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SHAMBHUNATH INSTITUTE OF ENGINEERING AND TECHNOLOGY

FLUID MACHINERY (RME-601)

B.TECH. SIXTH SEMESTER

FIRST SESSIONAL EXAMINATION, EVEN SEMESTER, (2019-2020)
BRANCH: MECHANICAL ENGINEERING

Time –1hr. 30 min **Maximum Marks – 30**

SECTION – A

1. Attempt all questions in brief: **(5*1 = 5)**

Q.N.	QUESTION	Marks	CO	BL
a.	State the impulse momentum principle. **Solution:** The impulse-momentum theorem states that the change in momentum of an object equals the impulse applied to it. The impulse-momentum theorem is logically equivalent to Newton's second law of motion (the force law).	1	1	1
b.	Define degree of reaction. **Solution:** Degree of reaction or reaction ratio (R) is defined as the ratio of the pressure energy change in the rotor to the total energy change..	1	1	1
c.	Differentiate between impulse turbine and a reaction turbine? **Solution:** The basic and main difference between impulse and reaction turbine is that there is pressure change in the fluid as it passes through runner of reaction turbine while in impulse turbine there is no pressure change in the runner. ... So it uses kinetic energy as well as pressure energy to rotate the turbine.	1	1	1
d.	What is the function of the nozzle in an impulse turbine? **Solution: Nozzle** is used to increase the kinetic energy of water. It increases the velocity and decreases the pressure.	1	2	1
e.	Write the specifications of francis turbine. **Solution:** It is a medium head, medium discharge, mixed flow, medium specific speed. It is a type of radially inward flow reaction turbine	1	2	1

SECTION - B

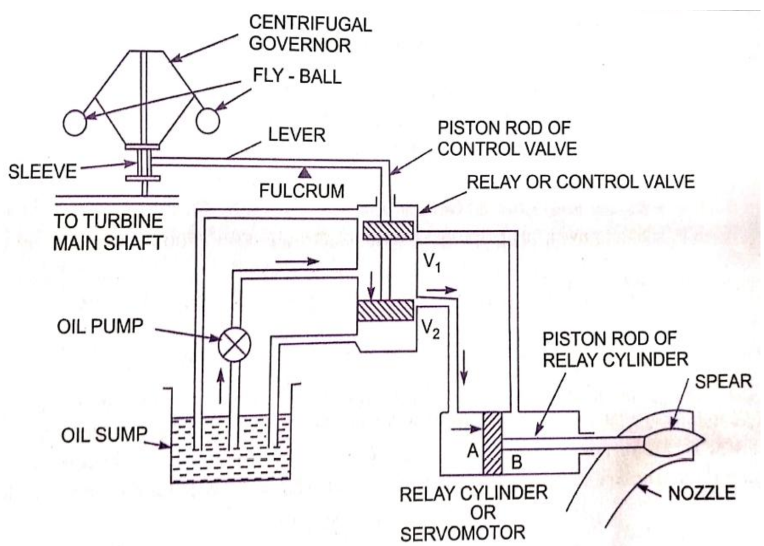
2. Attempt any two parts of the following: **(2*5 = 10)**

Q.N.	QUESTION	Marks	CO	BL
a.	Explain the Governing of a Pelton Turbine. Use neat sketch. **Solution:** **Governing of Pelton Turbine (Impulse Turbine)** Governing of Pelton turbine is done by means of oil pressure governor, which consists of the following parts : 1. Oil sump. 2. Gear pump also called oil pump, which is driven by the power obtained from turbine shaft 3. The Servomotor also called the relay cylinder. 4. The control valve or the distribution valve or relay valve. 5. The centrifugal governor or pendulum which is driven by belt or gear from the turbine shaft. 6. Pipes connecting the oil sump with the control valve and control valve with servomotor and 7. The spear rod or needle. Figure shows the position of the piston in the relay cylinder, position of control or relay raise and fly-balls of the centrifugal governor, when the turbine is running at the normal speed.	5	1	2

