Engineering Analysis

Lec.1

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3rd Year

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Differential Equations

Ordinary differential equations (ODEs) are differential equations that depend on a single variable.

The more difficult study of partial differential equations (PDEs), that is, differential equations that depend on several variables,

ODE

- An ordinary differential equation (ODE) is an equation that contains one or several derivatives of an unknown function, which we usually call y(x)

or sometimes y(t), if the independent variable is time t.

The equation may also contain y itself, known functions of x (or t), and constants. For example,

(1) $$y' = \cos x$$

(2) $$y'' + 9y = e^{-2x}$$

(3) $$y'y''' - \frac{3}{2}y'^2 = 0$$

are ordinary differential equations (ODEs). Here, as in calculus, y' denotes dy/dx, $y'' = d^2y/dx^2$, etc. The term *ordinary* distinguishes them from *partial differential equations* (PDEs), which involve partial derivatives of an unknown function of *two or more* variables. For instance, a PDE with unknown function u of two variables x and y is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

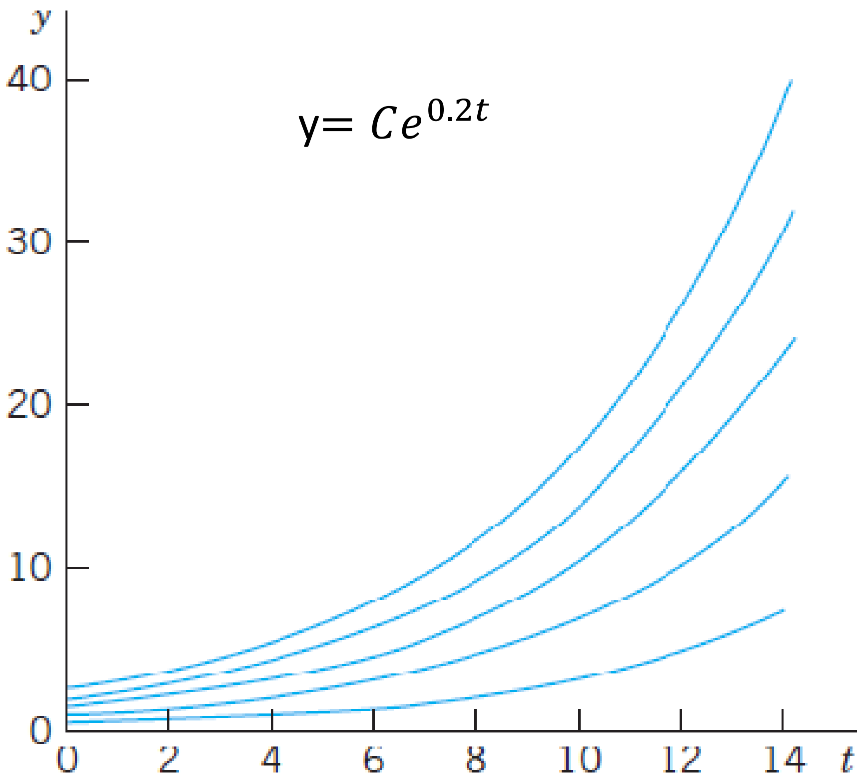


Fig. 4A. Solutions of $y' = 0.2y$ in Example 3 **(exponential growth)**

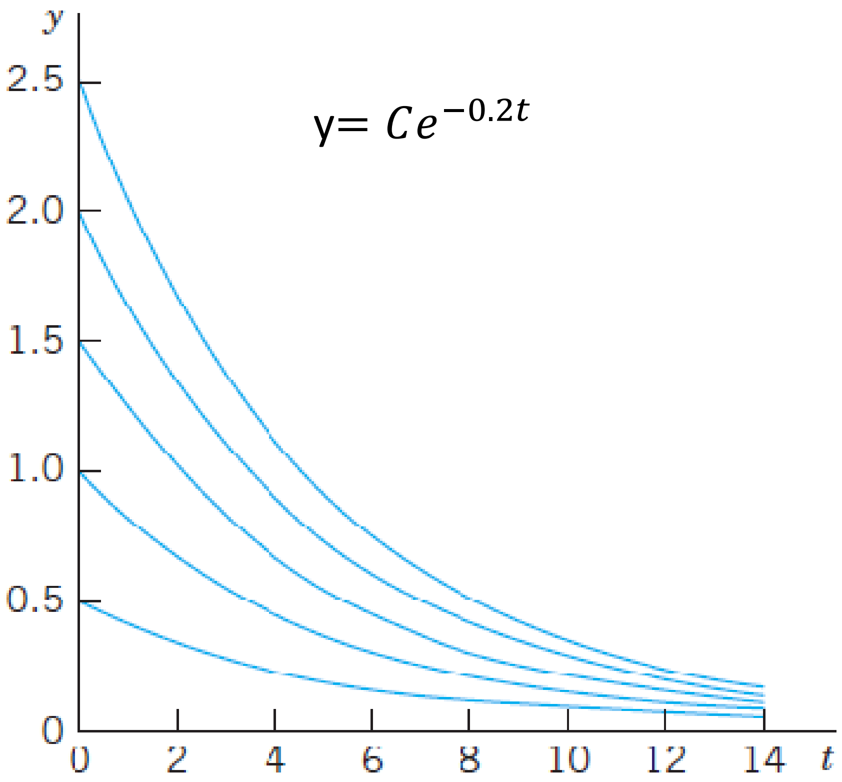


Fig. 4B. Solutions of $y' = -0.2y$ in Example 3 **(exponential decay)**

Geometrically, the general solution of an ODE is a family of infinitely many solution curves, one for each value of the constant *c*.

Initial value problem

by an **initial condition (X_0 , Y_0)**with given values X_0 and Y_0 that is used to determine a value of the arbitrary constant C.

, if the ODE is explicit, Y'=f(X , Y) and the initial value problem f(X_0)=Y_0

<u>Example</u>:

Solve the initial value problem

$$y' = \frac{dy}{dx} = 3y, \qquad y(0) = 5.7.$$

Solution. The general solution is $y(x) = ce^{3x}$; From this solution and the initial condition we obtain $y(0) = ce^0 = c = 5.7$. Hence the initial value problem has the solution $y(x) = 5.7e^{3x}$. This is a particular solution.

