V system by conductors having impedances $\mathbf{Z}_c = 0.4 + j0.3\ \Omega$. Obtain the line-to-line voltage at the load.

The single-line equivalent circuit is shown in Fig. 11-34. By voltage division, the voltage across the substitute Y-load is

$$\mathbf{V}_{AN} = \left(\frac{10\underline{/30^\circ}}{0.4 + j0.3 + 10\underline{/30^\circ}}\right)\left(\frac{250}{\sqrt{3}}\underline{/0^\circ}\right) = 137.4\underline{/-0.33^\circ}\ \text{V}$$

whence $V_L = (137.4)(\sqrt{3}) = 238.0$ V.

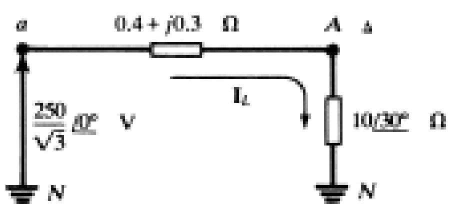

Fig. 11-34

Considering the magnitudes only, the line voltage at the load, 238.0 V, represents a drop of 12.0 V. The wire size and total length control the resistance in $\mathbf{Z}_c$, while the enclosing conduit material (e.g., steel, aluminum, or fiber), as well as the length, affects the inductive reactance.

11.17. Three impedances of $10.0\underline{/53.13^\circ}\ \Omega$ are connected in delta to a three-phase, *CBA* system with an affective line voltage 240 V. Obtain the line currents.

Ans. $\mathbf{I}_A = 58.8\underline{/-143.13^\circ}$ A, $\mathbf{I}_B = 58.8\underline{/-23.13^\circ}$ A, $\mathbf{I}_C = 58.8\underline{/96.87^\circ}$ A

11.18. Three impedances of $4.20\underline{/-35^\circ}\ \Omega$ are connected in delta to a three-phase, *ABC* system having $\mathbf{V}_{BC} = 495.0\underline{/0^\circ}$ V. Obtain the line currents.

Ans. $\mathbf{I}_A = 20.41\underline{/125^\circ}$ A, $\mathbf{I}_B = 20.41\underline{/5^\circ}$ A, $\mathbf{I}_C = 20.41\underline{/-115^\circ}$ A

11.19. A three-phase, three-wire system, with an effective line voltage 100 V, has currents

$$\mathbf{I}_A = 15.41\angle{-160^\circ}\ \text{A} \qquad \mathbf{I}_B = 15.41\angle{-40^\circ}\ \text{A} \qquad \mathbf{I}_C = 15.41\angle{80^\circ}\ \text{A}$$

What is the sequence of the system and what are the impedances, if the connection is delta?

Ans. *CBA*, $15.9\angle{70^\circ}\ \Omega$

11.20. A balanced Y-connected load, with impedances $6.0\angle{45^\circ}\ \Omega$, is connected to a three-phase, four-wire *CBA* system having effective line voltage 208 V. Obtain the four line currents.

Ans. $\mathbf{I}_A = 28.31\angle{-135^\circ}$ A, $\mathbf{I}_B = 28.31\angle{-15^\circ}$ A, $\mathbf{I}_C = 28.31\angle{105^\circ}$ A, $\mathbf{I}_N = 0$

11.21. A balanced Y-connected load, with impedances $65.0\angle{-20^\circ}\ \Omega$, is connected to a three-phase, three-wire, *CBA* system, where $\mathbf{V}_{AB} = 678.8\angle{-120^\circ}$ V. Obtain the three line currents.

Ans. $\mathbf{I}_A = 6.03\angle{-70^\circ}$ A, $\mathbf{I}_B = 6.03\angle{50^\circ}$ A, $\mathbf{I}_C = 6.03\angle{170^\circ}$ A

11.22. A balanced Δ-connected load, with $\mathbf{Z}_\Delta = 9.0\angle{-30^\circ}$, and a balanced Y-connected load, with $\mathbf{Z}_Y = 5.0\angle{45^\circ}\ \Omega$, are supplied by the same three-phase, *ABC* system, with effective line voltage 480 V. Obtain the line currents, using the single-line equivalent method.

Ans. $\mathbf{I}_A = 168.9\angle{93.36^\circ}$ A, $\mathbf{I}_B = 168.9\angle{-26.64^\circ}$ A, $\mathbf{I}_C = 168.9\angle{-146.64^\circ}$ A

11.23. A balanced Δ-connected load having impedances $27.0\ \angle{-25^\circ}\ \Omega$, and a balanced Y-connected load having impedances $10.0\ \angle{-30^\circ}\ \Omega$ are supplied by the same three-phase, *ABC* system, with $\mathbf{V}_{CN} = 169.8\angle{-150^\circ}$ V. Obtain the line currents.