When the load on the generator decreases, the speed of the generator increases. This increases the speed of the turbine beyond the normal speed. The centrifugal governor, which is connected to the turbine main shaft, will be rotating at an increased speed . Due to increase in the speed of the centrifugal governor, the fly-balls move upward due to the increased centrifugal force on them. Due to the upward movement of the fly-balls, the sleeve will also move upward. A horizontal lever, supported over a fulcrum, connects the sleeve and the piston rod of the control valve. As the sleeve moves up, the lever turns about the fulcrum and the piston rod of the control valve moves downward. This closes the V1 and opens the valve V2 as shown in Figure.

The oil, pumped from the oil pump to the control valve or relay valve, under pressure will flow through the valve V2 to the servomotor (or relay cylinder) and will exert force on the face **A** of the piston of the relay cylinder. The piston along with piston rod and spear will move towards right. This will decrease the area of flow of water at the outlet of the nozzle. This decrease of area of flow will reduce the rate of flow of water to the turbine which consequently reduces the speed of the turbine-When the speed of the turbine becomes normal, the fly-balls, sleeve, lever and piston rod of control valve come to its normal position as shown in Figure.

When the load on the generator increases, the speed of the generator and hence of the turbine decreases. The speed of the centrifugal governor also decreases and hence centrifugal force acting on the fly-balls also reduces. This brings the fly-balls in the downward direction. Due to this, the sleeve moves downward and the lever turns about the fulcrum, moving the piston rod of the control valve in the upward direction. This closes the valve V2 and opens the valve V1. The oil under pressure from the control valve, will move through valve V1 to the servomotor and will exert a force on the face B of the piston. This will move the piston along with the piston rod and spear towards left, increasing the area of flow of water at the outlet of the nozzle. This will increase the rate of flow of water to the turbine and consequently, the speed of the turbine will also increase, till the speed of the turbine becomes normal.



b.	A jet of water of diameter 50 mm having a velocity of 20 m/s strikes at inlet of a curved vane which is moving with a velocity of 10 m/s in the direction of the jet. The jet leaves the vane at an angle of 60^o to the direction of motion of vane at outlet. Determine the force exerted by the jet on the vane in the direction of motion and work done per second by the jet. **Solution:**	5	1	1

Solution. Given :

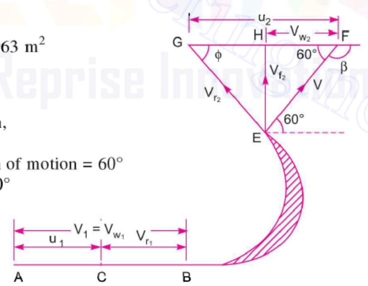


Fig. 17.18

Diameter of the jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

As jet and vane are moving in the same direction,

∴ $\alpha = 0$

Angle made by the leaving jet, with the direction of motion = 60°

∴ $\beta = 180° - 60° = 120°$

For this problem, we have

$$u_1 = u_2 = u = 10 \text{ m/s}$$
$$V_{r_1} = V_{r_2}$$

From Fig. 17.18, we have

$$V_{r_1} = AB - AC = V_1 - u_1 = 20 - 10 = 10 \text{ m/s}$$
$$V_{w_1} = V_1 = 20 \text{ m/s}$$
$$\therefore \quad V_{r_2} = V_{r_1} = 10 \text{ m/s}$$

