n-Hosoya Polynomials of Caterpillar Graphs

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Abstract

The n-diameter of a Caterpillar graph $T_{t,r}$ is obtained. The n-Hosoya Polynomials and n-Wiener indices of $T_{t,r}$ are obtained in this paper.

Keywords: Caterpillar graph, n-diameter, n-Hosoya polynomials, n-Wiener index ,Chemical tree.

1. Introduction:

In graph theory, a **caterpillar** or **caterpillar** tree is a tree in which all the vertices are within distance 1 of a central path. For two vertices u &v n a graph G, the distance from u to v is denoted by d (u, v) & defined as the length of a shortest u-v path in graph G.

We follow the terminology of [3], [4]. Let v be a vertex of a connected graph G, and let S be an (n-1)-subset of vertex set V(G), for $n \geq 2$, then the n-distance $d_n(v,S)$ is defined by [1]

$$d_n(v,S) = \min\{d(v,u) : u \in S\}. \qquad \text{.....(1.1)}$$

The n-diameter of G is defined by

$$\operatorname{diam}_n G = \max\{d_n(v,S) : v \in V(G), |S| = n-1, S \subseteq V(G)\}. \qquad \text{.....(1.2)}$$

The n-Wiener index of G is defined by

$$W_n(G) = \sum_{(v,S)} d_n(v,S). \qquad \text{.....(1.3)}$$

The n-Hosoya polynomial of connected graph G of order p is defined by

$$H_n(G;x) = \sum_{k=0}^{\delta_n} C_n(G,k)x^k, \qquad \text{.....(1.4)}$$

where $3 \leq n \leq p$, δ_n is the n-diameter of G , and $C_n(G,k)$ is the number of order pairs $(v,S), v \in V(G), S \subseteq V(G), |S| = n-1,$ such that $d_n(v,S) = k$.

One can easily show that [1].

$$C_n(G,0) = p\binom{p-1}{n-2},\ C_n(G,1) = p\binom{p-1}{n-1} - \sum_{v \in V(G)} \binom{p-1-\deg v}{n-1}. \qquad \text{.....}\quad (1.5)$$

The **n-Hosoya polynomial** of a vertex v in G, denoted by $H_n(v,G;x)$, is defined [1] by

$$H_n(v,G;x) = \sum_{k \geq 0} C_n(v,G,k)x^k, \qquad \text{.....(1.6)}$$

where $C_n(v,G,k)$ is the number of (n-1)-subsets of vertices S such that $d_n(v,S)=k$. It is clear that for each k, $0 \le k \le \delta_n$,

$$C_n(G,k) = \sum_{v\in V(G)} C_n(v,G,k), \qquad \text{….. (1.7)}$$

and

$$H_n(G;x) = \sum_{v\in V(G)} H_n(v,G;x), \qquad \text{….. (1.8)}$$

The following simple lemma is useful for obtaining $C_n(v,G,k)$ for every vertex v of a connected graph G.

Lemma 1.1: [2] Let t be the number of vertices of ordinary distance k from vertex v, and let s be the number of vertices of distance more than k from v in a connected graph G, then

$$C_n(v,G,k) = \binom{s+t}{n-1} - \binom{s}{n-1}, \qquad \text{….. (1.9)}$$

for $v \in V(G), 2 \le n \le p, 0 \le k \le \delta_n$. ■

Let T be a non-empty subset of vertices of G. We define

$$C_n(T,G,k) = \sum_{v\in T} C_n(v,G,k). \qquad \text{….. (1.10)}$$

We shall use this notation in our proofs.

Definition 1.2: A caterpillar graph $T_{t,r}$ is a tree obtained from a path $P_r: w_1, w_2, \ldots, w_r$, by attaching $(t-2)$, $t \ge 3$ **,** endvertices to each of the internal vertices of P_r ,and $(t-1)$ endvertices to each of its two terminals w_1 and w_r, so that $\deg v = t$ for each non end vertex of $T_{t,r}$, as shown in Fig.1.1.

