زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

The Average Distance Of Carpet Graph

Research Project

Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc. in Mathematics

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Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Date**: 4/4 /2023**

In view of available recommendations, I forward this word for debate by the examining committee.

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Scientific grade**: Assistant Professor**

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Date:**5/4/2023**

Acknowledgment

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My thanks appreciations go to Mathematical Department and all my valuable teachers.

Abstract

In this work we define the new special graph named Carpet graph $Cp(n)$ of order $2n$ as given in Figure 1 and we find the mean (average) distance $Cp(n)$ which is $\mu(Cp(n)) = \frac{2n^2+3n-2}{3(2n-1)}$ with two Remarks in which we find the mean distance of $Cp(2)$ and $Cp(3)$ which are $\frac{4}{3}$ and $\frac{5}{3}$ respectively Moreorer,we give same interesting examples.

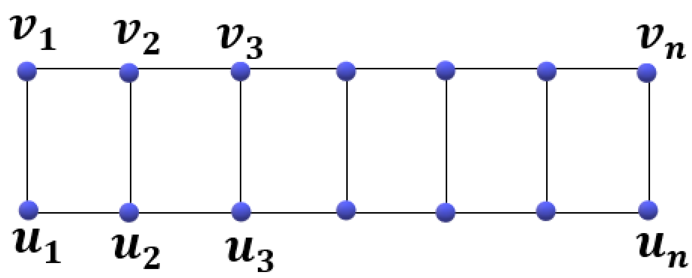

Figure (1) carpet graph $Cp(n)$

Contents

CHAPTER ONE

Introduction

Graph theory, is a branch of Mathematics concerned with networks of points connected by lines. The subject of graph theory had its beginnings in recreational math problems (see number game), but it has grown into a significant area of mathematical research, with applications in chemistry, operations research, social sciences, and computer science.

The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island—but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially he proved the first theorem in graph theory.

As used in graph theory, the term graph does not refer to data charts, such as line graphs or bar graphs. Instead, it refers to a set of vertices (that is, points or nodes) and of edges (or lines) that connect the vertices. When any two vertices are joined by more than one edge, the graph is called a multi graph. A graph without loops and with at most one edge between any two vertices is called a simple graph. Unless stated otherwise, graph is assumed to refer to a simple graph. When each vertex is connected by an edge to every other vertex, the graph is called a complete graph. When appropriate, a direction may be assigned to each edge to produce what is known as a directed graph, or digraph. ((Carlson 2022).

We think of a graph as a set of Points in a Plane or in 3-space and a set of line segments (possibly curved), each of which either joins two points or joins a point to itself.

Graphs are highly Versatile models for analyzing a wide range of Practical Problems in which Points and connections between them have some Physical or conceptual interpretation.

Placing such analysis on solid footing requires Precise definitions, terminology and notation (Behzad and Hartrand 1977).

CHAPTER TWO

Literature Review

Definition 2.1: A graphs *is* a finite non empty set of objects called *vertices* (the singular word is *vertex*) together with a (Possibly empty) set of un order pairs of distinct vertices of called edges. (Behzad and Hartrand 1977).

Definition 2.2: The cardinality or the number of the vertex set of a graph G is called the *order* of G, denoted by $p(G)$ or simply p . (Behzad and Hartrand 1977).

Definition 2.3: The cardinality or the number of the edge set of a graph G is called the *size* of G, is denoted by $q(G)$ or simply q. (Madhumangal 2021).

Definition 2.4: In graph theory, the distance between two vertices of a graph G is defined as the length of the shortest path between those vertices. The length of a path is the number of edges in the path, so the distance between two vertices is the minimum number of edges that must be traversed to get from one vertex to the other (Lin 2009).

Definition 2.5: (P. Dankelmann, 1997) Let G be a *connected* graph, and

$v \in V(G)$, the *transmission* of v is denoted by $\sigma(v)$ and defined by

$\sigma(v) = \sum_{u \epsilon V(G)} d(u, v)$. (Chen 1976)

Definition 2.6: Let G be a connected graph, then the average *distance* (*mean distance*) of G is denoted by $\mu(G)$ and defined by $\mu(G) = \frac{\sigma(G)}{p(p-1)}$,where p is the order of G .That is, the *average distance* of a graph G is the average value of the distance between all pairs of vertices in G . (Emmert-Streib 2014)

Definition 2.7: A path is walk that does not include any vertex twice, except that its first vertex might be the same as its least a cycle or circuit is a path. (June 1991).

Definition 2.8: Eccentricity of vertex v denoted by $\mathcal{E}(v)$ is the greatest geodesic distance between v and any other vertex it can be through of as far a vertex is from the vertex most. (Wallis 2007).

Definition 2.9: The Radius of graph G denoted by rad (G) is the minimum eccentricity of any vertex in the graph that is rad(G)= $\min\limits_{\mathrm{v}\in\mathrm{v(G)}} \mathcal{E}(v)$ (Wallis 2007).

