زانكۆى سەلاحەدین - هەولێر
Salahaddin University-Erbil

On the Wagner Special Graph Wag (n)

Research Project

Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc. in Mathematics

Prepared by:

Niga Burhan Abdulmajeed

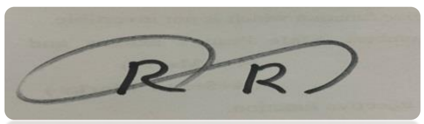
Supervised by:

Assist. Prof. Dr. Rashad Rashid Haji

April – 2023

Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.



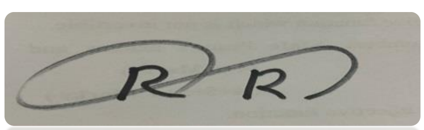
Signature:

Supervisor**: Dr. Rashad Rashid Haji**

Scientific grade**: Assistant Professor**

Date**: 7 / 4 /2023**

In view of available recommendations, I forward this word for debate by the examining committee.



Signature:

Name**: Dr. Rashad Rashid Haji**

Scientific grade**: Assistant Professor**

Chairman of the Mathematics Department

Date: **7** / **4** /**2023**

Acknowledgment

First of all I would like to thanks God for helping me to complete this project with success.

Secondly, I would like to express my special thanks to my supervisor Assist. Prof. Dr. Rashad Rashid Haji, it has been great honor to be his student.

It is great pleasure for me to undertake this project I have taken efforts however it would not have been possible without the support and help of many individuals.

Also I would like to express my gratitude towards my parents and my friends.

My thanks appreciations go to Mathematical Department and all my valuable teachers.

ABSTRACT

In this work we find the mean distance of a new special graph named Wagner graphW_n , which is $\mu(W_n) = \frac{2m^2+2m-1}{4m-1}$. Also, we study some graphical properties of the special graph W_n, such as regularity, non Eulerity, Hamiltonity. The radius of W_n , and the diameter of W_n.

List of contents

Certification of the Supervisor……………..ii

Acknowledgment…...iii

Abstract……………………………………………………………………………………...iv

List of contents………………………………………...…………………………….............v

Chapter One………………………………………………………………………………….1

Introduction………………………………………………………………………………1

Chapter Two………………………………………………………………………………….3

Literature Review………………………………..………………………………………3

Chapter Three……………………………………………………………………………...7

3.1. The mean distance of the special graph ***Wag(n)***……………………………….……………...7

3.2. Some Graphical properties of the special graph ***Wag(n)***……………………………..……..12

Reference……………………………………………………………………………….……..13

پوخته.………………………………………………………………………………….........a

CHAPTER ONE

INTRODUCTION

Graph theory, is a branch of Mathematics concerned with networks of points connected by lines. The subject of graph theory had its beginnings in recreational math problems (see number game), but it has grown into a significant area of mathematical research, with applications in chemistry, operations research, social sciences, and computer science.(C. Carlson, 2022).

The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island—but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially he proved the first theorem in graph theory. (C. Carlson, 2022).

As used in graph theory, the term graph does not refer to data charts, such as line graphs or bar graphs. Instead, it refers to a set of vertices (that is, points or nodes) and of edges (or lines) that connect the vertices. When any two vertices are joined by more than one edge, the graph is called a multi graph. A graph without loops and with at most one edge between any two vertices is called a simple graph. Unless stated otherwise, graph is assumed to refer to a simple graph. When each vertex is connected by an edge to every other vertex, the graph is called a complete graph. When appropriate, a direction may be assigned to each edge to produce what is known as a directed graph, or digraph. (C. Carlson, 2022).

We think of a graph as a set of Points in a Plane or in 3-space and a set of line segments (possibly curved), each of which either joins two points or joins a point to itself.

Graphs are highly Versatile models for analyzing a wide range of Practical Problems in which Points and connections between them have some Physical or conceptual interpretation.

Placing such analysis on solid footing requires Precise definitions, terminology and notation (Behzad & C. Hartrand, 1979).

CHAPTER TWO

Literature Review

Definition 2.1 :(Behzad & CHartrand, 1979) A *graphs* G is a finite non empty set of objects called *vertices* (the singular word is *vertex*) together with a (Possibly empty) set of un order pairs of distinct vertices of G called edges.

Definition 2.2: (Behzad & CHartrand, 1979) The cardinality or the number of the vertex set of a graph G is called the *order* of G, denoted by $p(G)$ or simply p .

Definition 2.3: (Behzad & CHartrand, 1979) The cardinality or the number of the edge set of a graph G is called the *size* of G, is denoted by $q(G)$ or simply q.