Now in ΔEFG, $EG = V_{r_2} = 10$ m/s,

$GF = u_2 = 10$ m/s

$\angle GEF = 180° - (60° + \phi) = (120° - \phi)$

From sine rule, we have

$$\frac{EG}{\sin 60°} = \frac{GF}{\sin(120° - \phi)} \quad \text{or} \quad \frac{10}{\sin 60°} = \frac{10}{\sin(120° - \phi)}$$

or $\sin 60° = \sin(120° - \phi)$

∴ $60° = 120° - \phi$ or $\phi = 120° - 60° = 60°$

Now $V_{w_2} = HF = GF - GH$

$$= u_2 - V_{r_2} \cos\phi = 10 - 10 \times \cos 60° = 10 - 5 = 5 \text{ m/s.}$$

(*i*) The force exerted by the jet on the vane in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}] \quad \text{(–ve sign is taken as } \beta \text{ is an obtuse angle)}$$
$$= 1000 \times .001963 \times 10 [20 - 5] \text{ N} = \mathbf{294.45 \text{ N. Ans.}}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 294.45 \times 10 = 2944.5 \text{ N m/s}$$
$$= \mathbf{2944.5 \text{ W. Ans.}} \quad [\because \text{Nm/s} = \text{W (watt)}]$$

c.	Determine the power given by the jet of water to the runner of a pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through the jet water is 0.03 m^3/s. The side clearance angle is 15° and take $c_v = 0.975$.	5	1	3

Solution:

Solution. Given :

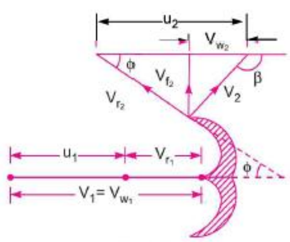


Fig. 18.9

Tangential velocity of wheel, $u = u_1 = u_2 = 20$ m/s

Net head, $H = 50$ m

Discharge, $Q = 0.03$ m³/s

Side clearance angle, $\phi = 15°$

Co-efficient of velocity, $C_v = 0.975$

Velocity of the jet, $V_1 = C_v \times \sqrt{2gH}$

$$= 0.975 \times \sqrt{2 \times 9.81 \times 50}$$
$$= 30.54 \text{ m/s}$$

From inlet triangle, $V_{w_1} = V_1 = 30.54$ m/s

$$V_{r_1} = V_{w_1} - u_1 = 30.54 - 20.0 = 10.54 \text{ m/s}$$

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 10.54 \text{ m/s}$$
$$V_{r_2} \cos\phi = 10.54 \cos 15° = 10.18 \text{ m/s}$$

As $V_{r_2} \cos\phi$ is less than u_2, the velocity triangle at outlet will be as shown in Fig. 18.9.

$$\therefore \quad V_{w_2} = u_2 - V_{r_2}\cos\phi = 20 - 10.18 = 9.82 \text{ m/s.}$$

Also as β is an obtuse angle, the work done per second on the runner,

$$= \rho a V_1 \left[V_{w_1} - V_{w_2}\right] \times u = \rho Q \left[V_{w_1} - V_{w_2}\right] \times u$$
$$= 1000 \times .03 \times [30.54 - 9.82] \times 20 = 12432 \text{ Nm/s}$$

$$\therefore \quad \text{Power given to the runner in kW} = \frac{\text{Work done per second}}{1000} = \frac{12432}{1000} = \mathbf{12.432 \text{ kW. Ans.}}$$

d.	An inward flow reaction turbine has external and internal diameters as 1 m and 0.5 m respectively. The velocity of flow through the runner is constant and is equal to 1.5 m/s. Determine: (i) Discharge through the runner, and (ii) Width of the turbine at outlet if the width of the turbine at inlet is 200 mm **Solution:** External diameter of turbine, $D_1 = 1$ m Internal diameter of turbine, $D_2 = 0.5$ m Velocity of flow at inlet and outlet, $V_{f_1} = V_{f_2} = 1.5$ m/s Width of turbine at inlet, $B_1 = 200$ mm $= 0.20$ m Let the width at outlet $= B_2$ Using equation (18.21) for discharge, $Q = \pi D_1 B_1 \times V_{f_1} = \pi \times 1 \times 0.20 \times 1.5 =$ **0.9425 m^3/s. Ans.** Also $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 B_1 = D_2 B_2$ $(\because \pi V_{f_1} = \pi V_{f_2})$ $\therefore B_2 = \frac{D_1 \times B_1}{D_2} = \frac{1 \times 0.20}{0.5} = 0.40$ m = **400 mm. Ans.**	5	2	3