First-order ODE

Equations contain only the first derivative y' and may contain *y* and any given functions of *x*. Hence written as

F(x,y,y')=0 ; implicit form

or

Y'=f(x,y) ; explicit form

Example1 : y=c/x , y'= -c/x^2 explicit form

or xy'+y=0 implicit form

a) Verify that y is a solution of the ODE.
(b) Determine from y the particular solution of the initial value problem.

$$y' + 4y = 1.4, \quad y = ce^{-4x} + 0.35, \quad y(0) = 2$$

$$y' + 5xy = 0, \quad y = ce^{-2.5x^2}, \quad y(0) = \pi$$

Type of equations

- Separated equations (y-term)dy =(x-term) dx +C
- Homogeneous methods check firstly if f(kx,ky)=f(x,y)

  Solving by **Reduction to Separable Form**
- Solution by Integration Factor
- Exact Differential Equation

Homogeneous Method Reduction to Separable Form

We discuss this technique for a class of ODEs of practical importance, namely, for equations $Y' = f\left(\frac{y}{x}\right)$, Here, f is any (differentiable) function of y/x, such as: **sin(y/x)** , $(\boldsymbol{y}/\boldsymbol{x})^{\boldsymbol{4}}$, ….

The form of such an ODE suggests that we set <u>**u= y/x**</u>; thus,

y=ux ; and by product differentiation y'= u'x + u

Substituting y'=f(y/x) =f(u) , so f(u)=u'x+u and u'x=f(u)-u

, know variables separated $\frac{du}{f(u)-u} = \frac{dx}{x}$

Note: or suggests that we set <u>**u= x/y**</u>

Example : Solve (x^2+y^2)dx+2xydy=0

Let u=y/x …

And then let u=x/y

Tutorial C.W.

Q://solve these deferential equations by reproduction using separable form:

- x dy – y dx $-\sqrt{x^2 - y^2}$ dx =0

  Ans. CX=$e^{\sin^{-1} y/x}$

- (2x sinh(y/x) + 3y cosh (y/x))dx -3xcosh(y/x)dy=0

  Ans. $x^2 =$Csinh $^3 y/x$

- (2x+3y)dx +(y-x) dy=0

  Ans. C=ln($y^2 + 2xy + 2x^2 - 4\tan^{-1}\frac{x+y}{x}$

ODEs. Integrating Factors

The general form of DE that must be solved by Integration factor :

$$Y' + P(x)y = Q(x)$$

IF: ʅ(x)= $e^{\int P(x)dx}$

The solution will be ʅ(x) y=$\int$ ʅ(x) $Q(x)dx + C$

Or ,

The DE that will written in the form $\frac{dx}{dy}$ +P(y) x=Q(y)