Definition 2.4 :(CHartarnd. *G*, 2016) Two vertices u and v in a graph G are *connected* if G contains a $u - v$ path. The grap G is itself *connected* if every two vertices of G are *connected.*

Definition 2.5 :(Behzad & CHartrand, 1979) A graph G is *complete* graph if every two of its vertices are adjacent and denoted by K_p if G_p is a (p, q) graph.

Definition 2.6: (P.Dankelmann., 1997)Let G be a *connected* graph, and $v \in V(G)$, the *transmission* of v is denoted by $\sigma(v)$ and defined by $\sigma(v) = \sum_{u \epsilon V(G)} d(u, v)$.

Definition 2.7: (P. Dankelmann, 1997) Let G be a *connected* graph, then the *transmission* of a graph G is denoted by $\sigma(G)$ and defined by $\sigma(G) = \sum_{v \epsilon V(G)} \sigma(v)$.

Definition 2.8: (P. Dankelmann, 1997) Let G be a connected graph, then the *mean distance* (*average distance*) of G is denoted by $\mu(G)$ and defined by $\mu(G) = \frac{\sigma(G)}{p(p-1)}$,where p is the order of G .That is ,The *average distance* of a graph G is the average value of the distance between all pairs of vertices in G .

Definition 2.9 :(Behzad & CHartrand, 1979) In a graph, The number of edges incident on a vertices V , With self-loops counted twice is called the *degree of the vertices* and is denoted by $deg(v)$ or simply $d(v)$.

Definition 2.10 :(Ping Zhang L. L. 2015) The *dimeter* of a graph , is denoted by $\dim(G)$, is the maximum eccentricity of any vertices in the graph , that is, $\dim(G) = \max\ e(v), v \epsilon V(G)$.

Definition 2.11:(Ping Zhang L. L. 2015) A graph is called a *regular* graph if degree of each vertices is equal a graph is called *K-regular* if degree of each vertices in the graph is K **.**

Definition 2.12:(Ping Zhang L. L. 2015) The *radius* of a graph G , denoted by $rad(G)$, is the minimum eccentricity of any vertices in the graph that is $rad(G) = \min e(v)$.

Definition 2.13 :(Ping Zhang L. L. 2015) If a graph has a closed trail (it starts and finishes at the same vertex) that uses every edge, it is called *Eulerian graph*.

Definition 2.14 : (Ping Zhang L. L. 2015) A connected graph G is called *Hamiltonian graph* if there is a cycle which includes every vertex of G and the cycle is called *Hamiltonian cycle*. Hamiltonian walk in graph G is a walk that passes through each vertex exactly once.

Definition 2.15: (P. Dankelmann, 1997) The *Eccentricity* of a vertex is the maximum distance from it to any other vertex.

Definition 2.16: (Akshay, 2020) In graph theory, a walk is defined as a finite length alternating sequence of vertices and edges. The total number of edges covered in a walk is called as *Length of the Walk*.

CHAPTER THREE

3.1. The mean distance of the special graph *Wag(n)*

Definition 3.1.1: The *Wagner graph* is a 3-regular connected graph has 12 vertices and 18 edges, as given in Figure 3.1

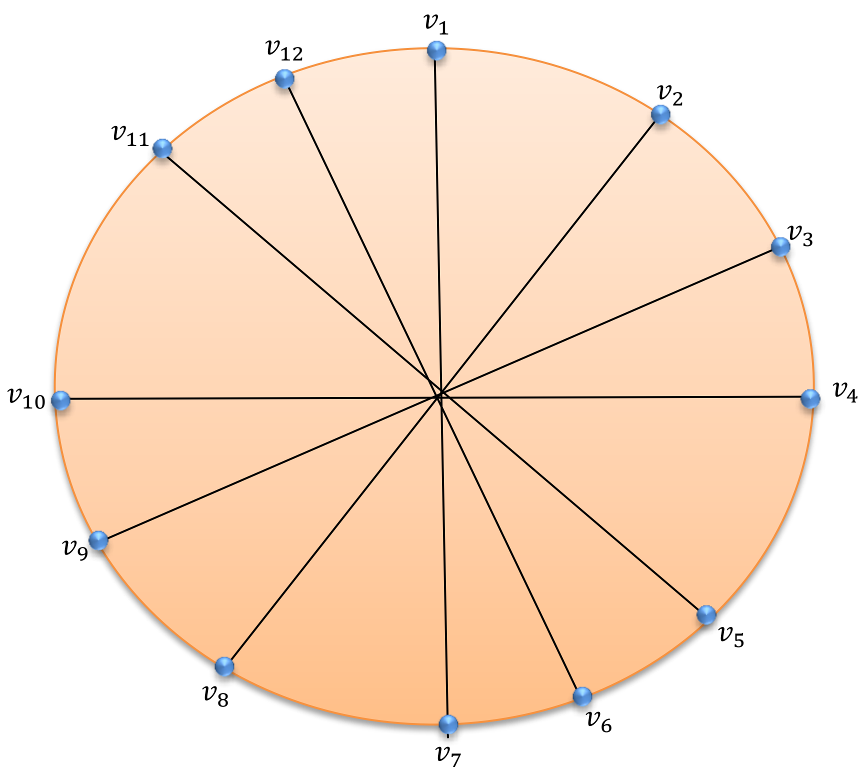


Figure 3.1 Wagner graph $Wag(n)$

Theorem 3.1.2: The mean distance of the special graph $W(n)$ is $\mu(W_n)=\frac{2m^2+2m-1}{4m-1}$.