SECTION - C

3. Attempt any one part of the following: **(1*5 = 5)**

Q.N.	QUESTION	Marks	CO	BL
a.	Discuss the classification of hydraulic turbines. **Solution:** **▶ 18.5 CLASSIFICATION OF HYDRAULIC TURBINES** The hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbines. Thus the following are the important classifications of the turbines : 1. According to the type of energy at inlet : (*a*) Impulse turbine, and (*b*) Reaction turbine. 2. According to the direction of flow through runner : (*a*) Tangential flow turbine, (*b*) Radial flow turbine, (*c*) Axial flow turbine, and (*d*) Mixed flow turbine. 3. According to the head at the inlet of turbine : (*a*) High head turbine, (*b*) Medium head turbine, and (*c*) Low head turbine. 4. According to the specific speed of the turbine : (*a*) Low specific speed turbine, (*b*) Medium specific speed turbine, and (*c*) High specific speed turbine. If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as **impulse turbine.** As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **reaction turbine**. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water. If the water flows along the tangent of the runner, the turbine is known as **tangential flow turbine.** If the water flows in the radial direction through the runner, the turbine is called **radial flow turbine**. If the water flows from outwards to inwards, radially, the turbine is known as **inward** radial flow turbine, on the other hand, if water flows radially from inwards to outwards, the turbine is known as **outward** radial flow turbine. If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine. If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called **mixed flow** turbine.	5	2	4
b.	Write about the main parts of the pelton turbine.Also describe its working. **Solution:**	5	1	2

Fig. 18.1 shows the layout of a hydroelectric power plant in which the turbine is Pelton wheel. The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner. The main parts of the Pelton turbine are :

1. Nozzle and flow regulating arrangement (spear),
2. Runner and buckets,
3. Casing, and
4. Breaking jet.

1. Nozzle and Flow Regulating Arrangement. The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. 18.2. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

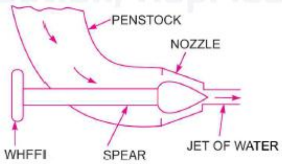


Fig. 18.2 *Nozzle with a spear to regulate flow.*

2. Runner with Buckets. Fig. 18.3 shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

3. Casing. Fig. 18.4 shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. Breaking Jet. When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

4. Attempt any one part of the following: (1*5 = 5)

Q.N.	QUESTION	Marks	CO	BL
a.	Derive the formula for the hydraulic efficiency of a pelton turbine.	5	1	4

Solution:

$\therefore$ **Hydraulic efficiency,** $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \quad \text{...(18.12)}$$

Now $V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

and $V_{w_2} = V_{r_2}\cos\phi - u_2 = V_{r_2}\cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (18.12),

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\left[1 + \cos\phi\right] u}{V_1^2}. \quad \text{...(18.13)}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

or $\dfrac{(1 + \cos\phi)}{V_1^2}\dfrac{d}{du}(2uV_1 - 2u^2) = 0$ or $\dfrac{d}{du}\left[2uV_1 - 2u^2\right] = 0$ $\left(\because \dfrac{1 + \cos\phi}{V_1^2} \neq 0\right)$

or $2V_1 - 4u = 0$ or $u = \dfrac{V_1}{2}$ ...(18.14)

Equation (18.14) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \dfrac{V_1}{2}$ in equation (18.13).

$$\therefore \quad \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + \cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1 + \cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1 + \cos\phi)}{2}. \quad \text{...(18.15)}$$

b.	A Francis turbine with an overall efficiency of 76% is required to produce 150 kW power. It is working under a head of 8 m. The peripheral velocity = $0.25\sqrt{2gh}$ and the radial velocity of flow at inlet is $0.95\sqrt{2gh}$. The wheel runs at 150 r.p.m. and the hydraulic losses in the turbine are 20 % of the available energy. Assuming radial discharge, determine : (i) The guide blade angle, (ii) The wheel vane angle at inlet, (iii) Diameter of the wheel at inlet, and (iv) Width of the wheel at inlet.	5	2	6

Solution:

Solution. Overall efficiency, $\eta_0 = 76\%$

Shaft power produced, $P = 150$ kW.