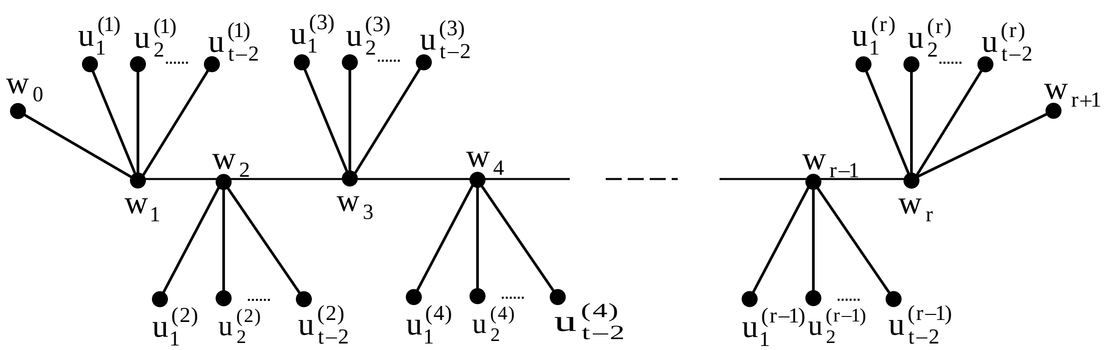

Fig.1.1. A caterpillar $T_{t,r}$

The vertices of $T_{t,r}$ are so labeled as in Fig.1.1. It is clear that $p(T_{t,r}) = r(t-1)+2$, $q(T_{t,r}) = r(t-1)+1$, and $\operatorname{diam} T_{t,r} = r+1$.

In this paper we determine the-diameter of $T_{t,r}$, the n-Hosoya polynomial of $T_{t,r}$.

2. The n-diameter of a caterpillar:

The following propostion determines the n-diameter of a caterpillar $T_{t,r}$.

Proposition 2.1: For $r, t \geq 2$, and $m(t-1)+2 \leq n \leq m(t-1)+t$, $0 \leq m \leq r$,

$$\mathrm{diam}_n T_{t,r} = r+1-m. \qquad ..... \quad (2.1)$$

Proof: For $1 \leq i \leq r$, let $A_i = \{w_{i+1}, u_1^{(i)}, u_2^{(i)}, \;....\;, u_{t-2}^{(i)}\}$. Moreover, let $B = \{w_0, u_1^{(1)}, u_2^{(1)}, \;....\;, u_{t-2}^{(1)}\}$.

From the definition of n-diameter, we notice that for $2 \leq n \leq t$, $\mathrm{diam}_n T_{t,r} = r+1$, that is (2.1) is true for $m = 0$. This is the case when we take $v \in B$ and $S \subseteq A_r$ (or we take $v \in A_{r+1}$, $S \subseteq B$) which implies that

$\mathrm{diam}_n T_{t,r} = d_n(v,S) = r+1$.

If $m = 1$, we take $v \in B$ and $S \subseteq A_r \cup A_{r-1}$, then $\mathrm{diam}_n T_{t,r} = d_n(v,S) = r$, $t+1 \leq n \leq 2t-1$.

If $m = 2$, we may take $v \in B$ and $S \subseteq A_r \cup A_{r-1} \cup A_{r-2}$, then $\mathrm{diam}_n T_{t,r} = d_n(v,S) = r-1$, $2t \leq n \leq 3t-2$.

So on, $m = j \leq r$, then we take $v \in B$, and $S \subseteq \bigcup_{i=r-j}^{r} A_i$, which implies

$\mathrm{diam}_n T_{t,r} = d_n(v,S) = r+1-j$, $2(t-1)+2 \leq n \leq j(t-1)+t$.

Thus (2.1) is true for all values of m, $0 \leq m \leq r$, which completes the proof. #

The following result determines the exact value of m for given n.