Ans. $\mathbf{I}_A = 35.8\angle{117.36^\circ}$ A, $\mathbf{I}_B = 35.8\angle{-2.64^\circ}$ A, $\mathbf{I}_C = 35.8\angle{-122.64^\circ}$ A

11.24. A balanced Δ-connected load, with impedances $10.0\angle{-36.9^\circ}\ \Omega$, and a balanced Y-connected load are supplied by the same three-phase, *ABC* system having $\mathbf{V}_{CA} = 141.4\angle{240^\circ}$ V. If $\mathbf{I}_B = 40.44\angle{13.41^\circ}$ A, what are the impedances of the Y-connected load? *Ans.* $5.0\angle{-53.3^\circ}$

11.25. A three-phase, *ABC* system, with effective line voltage 500 V, has a Δ-connected load for which

$$\mathbf{Z}_{AB} = 10.0\angle{30^\circ}\ \Omega \qquad \mathbf{Z}_{BC} = 25.0\angle{0^\circ}\ \Omega \qquad \mathbf{Z}_{CA} = 20.0\angle{-30^\circ}\ \Omega$$

Obtain the line currents.

Ans. $\mathbf{I}_A = 106.1\angle{90.0^\circ}$ A, $\mathbf{I}_B = 76.15\angle{-68.20^\circ}$ A, $\mathbf{I}_C = 45.28\angle{-128.65^\circ}$ A

11.26. A three-phase, *ABC* system, with $\mathbf{V}_{BC} = 294.2\angle{0^\circ}$ V, has the Δ-connected load

$$\mathbf{Z}_{AB} = 5.0\angle{0^\circ}\ \Omega \qquad \mathbf{Z}_{BC} = 4.0\angle{30^\circ}\ \Omega \qquad \mathbf{Z}_{CA} = 6.0\angle{-15^\circ}$$

Obtain the line currents.

Ans. $\mathbf{I}_A = 99.7\angle{99.7^\circ}$ A, $\mathbf{I}_B = 127.9\angle{-43.3}$ A, $\mathbf{I}_C = 77.1\angle{-172.1^\circ}$ A

11.27. A three-phase, four-wire, *CBA* system, with effective line voltage 100 V, has Y-connected impedances

$$\mathbf{Z}_A = 3.0\angle{0^\circ}\ \Omega \qquad \mathbf{Z}_B = 3.61\angle{56.31^\circ}\ \Omega \qquad \mathbf{Z}_C = 2.24\angle{-26.57^\circ}\ \Omega$$

Obtain the currents $\mathbf{I}_A$, $\mathbf{I}_B$, $\mathbf{I}_C$, and $\mathbf{I}_N$.

Ans. $27.2\angle{-90^\circ}$ A, $22.6\angle{-26.3^\circ}$ A, $36.4\angle{176.6^\circ}$ A, $38.6\angle{65.3^\circ}$ A

11.28. A three-phase, four-wire, *ABC* system, with $\mathbf{V}_{BC} = 294.2\angle{0^\circ}$ V, has Y-connected impedances

$$\mathbf{Z}_A = 12.0\angle{45^\circ}\ \Omega \qquad \mathbf{Z}_B = 10.0\angle{30^\circ}\ \Omega \qquad \mathbf{Z}_C = 8.0\angle{0^\circ}\ \Omega$$

Obtain the currents $\mathbf{I}_A$, $\mathbf{I}_B$, $\mathbf{I}_C$, and $\mathbf{I}_N$.

Ans. $14.16\angle{45^\circ}$ A, $16.99\angle{-60^\circ}$ A, $21.24\angle{-150^\circ}$ A, $15.32\angle{90.4^\circ}$ A

11.29. A Y-connected load, with $\mathbf{Z}_A = 10\angle 0^\circ\ \Omega$, $\mathbf{Z}_B = 10\angle 60^\circ$, and $\mathbf{Z}_C = 10\angle -60^\circ\ \Omega$, is connected to a three-phase, three-wire, *ABC* system having effective line voltage 141.4 V. Find the load voltages $\mathbf{V}_{AO}$, $\mathbf{V}_{BO}$, $\mathbf{V}_{CO}$ and the displacement neutral voltage $\mathbf{V}_{ON}$. Construct a phasor diagram similar to Fig. 11-18.