IF: ʅ(y)= $e^{\int P(y)dy}$

The solution will be ʅ(y) x=$\int$ ʅ(y) $Q(y)dy + C$

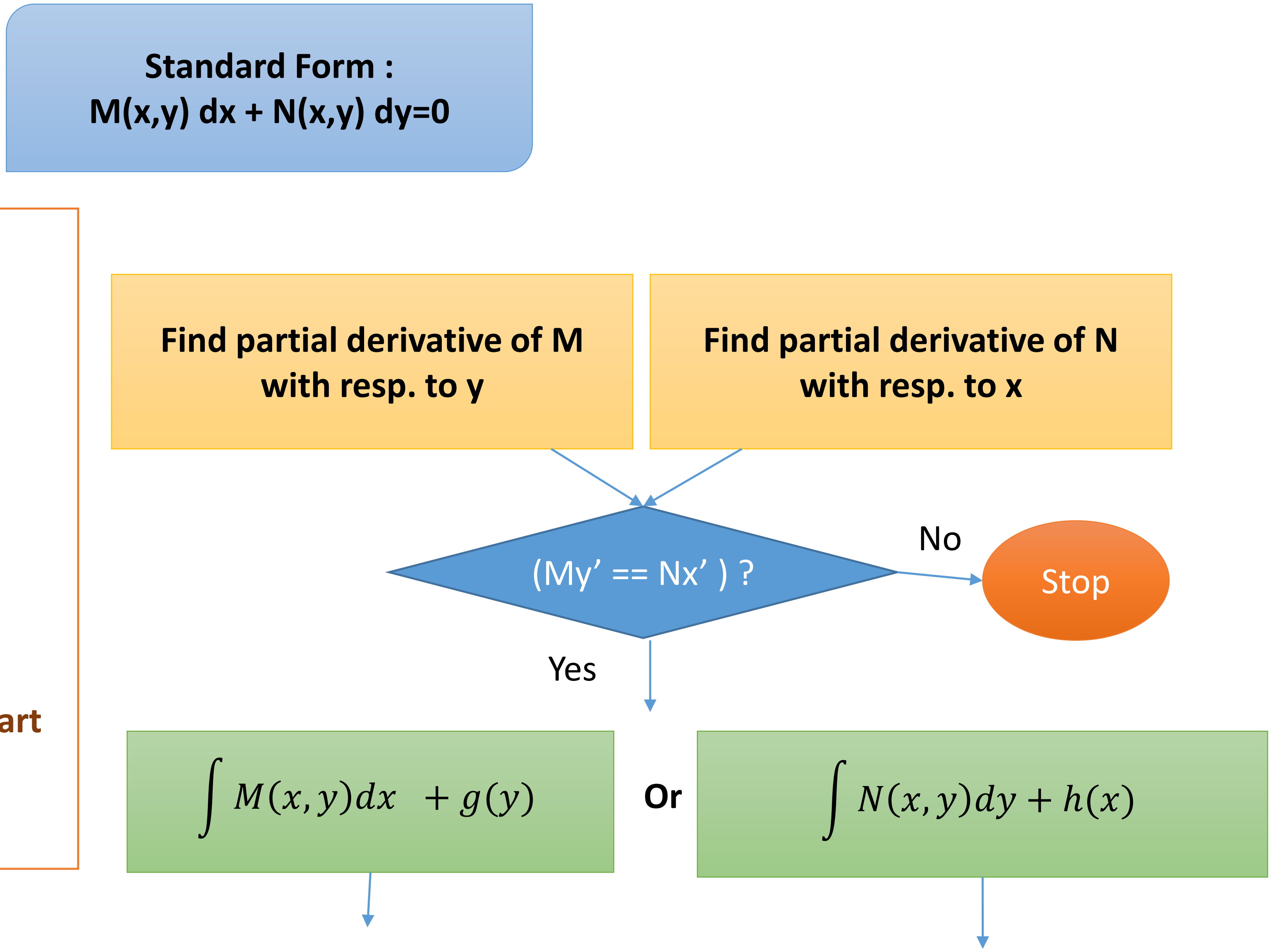
Solve the following DE using integration factor.