Corollary 2.2: For $r, t \geq 2$, and $2 \leq n \leq p$,

$$\mathrm{diam}_n T_{t,r} = r+1-\left\lfloor \frac{n-2}{t-1} \right\rfloor. \qquad ..... \quad (2.2)$$

Proof: From Propostion 2.1, we have $m(t-1)+2 \leq n \leq m(t-1)+t$, which implies $\frac{n-t}{t-1} \leq m \leq \frac{n-2}{t-1}$.

For $n \leq t$, we have $m = 0$, and for $n > t$, we have $\frac{n-2}{t-1} - \frac{t-2}{t-1} \leq m \leq \frac{n-2}{t-1}$.

Since $t \geq 2$ and m is nonnegative integer then $m = \left\lfloor \frac{n-2}{t-1} \right\rfloor$

Thus, from (2.1) we obtain (2.2). #

3.The n-Hosoya Polynomial of $T_{t,r}$:

In this section, we obtain $C_n(T_{t,r},k)$ for $2 \le k \le \mathrm{diam}_n T_{t,r}$ in a series of four propositions .

Let A_i, $1 \le i \le r$, be the set of $t-1$ vertices as defined in the proof of Propostion 2.1 , and let $B_i = \{w_{r-i}, u_1^{(r-i+1)}, u_2^{(r-i+1)}, \ \dots \ , u_{t-2}^{(r-i+1)}\}$, for $i = 1, 2, \ \dots \ , r$.

Propostion 3.1: For $2 \le n \le p (= r(t-1)+2)$, $r, t \ge 2$,

$$C_n(T_{t,r},2) = (p-r)\binom{p-2}{n-1} - \left[(p-2r)\binom{p-t-1}{n-1} + 2\binom{p-2t}{n-1} + (r-2)\binom{p-3t+1}{n-1}\right] \ldots\ldots \ (3.1)$$

Proof: One may check that for $v \in D = \{w_0, w_{r+1}\} \cup \{u_i^{(\ell)} : i = 1, 2, \ \dots \ , t-2 \ ; \ \ell = 1, 2, \ \dots \ , r\}$, there are $t-1$ vertices each of distance 2 from v , and there are $p-t-1$ vertices each of distance more that 2 from v . Thus , by Lemma 1.1 ,

$$C_n(D, T_{t,r}, 2) = [2 + r(t-2)]\left[\binom{p-2}{n-1} - \binom{p-t-1}{n-1}\right] . \qquad \ldots\ldots(3.2)$$

For $v \in \{w_1, w_r\}$, there are $t-1$ vertices of distance 2 from v , and there $p-2t$ vertices of distance more than 2 . Thus

$$C_n(v, T_{t,r}, 2) = \binom{p-t-1}{n-1} - \binom{p-2t}{n-1}, \text{ for } v = w_1, w_r . \qquad \ldots\ldots(3.3)$$

Finally , for $v \in D' = \{w_2, w_3, \ \dots \ , w_{r-1}\}$, there are $2t-2$ vertices each of distance 2 from v , and there are $p-3t+1$ vertices each of distance more than 2 from v . Thus by Lemma 1.1 ,

$$C_n(D', T_{t,r}, 2) = (r-2)\left[\binom{p-t-1}{n-1} - \binom{p-3t+1}{n-1}\right]. \qquad \ldots\ldots \ (3.4)$$

Therefore , from (3.2) , (3.3) , and (3.4) , we get (3.1). #

Remark(1): One can easily show that :

- $C_n(T_{2,r},2) = 2(r-1)\binom{r-1}{n-1}$.
- $C_n(T_{3,r},3) = (3r-4)\binom{2r-2}{n-1} - (2r-5)\binom{r-1}{n-1}$.
- $C_n(T_{3,r},4) = 2(r-1)\binom{r-1}{n-1}$.