Head, $H = 8$ m

Peripheral velocity, $u = 0{\cdot}25\sqrt{2gH}$

Radial velocity of flow at inlet, $V_{f1} = 0{\cdot}95\sqrt{2gH}$

Wheel speed, $N = 150$ r.p.m.

Since discharge at the outlet is *radial*; $V_{w2} = 0$, $V_{f2} = V_2$

Hydraulic losses in the turbine = 20% of available energy

Now,
$$u_1 = 0{\cdot}25\sqrt{2 \times 9{\cdot}81 \times 8} = 3{\cdot}13 \text{ m/s}$$
$$V_{f1} = 0{\cdot}95\sqrt{2 \times 9{\cdot}81 \times 8} = 11{\cdot}9 \text{ m/s}$$

Hydraulic efficiency,
$$\eta_h = \frac{\text{Total head at inlet} - \text{hydraulic losses}}{\text{Total head at inlet}}$$
$$= \frac{H - 0{\cdot}2H}{H} = 0{\cdot}8$$

Also,
$$\eta_h = \frac{V_{w1}u_1}{gH} \qquad [\because V_{w2} = 0]$$

$\therefore$
$$0{\cdot}8 = \frac{V_{w1} \times 3{\cdot}13}{9{\cdot}81 \times 8} \quad \text{or} \quad V_{w1} = \frac{0{\cdot}8 \times 9{\cdot}81 \times 8}{3{\cdot}13} = 20{\cdot}0 \text{ m/s}$$

(*i*) **The guide blade angle, α :**

From inlet velocity triangle (Fig. 18·24),
$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{11{\cdot}9}{20{\cdot}0} = 0{\cdot}595$$
$\therefore$
$$\alpha = \tan^{-1} 0{\cdot}595 = \mathbf{30{\cdot}75^\circ} \textbf{ (Ans.)}$$

(*ii*) **The wheel vane angle at inlet, θ :**
$$\tan\theta = \frac{V_{f1}}{(V_{w1} - u_1)} = \frac{11{\cdot}9}{(20{\cdot}0 - 3{\cdot}13)} = 0{\cdot}705$$
$\therefore$
$$\theta = \tan^{-1} 0{\cdot}705 = \mathbf{35{\cdot}18^\circ} \textbf{ (Ans.)}$$

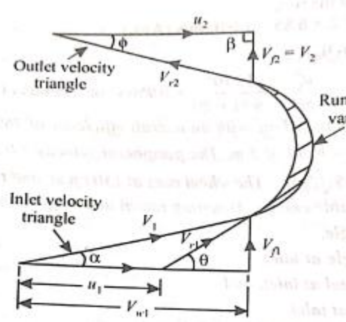


Fig. 18.24

(*iii*) **Diameter of the wheel at inlet, D_1 :**

Using the relation : $u_1 = \frac{\pi D_1 N}{60}$, we get
$$D_1 = \frac{60u_1}{\pi N} = \frac{60 \times 3{\cdot}13}{\pi \times 150} = \mathbf{0{\cdot}398 \text{ m}} \textbf{ (Ans.)}$$

(*iv*) **Width of the wheel at inlet, B_1 :**

Overall efficiency,
$$\eta_0 = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{wQH}$$

or,
$$0{\cdot}76 = \frac{150}{9{\cdot}81 \times Q \times 8}$$

or,
$$Q = \frac{150}{0{\cdot}76 \times 9{\cdot}81 \times 8} = 2{\cdot}515 \text{ m}^3\text{/s}$$