Ans. $173.2\angle 90^\circ$ V, $100\angle 0^\circ$ V, $100\angle 180^\circ$ V, $57.73\angle -90^\circ$ V

11.30. A Y-connected load, with $\mathbf{Z}_A = 10\angle -60^\circ\ \Omega$, $\mathbf{Z}_B = 10\angle 0^\circ\ \Omega$, and $\mathbf{Z}_C = 10\angle 60^\circ\ \Omega$, is connected to a three-phase, three-wire, *CBA* system having effective line voltage 147.1 V. Obtain the line currents $\mathbf{I}_A$, $\mathbf{I}_B$, and $\mathbf{I}_C$.

Ans. $20.8\angle -60^\circ$ A, 0, $20.8\angle 120^\circ$ A

11.31. A three-phase, three-wire, *ABC* system with a balanced load has effective line voltage 200 V and (maximum) line current $\mathbf{I}_A = 13.61\angle 60^\circ$ A. Obtain the total power. *Ans.* 2887 W

11.32. Two balanced Δ-connected loads, with impedances $20\angle -60^\circ\ \Omega$ and $18\angle 45^\circ$, respectively, are connected to a three-phase system for which a line voltage is $\mathbf{V}_{BC} = 212.1\angle 0^\circ$ V. Obtain the phase power of each load. After using the single-line equivalent method to obtain the total line current, compute the total power, and compare with the sum of the phase powers.

Ans. 562.3 W, 883.6 W, 4337.5 W = 3(562.3 W) + 3(883.6 W)

11.33. In Problem 11.5, a balanced Δ-connected load with $\mathbf{Z} = 20\angle 45^\circ\ \Omega$ resulted in line currents 8.65 A for line voltages 100 V, both maximum values. Find the readings of two wattmeters used to measure the total average power. *Ans.* 111.9 W, 417.7 W

11.34. Obtain the readings of two wattmeters in a three-phase, three-wire system having effective line voltage 240 V and balanced, Δ-connected load impedances $20\angle 80^\circ\ \Omega$. *Ans.* −1706 W, 3206 W

11.35. A three-phase, three-wire, *ABC* system, with line voltage $\mathbf{V}_{BC} = 311.1\angle 0^\circ$ V, has line currents

$$\mathbf{I}_A = 61.5\angle 116.6^\circ \text{ A} \qquad \mathbf{I}_B = 61.2\angle -48.0^\circ \text{ A} \qquad \mathbf{I}_C = 16.1\angle 218^\circ \text{ A}$$

Find the readings of wattmeters in lines (*a*) *A* and *B*, (*b*) *B* and *C*, and (*c*) *A* and *C*.

Ans. (*a*) 5266 W, 6370 W; (*b*) 9312 W, 2322 W; (*c*) 9549 W, 1973 W

11.36. A three-phase, three-wire, *ABC* system has an effective line voltage 440 V. The line currents are

$$\mathbf{I}_A = 27.9\angle 90^\circ \text{ A} \qquad \mathbf{I}_B = 81.0\angle -9.9^\circ \text{ A} \qquad \mathbf{I}_C = 81.0\angle 189.9^\circ \text{ A}$$

Obtain the readings of wattmeters in lines (*a*) *A* and *B*, (*b*) *B* and *C*.

Ans. (*a*) 7.52 kW, 24.8 kW; (b) 16.16 kW, 16.16 kW

11.37. Two wattmeters in a three-phase, three-wire system with effective line voltage 120 V read 1500 W and 500 W. What is the impedance of the balanced Δ-connected load? *Ans.* $16.3\angle +40.9^\circ\ \Omega$

11.38. A three-phase, three-wire, *ABC* system has effective line voltage 173.2 V. Wattmeters in lines *A* and *B* read −301 W and 1327 W, respectively. Find the impedance of the balanced Y-connected load. (Since the sequence is specified, the sign of the impedance angle can be determined.)