Propostion 3.2: For $3 \le k \le \left\lfloor \frac{r}{2} \right\rfloor + 1$, $r \ge 4$, the coefficient $C_n(T_{t,r},k)$ of the n-Hosoya polynomial of the caterpillar $T_{t,r}$ is given by :

$$C_n(T_{t,r},k) = 2\left[\binom{p-(k-2)(t-1)-2}{n-1} - \binom{p-2(k-1)(t-1)-2}{n-1}\right]$$

$$+\binom{r-2k+2}{1}\left[\binom{p-(2k-3)(t-1)-2}{n-1}-\binom{p-(2k-1)(t-1)-2}{n-1}\right]$$

$$+2(t-2)\left[\binom{p-(k-2)(t-1)-2}{n-1}-\binom{p-2(k-2)(t-1)-2}{n-1}\right]$$

$$+(t-2)\binom{r-2k+4}{1}\left[\binom{p-(2k-5)(t-1)-2}{n-1}-\binom{p-(2k-3)(t-1)-2}{n-1}\right] \quad \text{...... (3.5)}$$

Proof: To find $C_n(w_i, T_{t,r}, k)$, $w_i \in W$, for $3 \le k \le \left\lfloor \frac{r}{2} \right\rfloor + 1$, we consider two cases :

Case (1): (a) $0 \le i \le k-1$; **(b)** $r-k+2 \le i \le r+1$.

(a) For $0 \le i \le k-1$, there are exactly $t-1$ vertices , namely $u_1^{(k+i-1)}, u_2^{(k+i-1)}, \ \ldots\ , u_{t-2}^{(k+i-1)}, w_{k+i}$ each of distance k from w_i , and there are $(p-(k+i-1)(t-1)-2)$ vertices each of distance more than k from w_i . Hence, by Lemma 1.1 ,

$$C_n(w_i, T_{t,r}, k) = \binom{p-(k+i-2)(t-1)-2}{n-1} - \binom{p-2(k+i-1)(t-1)-2}{n-1}. \quad \text{.... (3.6)}$$

(b) Because of the symmetry of $T_{t,r}$, we have

$$C_n(w_i, T_{t,r}, k) = C_n(w_{r+1-i}, T_{t,r}, k),\ 0 \le i \le k-1 . \quad \text{..... (3.7)}$$

Case (2): $i = k, k+1, \ \ldots\ , r-k+1$. For such values of i , there are exactly $2(t-1)$ vertices , namely $u_1^{(i-k+1)}, u_2^{(i-k+1)}, \ \ldots\ , u_{t-2}^{(i-k+1)}, w_{i-k}, u_1^{(i+k-1)}, u_2^{(i+k-1)}, \ \ldots\ , u_{t-2}^{(i+k-1)}, w_{i+k}$ each of distance k from w_i ; and there are $(p-(2k-1)(t-1)-2)$ vertices each of distance more than k from w_i . Hence , by Lemma 1.1 ,

$$C_n(w_i, T_{t,r}, k) = \binom{p-(2k-3)(t-1)-2}{n-1} - \binom{p-(2k-1)(t-1)-2}{n-1}, \text{ for}$$

$$i = k, k+1, \ \ldots\ , r-k+1 . \quad \text{..... (3.8)}$$

From (3.6) , (3.7) , and (3.8) , we have

$$C_n(W, T_{t,r}, k) = 2\sum_{i=0}^{k-1}\left[\binom{p-(k+i-2)(t-1)-2}{n-1} - \binom{p-2(k+i-1)(t-1)-2}{n-1}\right]$$

$$+\binom{r-2k+2}{1}\left[\binom{p-(2k-3)(t-1)-2}{n-1}-\binom{p-(2k-1)(t-1)-2}{n-1}\right]$$

$$=2\left[\binom{p-(k-2)(t-1)-2}{n-1}-\binom{p-2(k-1)(t-1)-2}{n-1}\right]$$

$$+\binom{r-2k+2}{1}\left[\binom{p-(2k-3)(t-1)-2}{n-1}-\binom{p-(2k-1)(t-1)-2}{n-1}\right]. \quad \text{.....(3.9)}$$

Also, there are two cases to find $C_n(u_i^{(\ell)}, T_{t,r}, k)$, $1 \le i \le t-2$, $1 \le \ell \le r$, for $3 \le k \le \left\lfloor \frac{r}{2} \right\rfloor + 1$.