Also,
$$Q = \pi D_1 B_1 \times V_{f1}$$
$$2{\cdot}515 = \pi \times 0{\cdot}398 \times B_1 \times 11{\cdot}9$$
$\therefore$
$$B_1 = \frac{2{\cdot}515}{\pi \times 0{\cdot}398 \times 11{\cdot}9} = \mathbf{0{\cdot}169 \text{ m}} \textbf{ (Ans.)}$$

5. Attempt any one part of the following: **(1*5 = 5)**

Q.N.	QUESTION	Marks	CO	BL
a.	An Inward flow reaction turbine works at 450 rpm under a head of 120 m. Its diameter at inlet is 120 cm and the flow area is 0.4 m^2.The angles made by absolute and relative velocities at inlet are 20°and 60° respectively with the tangential velocity. Determine: (i)Volume flow rate, (ii) The power developed and (iii) Hydraulic efficiency .Assume Whirl at outlet to be zero.	5	2	5

Solution:

Solution. Given :

Speed of turbine, $N = 450$ r.p.m.

Head, $H = 120$ m

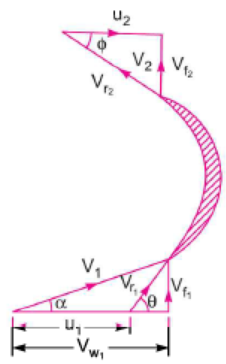
Diameter at inlet, $D_1 = 120$ cm $= 1.2$ m

Flow area, $\pi D_1 \times B_1 = 0.4\ m^2$

Angle made by absolute velocity at inlet, $\alpha = 20°$

Angle made by the relative velocity at inlet, $\theta = 60°$

Whirl at outlet, $V_{w_2} = 0$



$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle,

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \text{ or } \tan 20° = \frac{V_{f_1}}{V_{w_1}} \text{ or } \frac{V_{f_1}}{V_{w_1}} = \tan 20° = 0.364$$

$$\therefore \quad V_{f_1} = 0.364\, V_{w_1}$$

Also
$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364\, V_{w_1}}{V_{w_1} - 28.27} \qquad (\because \; V_{f_1} = 0.364\, V_{w_1})$$

or
$$\frac{0.364\, V_{w_1}}{V_{w_1} - 28.27} = \tan\theta = \tan 60° = 1.732$$

$$\therefore \quad 0.364 V_{w_1} = 1.732(V_{w_1} - 28.27) = 1.732 V_{w_1} - 48.96$$

or
$$(1.732 - 0.364)\, V_{w_1} = 48.96$$

$$\therefore \quad V_{w_1} = \frac{48.96}{(1.732 - 0.364)} = 35.789 = 35.79 \text{ m/s.}$$

From equation (i), $V_{f_1} = 0.364 \times V_{w_1} = 0.364 \times 35.79 = 13.027$ m/s.

(a) Volume flow rate is given by equation (18.21) as $Q = \pi D_1 B_1 \times V_{f_1}$

But $\pi D_1 \times B_1 = 0.4\ m^2$ (given)

$Q = 0.4 \times 13.027 =$ **5.211 m^3/s. Ans.**

(b) Work done per sec on the turbine is given by equation (18.18),

$$= \rho Q\, [V_{w_1} u_1] \qquad (\because \; V_{w_2} = 2)$$

$$= 1000 \times 5.211\, [35.79 \times 28.27] = 5272402 \text{ Nm/s}$$

$$\therefore \quad \text{Power developed in kW} = \frac{\text{Work done per second}}{1000} = \frac{5272402}{1000} = \mathbf{5272.402\ kW.\ Ans.}$$

(c) The hydraulic efficiency is given by equation (18.20B) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = \mathbf{85.95\%.\ Ans.}$$

Q.N.	QUESTION	Marks	CO	BL
b.	Establish the relation $R = 1 - \left(\frac{v_1{}^2 - v_2{}^2}{2gH_e}\right)$.Where R is the degree of reaction.	5	2	4