Proof: If $n=4m$, for $m \in Z^+$, we have to find the transmission of the vertices of the graph W_n as follows.

$\sigma(v_1) = 1+2+\cdots+m+m+\cdots+2+1+2+\cdots+m+m+(m-1)+\cdots+2+1$

Then $\sigma(v_1)=4\sum_{i=1}^{m} i+(-1)$

$$=4\left(\frac{m(m+1)}{2}\right)+(-1)$$

$$=2m(m+1)-1$$

Since all vertices of the Wagner Special graph (W_n) are symmetric in adjacency.

$$\sigma(W_n)=\sigma(W_{4m})$$

$$=\sum_{i=1}^{4m}\sigma(v_i)$$

$$=4m(2m(m+1)-1)$$

Then, $\mu(W_n)=\mu(W_{4m})$

$$=\frac{\delta(W_{4m})}{4m(4m-1)}$$

$$=\frac{4m(2m(m+1)-1)}{4m(4m-1)}$$

$$=\frac{2m^2+2m-1}{4m-1}.$$

Example 3.1: The mean distance of the Wagner graph W_6 as given in Figure 3.2 can be found as follows

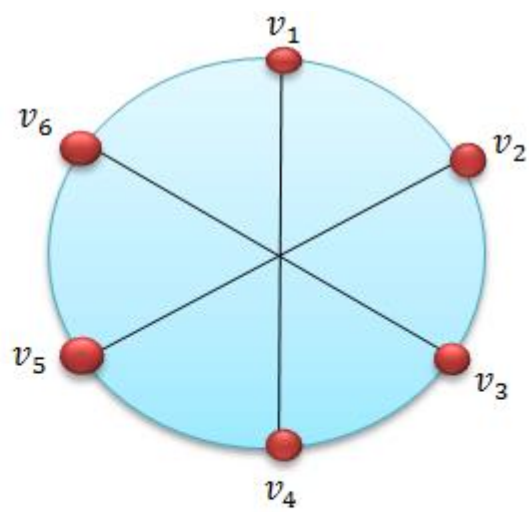


Figure 3.2 Wagner graph $Wag(n)$

The transmission of the vertices of the Wagner graph W_6 are

$$\sigma(v_1) = \sum_{u \epsilon W_6} d(v_1, u) = 1 + 2 + 1 + 2 + 1 = 7$$

$$= \sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \sigma(v_5) = \sigma(v_6)$$

And the transmission of the graph W_6 is

$\sigma(W_6) = \sum_{i=1}^{6} \sigma(v_i) = 6(7) = 42.$

So the mean distance of the special graph W_6 is

$\mu(W_6) = \frac{\sigma(G)}{p(p-1)} = \frac{42}{6(6-1)} = \frac{42}{6(5)} = \frac{7}{5}.$

Example 3.2: The mean distance of the Wagner graph W_8 as given in Figure 3.3 can be find as follows

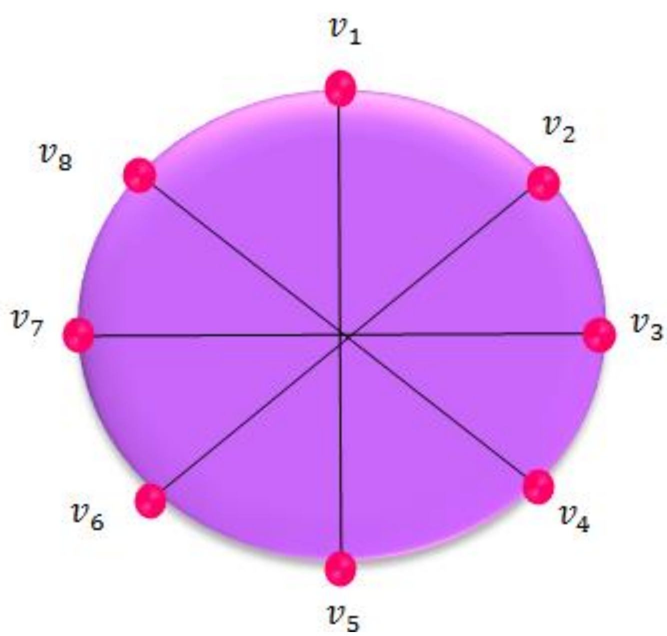


Figure 3.3 Wagner graph $Wag(n)$

The transmission of the vertices of the special graph W_8 are

$\sigma(v_1) = \sum_{u \epsilon W_8} d(v_1, u) = 1 + 2 + 2 + 1 + 2 + 2 + 1 = 11$

$= \sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \sigma(v_2) = \sigma(v_2) = \sigma(v_2) = \sigma(v_2)$