Case-1: (a) $1 \le \ell \le k-2$; **(b)** $r-k+3 \le \ell \le r$.

(a) $1 \le \ell \le k-2$, there are exactly $t-1$ vertices, namely $u_1^{(k+\ell-2)}, u_2^{(k+\ell-2)}, \;\ldots\;, u_{t-2}^{(k+\ell-2)}, w_{k+\ell-1}$ each of distance k from $u_i^{(\ell)}$; and there are $(p-(k+\ell-2)(t-1)-2)$ vertices each of distance more than k from $u_i^{(\ell)}$. Therefore,

$$C_n(u_i^{(\ell)}, T_{t,r}, k) = \binom{p-(k+\ell-3)(t-1)-2}{n-1} - \binom{p-(k+\ell-2)(t-1)-2}{n-1}. \qquad \ldots\ldots (3.10)$$

(b) Because of the symmetry of $T_{t,r}$, we have

$$C_n(u_i^{(\ell)}, T_{t,r}, k) = C_n(u_i^{(r+1-\ell)}, T_{t,r}, k), \text{ for } 1 \le i \le t-2,\ 1 \le \ell \le k-2. \qquad \ldots\ldots (3.11)$$

Case -2 : For $i = k-1, k, \;\ldots\;, r-k+2$, and $1 \le i \le t-2$, there are exactly $2(t-1)$ vertices, namely $u_1^{(\ell-k+2)}, u_2^{(\ell-k+2)}, \;\ldots\;, u_{t-2}^{(\ell-k+2)}, w_{\ell-k+1}, u_1^{(\ell+k-2)}, u_2^{(\ell+k-2)}, \;\ldots\;, u_{t-2}^{(\ell+k-2)}, w_{\ell+k-1}$ each of distance k from $u_i^{(\ell)}$; and there are $(p-(2k-3)(t-1)-2)$ vertices each of distance more than k from $u_i^{(\ell)}$. Therefore, by Lemma 1.1,

$$C_n(u_i^{(\ell)}, T_{t,r}, k) = \binom{p-(2k-5)(t-1)-2}{n-1} - \binom{p-(2k-3)(t-1)-2}{n-1}, \text{ for}$$

$i = k-1, k, \;\ldots\;, r-k+2$, and $1 \le i \le t-2$ . ..... (3.12)

From (3.10) , (3.11) , and (3.12) , we have

$$C_n(U, T_{t,r}, k) = 2(t-2)\sum_{\ell=0}^{k-2}\left[\binom{p-(k+\ell-3)(t-1)-2}{n-1} - \binom{p-(k+\ell-2)(t-1)-2}{n-1}\right]$$
$$+ (t-2)(r-2k+4)\left[\binom{p-(2k-5)(t-1)-2}{n-1} - \binom{p-(2k-3)(t-1)-2}{n-1}\right]$$
$$= 2(t-2)\left[\binom{p-(k-2)(t-1)-2}{n-1} - \binom{p-2(k-1)(t-1)-2}{n-1}\right]$$
$$+ (t-2)(r-2k+4)\left[\binom{p-(2k-5)(t-1)-2}{n-1} - \binom{p-(2k-3)(t-1)-2}{n-1}\right]. \qquad \ldots\ldots (3.13)$$

Finally, from (3.9) and (3.13) , we have (3.5) . #

Propostion 3.3: For $\left\lfloor \frac{r}{2} \right\rfloor + 2 \le k \le \operatorname{diam}_n T_{t,r}$, $r \ge 4$, the coefficient $C_n(T_{t,r}, k)$ of the n-Hosoya polynomial of the caterpillar $T_{t,r}$ is given by :

$$C_n(T_{t,r}, k) = 2(t-1)\binom{p-(k-2)(t-1)-2}{n-1}. \qquad \ldots\ldots (3.14)$$

Proof: We have to consider two cases:

Case (1): (a) $0 \le i \le r+1-k$; **(b)** $k \le i \le r+1$.