A) xy’ +3y= $\frac{\sin x}{x^2}$

B) $(y^2 + 1)$dx+(2xy+1)dy=0

C) $(x-1)^3\frac{dy}{dx} + 4(x-1)^2 y = x + 1$

Exact DE

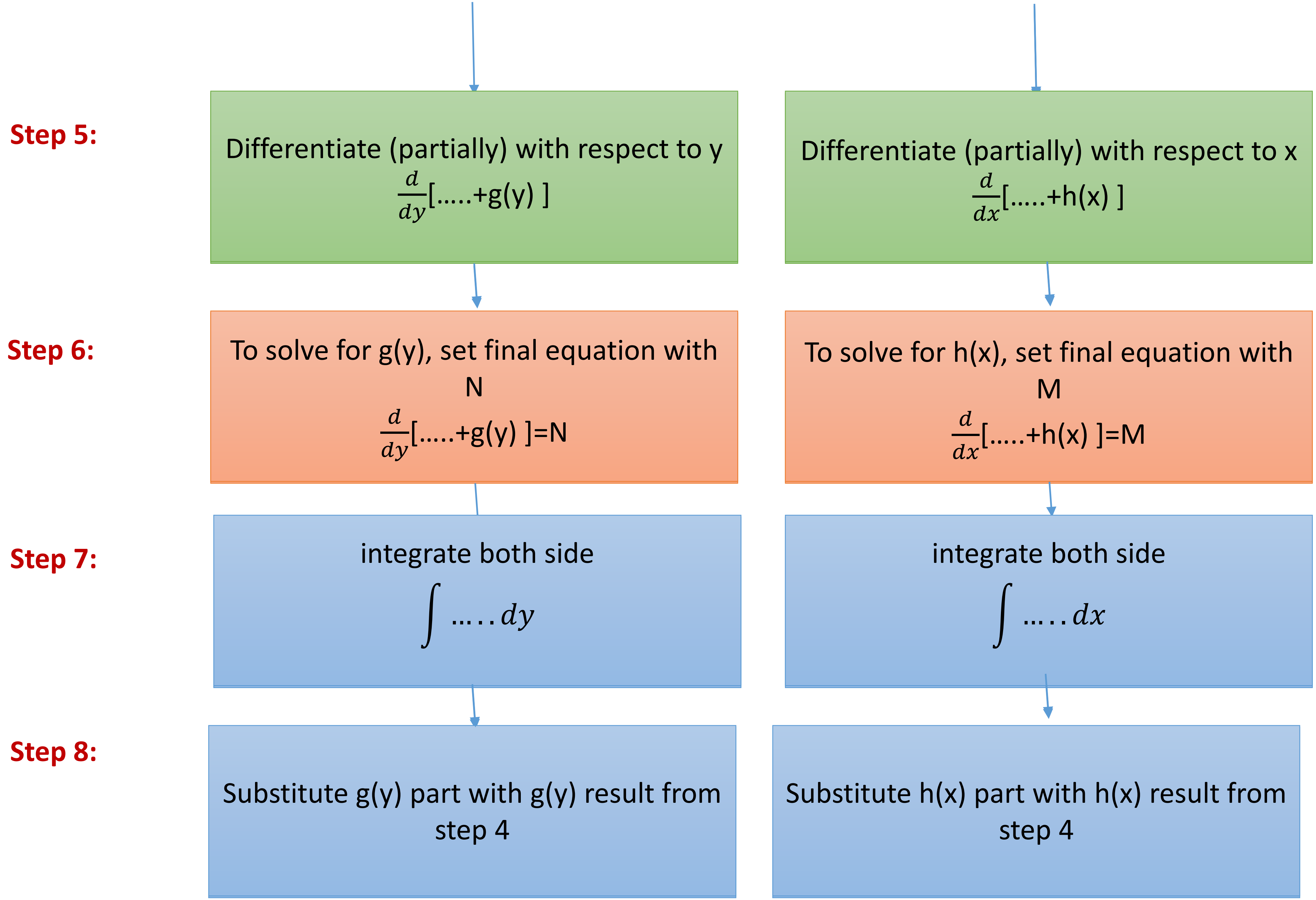


Step1:
Observe if the equation in S.F.

Step2:
Identify M(x,y), N(x,y), or My ,Nx

Step3:
Check if its Exact DE cont. , else stop

Step 4:
Integrate either M(x,y) part or N(x,y) part



Step 5:
Differentiate (partially) with respect to y
$\frac{d}{dy}$[…..+g(y)]
Differentiate (partially) with respect to x
$\frac{d}{dx}$[…..+h(x)]
Step 6:
To solve for g(y), set final equation with N
$\frac{d}{dy}$[…..+g(y)]=N
To solve for h(x), set final equation with M
$\frac{d}{dx}$[…..+h(x)]=M
Step 7:
integrate both side
$\int \ldots.. dy$
integrate both side
$\int \ldots.. dx$
Step 8:
Substitute g(y) part with g(y) result from step 4
Substitute h(x) part with h(x) result from step 4

Solve the following ODE

- Cos(x+y) dx+$(3y^2 + 2y + \cos(x + y))\, dy = 0$

- $(3x^2 + 1) + (3y^2 + 2y)$y'=0