Solution:

$$\therefore \quad \text{Change in pressure energy inside the runner per unit weight} = \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r_2}^2 - V_{r_1}^2}{2g} \qquad \ldots(iii)$$

Now the equation (18.20C) becomes as

$$R = \frac{\text{Change of pressure energy inside the runner per unit weight}}{\text{Change of total energy inside the runner per unit weight}}$$

$$= \left(\left(\frac{u_1^2 - u_2^2}{2g}\right) + \frac{V_{r_2}^2 - V_{r_1}^2}{2g}\right) \Bigg/ \left[\left(\frac{V_1^2 - V_2^2}{2g}\right) + \left(\frac{u_1^2 - u_2^2}{2g}\right) + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2g}\right)\right]$$

or
$$R = \frac{\left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)} \qquad \ldots(18.20F)$$

or
$$R = \frac{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right) - \left(V_1^2 - V_2^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)}$$

$$= 1 - \frac{\left(V_1^2 - V_2^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)} \qquad \ldots(18.20G)$$

From equation (18.20E) , we know that

$$H_e = \frac{V_1^2 - V_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r_2}^2 - V_{r_1}^2}{2g}$$

or $$2gH_e = (V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)$$

Now the equation (18.20*G*) can be written as

$$R = 1 - \frac{\left(V_1^2 - V_2^2\right)}{2g\ H_e} \qquad ...(18.20H)$$

(*ii*) *For an actual reaction turbine,* generally, the angle β is 90° so that the loss of kinetic energy at outlet is minimum (*i.e.*, V_2 is minimum).

Hence in outlet velocity triangle, V_{w_2} becomes zero (*i.e.*, $V_{w_2} = 0$). Also $V_2 = V_{f_2}$ [Refer to Fig. 18.11(*b*)]

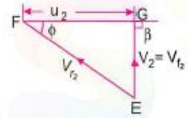


Fig. 18.11 (*b*)

Also there is not much change in velocity of flow. This means $V_{f_1} = V_{f_2}$

From equation (18.20*D*), we know that

$$H_e = \frac{1}{g}[V_{w_1}u_1 + V_{w_2}u_2]$$

$$= \frac{1}{g}V_{w_1}u_1 \qquad \left(\because\ V_{w_2} = 0\right)$$

$$= \frac{1}{g}[V_{f_1}\cot\alpha]\,[V_{f_1}\cot\alpha - V_{f_1}\cot\theta] \qquad \text{[Refer to Fig. 18.11(a)]}$$

$$[\because\ V_{w_1} = V_{f_1}\cot\alpha \text{ and } u_1 = V_{w_1} - V_{f_1}\cot\theta = V_{f_1}\cot\alpha - V_{f_1}\cot\theta]$$

$$= \frac{1}{g}V_{f_1}^2\cot\alpha\,[\cot\alpha - \cot\theta]$$

Now $$V_1^2 - V_2^2 = (V_{f_1}\operatorname{cosec}\alpha)^2 - V_{f_2}^2 \qquad (\because\ V_2 = V_{f_2})$$

$$= V_{f_1}^2\operatorname{cosec}^2\alpha - V_{f_1}^2 \qquad (\because\ V_{f_2} = V_{f_1})$$

or $$V_1^2 - V_2^2 = V_{f_1}^2(\operatorname{cosec}^2\alpha - 1)$$

$$= V_{f_1}^2\cot^2\alpha \qquad (\because\ \operatorname{cosec}^2\alpha - 1 = \cot^2\alpha)$$

Substituting the value of H_e and $(V_1^2 - V_2^2)$ in equation (18.20*H*), we get

$$R = 1 - \frac{V_{f_1}^2\cot^2\alpha}{2g\times[\frac{1}{g}V_{f_1}^2\cot\alpha(\cot\alpha - \cot\theta)]}$$

$$= 1 - \frac{\cot\alpha}{2(\cot\alpha - \cot\theta)} \qquad ...(18.20I)$$