Ans. $10\angle -70^\circ\ \Omega$

11.39. A three-phase, three-wire system, with a line voltage $\mathbf{V}_{BC} = 339.4\angle 0^\circ$ V, has a balanced Y-connected load of $\mathbf{Z}_Y = 15\angle 60^\circ\ \Omega$. The lines between the system and the load have impedances $2.24\angle 26.57^\circ\ \Omega$. Find the line-voltage magnitude at the load. *Ans.* 301.1 V

11.40. Repeat Problem 11.39 with the load impedance $\mathbf{Z}_Y = 15\angle -60^\circ\ \Omega$. By drawing the voltage phasor diagrams for the two cases, illustrate the effect of the load impedance angle on the voltage drop for a given line impedance. *Ans.* 332.9 V

11.41 A three-phase generator with an effective line voltage of 6000 V supplies the following four balanced loads in parallel: 16 kW at pf = 0.8 lagging, 24 kW at pf = 0.6 lagging, 4 kW at pf = 1, and 1 kW at pf = 0.1 leading. (*a*) Find the total average power (P) supplied by the generator, reactive power (Q), apparent power (S), power

factor, and effective value of line current. (*b*) Find the amount of reactive load Q_c to be added in parallel to produce an overall power factor of 0.9 lagging. Then find apparent power and effective value of line current.

Ans. (*a*) $P = 45$ kW, $Q = 21.81$ kVAR, $S = 52.9$ kVA, pf = 0.85 lagging, $I_L = 5.09$ A, (*b*) $Q_C = -6$ kVAR, $S = 50$ kVA, $I_L = 4.81$ A

11.42. A balanced Δ-connected load with impedances $Z_\Delta = 6 + j9\ \Omega$ is connected to a three-phase generator with an effective line voltage of 400 V. The lines between the load and the generator have resistances of 1 Ω each. Find the effective line current, power delivered by the generator, and power absorbed by the load.

Ans. $I_L = 54.43$ A, $P_g = 26666$ W, $P_l = 17777$ W

11.43. In Problem 11.42, find the effective line voltage at the load. *Ans.* $V_L = 340$ V

11.44. A three-phase generator feeds two balanced loads (9 kW at pf = 0.8 and 12 kW at pf = 0.6, both lagging) through three cables (0.1 Ω each). The generator is regulated such that the effective line voltage at the load is 220 V. Find the effective line voltage at the generator. *Ans.* 230 V

11.45. A balanced Δ-connected load has impedances $45 + j60\ \Omega$. Find the average power delivered to it at an effective line voltage of: (*a*) 400 V, (*b*) 390 V. *Ans.* (*a*) 3.84 kW, (*b*) 3.65 kW

11.46. Obtain the change in average power delivered to a three-phase balanced load if the line voltage is multiplied by a factor α. *Ans.* Power is multiplied by the factor α^2

11.47. A three-phase, three-wire source supplies a balanced load rated for 15 kW with pf = 0.8 at an effective line voltage of 220 V. Find the power absorbed by the load if the three wires connecting the source to the load have resistances of 0.05 Ω each and the effective line voltage at the source is 220 V. Use both a simplified approximation and also an exact method.

Ans. 14.67 kW (by an approximate method), 14.54 kW (by an exact method)

11.48. In Problem 11.47 determine the effective value of the line voltage such that the load operates at its rated values. *Ans.* 222.46 V (by an approximate method), 221.98 V (by an exact method)

11.49. What happens to the quantity of power supplied by a three-phase, three-wire system to a balanced load if one phase is disconnected? *Ans.* Power is halved.

11.50. A three-phase, three-wire generator with effective line voltage 6000 V is connected to a balanced load by three lines with resistances of 1 Ω each, delivering a total of 200 kW. Find the efficiency (the ratio of power absorbed by the load to power delivered by the system) if the power factor of the generator is (*a*) 0.6, (*b*) 0.9

Ans. (*a*) 98.5 percent (*b*) 99.3 percent.

11.51. A 60-Hz three-phase, three-wire system with terminals labeled 1, 2, 3 has an effective line voltage of 220 V. To determine if the system is ABC or CBA, the circuit of Fig. 11-35 is tested. Find the effective voltage between node 4 and line 2 if the system is (*a*) ABC, (*b*) CBA.

Ans. (*a*) 80.5 V; (*b*) 300.5 V

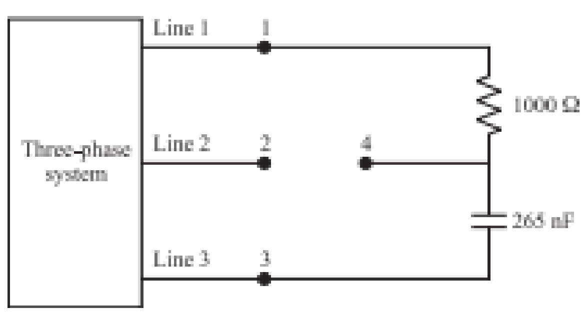

Fig. 11-35