And the transmission of the special graph W_8 is

$$\sigma(W_8) = \sum_{i=1}^{8} \sigma(\mathrm{v_i}) = 8(11) = 88.$$

So the mean distance of the special graph W_8 is

$$\mu(G) = \frac{\sigma(G)}{p(p-1)} = \frac{88}{8(8-1)} = \frac{88}{8(7)} = \frac{11}{7}.$$

3.2. Some Graphical properties of the special graph $\boldsymbol{Wag(n)}$

3.2.1 Degree of vertices of $\boldsymbol{W_n}$ The Degree of vertices of the special graph W_n is $\deg(v) = 3$, for all $v \in V(W_n)$.

3.2.2 Regularity The special graph W_n is a 3-regular graph.

3.2.3 Non Eulerian of $\boldsymbol{W_n}$ The special graph W_n is not Eulerian, since the degree of vertices of W_n are odd.

3.2.4 Hamiltonian of $\boldsymbol{W_n}$ The special graph W_n is a Hamiltonian graph, since the cycle C_n passing throng all vertices of W_n.

3.2.5 Radius of $\boldsymbol{W_n}$ The Radius of the special graph W_n is $rad\ (W_n)$=$\min_{v\in V(W_n)} e(v)$, for all $v \in V(W_n)$. Then

$rad\ (W_6)$=$\min_{v\in V(W_6)} e(v) = 2$ = $rad\ (W_8)$=$\min_{v\in V(W_6)} e(v)$.

And $rad\ (W_{10})$=$\min_{v\in V(W_6)} e(v) = 2$.

But $rad\ (W_{12})$=$\min_{v\in V(W_6)} e(v) = 3$.

3.2.6 Diameter of $\boldsymbol{W_n}$ The Diameter of the special graph W_n is $diam\ (W_n)$=$\max_{v\in V(W_n)} e(v)$, for all $v \in V(W_n)$.Then

$diam\ (W_6)$=$\max_{v\in V(W_6)} e(v) = 2$= $diam\ (W_8)$=$\max_{v\in V(W_6)} e(v) = 2$.

and $diam\ (W_{10})$=$\max_{v\in V(W_6)} e(v) = 2$.

But $diam\ (W_{12})$=$\max_{v\in V(W_6)} e(v) = 3$.

Reference

Adrian Bondy, U. M. (2008). Graph Theory. London: Springer London.

Akshay, S. (2020, feb 2). Walk in Graph Theory | Path | Trail | Cycle | Circuit . Retrieved feb 2, 2020, from https://www.gatevidyalay.com/walk- in-graph-theory/: https://www.gatevidyalay.com/walk-in-graph-theory/

Behzad, M., & CHartrand, G. (1979). Graph and Digraphs. California: Wodsworth.

C.Carlson, S. (2022). Graph Theory. Encyclopædia Britannica, Inc.

CHartarnd. G, L. (2016). Graph and Digraph. london: Taylor and Francis group. LLc.

Jonathan, L. G., & Jay, Y. (2019). Graph THeory and it's Applications. london: Taylor and Francis group.LLC.

Li, R. (2011). A note on the average distance of a graph. JDMSC, 14 (6): 531-536.

P.Dankelmann. (1997). Average distance and domination number. Germany: Discrete Applied Mathematics.

Ping Zhang, L. L. (2015). Graph and Digraphs. Chapman&Hall/CRC.

Thomassen, P. S. (2022). Jornal of Graph Theory. Wiley Periodcals,LLC.

پوختە

لەم کارەدا تێکرای دووری گرافێکی تایبەت بە ناوی(Wagner graph) دەدۆزینەوە کە بە $Wag(n)$ هێمامان کردوە .کە تێدا دووری ئەو گرافە تایبەتە بریتییە لە

$\mu(W_n)$=$\frac{2m^2+2m-1}{4m-1}$.

لەگەڵ دیراسە کردنی هەندێك سیفاتی گرافی ئەم گرافە تایبەتەمان وەك دۆزینەوەی (رێکی، نا ئۆیلەری،هامڵتۆنی، نیوە تیرە ، تیرەی گرافی تایبەتی $Wag(n)$).