(a)For $0 \le i \le r+1-k$, there are exactly $t-1$ vertices, namely $u_1^{(k+i-1)}, u_2^{(k+i-2)}, \ \ldots\ , u_{t-2}^{(k+i-2)}, w_{k+i}$ each of distance k from w_i , and there are $(p-(k+i-1)(t-1)-2)$ vertices each of distance more than k from w_i . Hence , by Lemma 1.1,

$$C_n(w_i, T_{t,r}, k) = \binom{p-(k+i-2)(t-1)-2}{n-1} - \binom{p-2(k+i-1)(t-1)-2}{n-1}.$$

(b) Because of the symmetry of $T_{t,r}$, we have

$C_n(w_i, T_{t,r}, k) = C_n(w_{r+1-i}, T_{t,r}, k)$, $0 \le i \le r+1-k$. Thus

$$C_n(W, T_{t,r}, k) = 2\sum_{i=0}^{r+1-k}\left[\binom{p-(k+i-2)(t-1)-2}{n-1} - \binom{p-2(k+i-1)(t-1)-2}{n-1}\right]$$

$$= 2\binom{p-(k-2)(t-1)-2}{n-1}. \qquad ..... \ (3.15)$$

Case -2: (c) $1 \le \ell \le r+2-k$; **(d)** $k-2 \le \ell \le r$.

(c) For $1 \le \ell \le r+2-k$, there are exactly $t-1$ vertices , namely $u_1^{(k+\ell-2)}, u_2^{(k+\ell-2)}, \ \ldots\ , u_{t-2}^{(k+\ell-2)}, w_{k+\ell-1}$ each of distance k from $u_i^{(\ell)}$; and there are $(p-(k+\ell-2)(t-1)-2)$ vertices each of distance more than k from $u_i^{(\ell)}$. Therefore,

$$C_n(u_i^{(\ell)}, T_{t,r}, k) = \binom{p-(k+\ell-3)(t-1)-2}{n-1} - \binom{p-(k+\ell-2)(t-1)-2}{n-1}.$$

Because of the symmetry of $T_{t,r}$, we have

$C_n(u_i^{(\ell)}, T_{t,r}, k) = C_n(u_i^{(r+1-\ell)}, T_{t,r}, k)$, for $1 \le i \le t-2$, $1 \le \ell \le r+2-k$.

Thus

$$C_n(U, T_{t,r}, k) = 2(t-2)\sum_{\ell=1}^{r+2-k}\left[\binom{p-(k+\ell-3)(t-1)-2}{n-1} - \binom{p-(k+\ell-2)(t-1)-2}{n-1}\right]$$

$$= 2(t-2)\binom{p-(k-2)(t-1)-2}{n-1}. \qquad ..... \ (3.16)$$

From (3.15) and (3.16) we obtain (3.14) . #

Theorem 3.4: The n-Hosoya Polynomial of the Caterpillar graph $T_{t,r}$ of order $p = r(t-1)+2$, is given by :

$$H_n(T_{t,r}; x) = p\binom{p-1}{n-2} + \left[p\binom{p-1}{n-1} - (p-r)\binom{p-2}{n-1} - r\binom{p-t-1}{n-1}\right]x + \sum_{k=2}^{\delta_n} C_n(T_{t,r}, k)x^k .$$

And the n-Wiener index of $T_{t,r}$ is

$$W_n(T_{t,r}) = p\binom{p-1}{n-1} - (p-r)\binom{p-2}{n-1} - r\binom{p-t-1}{n-1} + \sum_{k=2}^{\delta_n} kC_n(T_{t,r}, k),$$

where $C_n(T_{t,r},k)$, $r \geq 2$, $2 \leq k \leq \delta_n$ ($\delta_n = diam_n T_{t,r}$) given in Propositions 3.1, 3.2, 3.3 and Remark (1). #

Putting t=4 in Theorem 3.4, we get the n-Hosoya polynomial of the chemical tree $T_{4,r}$ in the next corollary.

