Q1 / If V and W are finite dimensional vector spaces, then V isomorphic to W if and only if dimV=dim W, prove.
10 marks

Q2/ Let T∈ L(R^3, R^2) defined by T(x,y,z) = (x+y, x+z). Find the basis for R^3 and R^2 which the matrix M_T for T relative to this basis is normal form.
10 marks

Q3/A- Define linear transformation, Is T: $M_{2\times2}$(R) → R^4, defined by T$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$= (a, -b, 0, d), linear transformation?

B- Let V and W be vector spaces over F and T: V→ W be a linear transformation, then T is one to one if kernT ={0}. Prove.
5+5 marks

Q4/ Define kernel, and Image of linear transformation, if T: P_2(R)→ R^4 be a linear transformation defined by T(a+bx+cx^2) = (a+b, 0, a+b, 3c), then find the following:

1- kernT and ImagT.

2- rank and nullity of T

3- Is T bijective (one to one and on to)?
12 marks

Q5/ A- If T : P_1(R) → P_2(R) defined by T(a+bx) = ax +bx^2, find the matrix M_T relative to the basis {2, 1-x} for P_1(R) and { -1, 3x, -2x^2} for P_2(R).
8 marks

Q6/ Define the inverse of linear transformation T, If T: R^2 → R^3 defined by T(x,y) =(x, x+2y, x-y), and S: R^3 → R^2 defined by S(x, y, z) =(x, y-2x+z), show that S is left inverse of T. Is S right inverse of T?
10 marks