Q1 State the existence and uniqueness theorem of fixed point, then use it to **show that**

$g(x) = (x^2 - 1)/3$ has a unique fixed point on the interval $[-1,1]$.

Q2// Let $p(x) = x^5 - 2x^4 - x^2 + 2x + 3$. Use synthetic division method to find

1. the **first** approximation $(x_1)$of a root of $p(x)$ (put $x_0 = 1$).
   the value of $(p' \circ p)(1)$.

Q3// Define an $(n \times n)$ strictly diagonally dominant matrix, then use it in **Gauss-Seidle** method to solve the linear system:

$$4x + 3y = 3 - 8z$$

$$6x - 4z = 4 + y$$

$$3x - 6y + 2z = 2$$

where $(x, y, z) = (0, 0, 0)$. (Stop iteration after one steps)

Q4// Use Modified Newton-Raphson Method to calculate (x_1, y_1, z_1) for the system:

$$f(x, y, z) = 2x^2 + y^4 - z^3$$

$$g(x, y, z) = x - 3y^3 + 5z$$

$h(x, y, z) = -x^2 - y + z^2$; put $(x_0, y_0, z_0) = (1,0,0)$.

Q5// The iteration form $x_{n+1} = g(x_n), n = 0, 1, 2, \ldots$ is used to find the root $x = \lambda$ of the equation $f(x)=0$, if $g(x) = x - h_1(x)f(x) - h_2(x)[f(x)]^2 - h_3(x)[f(x)]^3$, and the derivatives of g & f are exists , then prove that ***g(x)* is of order two** if $(1 - h_1(x)f'(x)) = 0$

Q6 Use Modified Newton Raphson method to find the first approximation (x_1, y_1) of the system:

$$x^2 - 3xy - 1 = 0$$

$xy^2 + 3x^2 = 0$, with $(x_0, y_0) = (1, 1)$.

Q7 Use the best x_0 and the best method among the FD.I.F, BD.I.F., and Bsseel method to estimate the value of $f(4)$ from the data (2.5,3), (3.5,5), (4.5, 6), and (5.5, 8)**.**

Q8\ Define k^{th} degree Spline function, then from the values (1,2), (3,5), (6,7), and (7, 10) determine *f*(5), using second degree Spline function.

Q9\ Consider a linear approximation $f(x) = Ax + B$ of the nodes $(x_i, y_i); i = 0,1, \dots, n$. Find the best value of A, and B such that the error $E_2 = \sqrt{\frac{(y_i - f(x_i))^2}{n}}$ is minimize.

Q1\ Let $\left.\begin{matrix} f(x,y) = 0 \\ g(x,y) = 0 \end{matrix}\right\}$ be a non-linear system of equations, and

$\left.\begin{matrix} x = F(x,y) \\ y = G(x,y) \end{matrix}\right\}$ be a Fixed-Point iteration form of it. Show that the sufficient condition for convergence of this iteration is $|F_x| + |G_x| < 1$ & $|F_y| + |G_y| < 1$.

Q2\ Find the approximate solution of the following system

$3x_1 + x_2 + x_3 = 2$

$x_1 + 5x_2 + 3x_3 = 3$

$4x_1 + 2x_2 + 8x_3 = 5$, using Triangular factorization method.

Q3\ **a-** Derive Lagrange interpolation polynomial of degree one.

b- Use the best method and best x_0 to estimate the value of *f*(1.9) and *f*(3) from the data (0.5,3), (1.5,5), (2.5, 6), and (3.5, 8)**.**