Corollary 3.5 : The n-Hosoya polynomial of $T_{4,r}$

$$H_n(T_{4,r};x) = p\binom{p-1}{n-2} + \left[p\binom{p-1}{n-1} - (p-r)\binom{p-2}{n-1} - r\binom{p-5}{n-1}\right]x + \sum_{k=2}^{\delta_n} C_n(T_{4,r},k)x^k \text{, where}$$

$$C_n(T_{4,r},2) = (p-r)\binom{p-2}{n-1} - \left[(r+2)\binom{p-5}{n-1} + 2\binom{p-8}{n-1} + (r-2)\binom{p-11}{n-1}\right]$$

$$C_n(T_{4,r},k) = 6\binom{p-3k+4}{n-1} + \binom{r-2k+2}{1}\left[\binom{p-6k+7}{n-1} - \binom{p-6k+1}{n-1}\right] - 4\binom{p-6k+10}{n-1}$$
$$- 2\binom{p-6k+4}{n-1} + 2\binom{r-2k+4}{1}\left[\binom{p-6k+13}{n-1} - \binom{p-6k+7}{n-1}\right],$$

$$\text{for } 3 \leq k \leq \left\lfloor \frac{r}{2} \right\rfloor + 1, \qquad \text{..... (3.17)}$$

$$C_n(T_{4,r},k) = 6\binom{p-3k+4}{n-1}, \text{ for } \left\lfloor \frac{r}{2} \right\rfloor + 2 \leq k \leq r+1-\left\lfloor \frac{n-2}{3} \right\rfloor, \qquad \text{..... (3.18)}$$

Corollary 3.6: The n-Wiener index of $T_{4,r}$ is :

$$W_n(T_{4,r}) = p\binom{p-1}{n-1} + (p-r)\binom{p-2}{n-1} - \left[(p+2)\binom{p-5}{n-1} + 4\binom{p-8}{n-1} + 2(r-2)\binom{p-11}{n-1}\right]$$
$$+ \sum_{k=3}^{\delta_n} kC_n(T_{4,r},k),$$

Where $C_n(T_{4,r},k)$ are given in (3.17) and (3.18), $3 \leq k \leq \delta_n = r+1-\left\lfloor \frac{n-2}{3} \right\rfloor$. #

Corollary 3.7 : The Hosoya polynomial of $T_{t,r}$ of order $r(t-1)+2$ is

$$H(T_{t,r};x) = p + qx + \frac{1}{2}rt(t-1)x^2 + (t-1)^2\sum_{k=3}^{r+1}(r-k+2)x^k.$$

Its Wiener index is :

$$W(T_{t,r}) = 1 + r(t^2-1) + (t-1)^2\left[\binom{r+3}{3} - (3r+1)\right].$$

Proof : Follows from Theorem 3.4. #

Remark (2): We notice that the coefficients of $H(T_{t,r};x)$ satiffy $d(T_{t,r},0) > d(T_{t,r},1) < d(T_{t,r},2) < d(T_{t,r},3) > d(T_{t,r},4) > \ ... \ > d(T_{t,r},r+1)$, which is strong-unimodal, and strictly decreasing from the third term [5] .

Corollary 3.8 : For chemical tree $T_{t,r}$,we have

$$H(T_{4,r};x) = (3r+2) + (3r+1)x + 6rx^2 + 9\sum_{k=3}^{r+1}(r-k+2)x^k \ ,$$

and

$$W(T_{t,r}) = 9\binom{r+3}{3} - 4(3r+2) . \quad \#$$

متعددات حدود هوسويا – n لبيانات التراكتورات

<u>الملخص</u>

يضمن هذا البحث إيجاد القطر-n لبيان بشكل تراكتورات $T_{t,r}$ ، كما تم إيجاد متعددات حدود هوسويا-n ودليل وينر-n للبيان $T_{t,r}$.

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