Definition 2.10: The diameter of a graph G denoted by diam(G) is maximum eccentricity of any vertex in the graph that is diam(G)= $\max\limits_{\mathrm{v}\in\mathrm{v(G)}} \mathcal{E}(v)$ (Wallis 2007).

Definition 2.12:A Eulerian cycle or Eulerian circuit or Euler tour in undirected graph is a cycle that use edge exactly once if such an Euler cycle exists in the graph is called an Eulerian graph. (Michael A.Henning 2022)

Definition 2.13: A Hamiltonian cycle or Hamiltonian circuit or vertex tour a graph cycle is a cycle that visits each vertex exactly once (except for the vertex that is both the start and end witch is visited twice). A graph that contains a Hamiltonian cycle is called Hamiltonians graphs. (Ashay Dharwadker 2011).

CHAPTER THREE

3.1.The Average Distance Of The Carpet Special Graph $Cp(n)$

Definition 3.1.1: carpet graph $\boldsymbol{Cp}(\boldsymbol{n})$ is the connected graph of order $2n$ Consists of two path $\boldsymbol{P_n}$ in witch the symmetric vertices in places are adjacent as given in Figure (2)

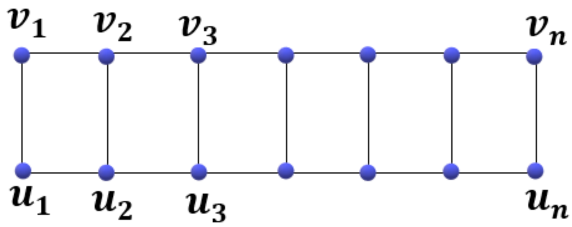

Figure (2) The graph of $Cp(n)$

Theorem 3.1.2.

The average *distance* (mean) distance of the carpet graph $Cp(n)$is

$\mu(Cp(n)) = \frac{2n^2+3n-2}{3(2n-1)}$ for *n, m* >1

Let G be a connected graph, then the average *distance* (*mean distance*) of G is denoted by $\mu(G)$ and defined by $\mu(G) = \frac{\sigma(G)}{p(p-1)}$,where p is the order of G .That is ,The *average distance* of a graph G is the average value of the distance between all pairs of vertices in G .

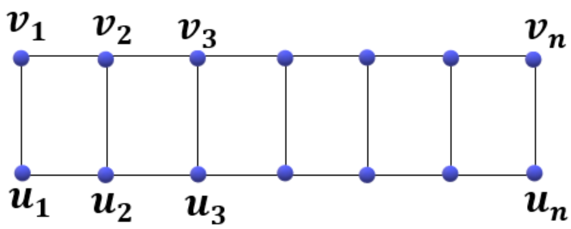

Figure (3) The graph of $Cp(n)$

Proof: The transmission of vertices of $Cp(n)$ are

$$\sigma(v_1) = \frac{n(n-1)}{2} + \frac{n(n+1)}{2} + \frac{(1)(2)}{2} + (-1)$$

$$\sigma(v_2) = \frac{(1)(2)}{2} + \frac{(n-2)(n-1)}{2} + \frac{(2)(3)}{2} + \frac{n(n-1)}{2} + (-1)$$

$$\sigma(v_3) = \frac{(2)(3)}{2} + \frac{(n-3)(n-2)}{2} + \frac{(3)(4)}{2} + \frac{(n-2)(n-1)}{2} + (-1)$$

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$$\sigma(v_{n-1}) = \frac{(n-2)(n-1)}{2} + \frac{(1)(2)}{2} + \frac{(n-1)(n)}{2} + \frac{(2)(3)}{2} + (-1)$$

$$\sigma(v_n) = \frac{(n)(n-1)}{2} + \frac{(n+1)(n)}{2} + \frac{(1)(2)}{2} + (-1)$$

$$\sigma\big(Cp(n)\big) = \sum_{i=1}^{n} \sigma(v_i) + \sum_{i=1}^{n} \sigma(u_i)$$

$$\sigma\big(Cp(n)\big) = \sum_{i=1}^{n-1} \frac{i(i+1)}{2} + \sum_{i=1}^{n-1} \frac{i(i+1)}{2} + \sum_{i=1}^{n} \frac{i(i+1)}{2} + \sum_{i=1}^{n} \frac{i(i+1)}{2} + n(-1)$$

$$\sigma\big(Cp(n)\big) = 2\left[2\sum_{i=1}^{n-1} \frac{i(i+1)}{2} + 2\sum_{i=1}^{n} \frac{i(i+1)}{2} + n(-1)\right]$$

$$\sigma\big(Cp(n)\big) = 2\left[\sum_{i=1}^{n} i(i+1) + \sum_{i=1}^{n-1} i(i+1) - n\right]$$

Since$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}and\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$

So transmission of the carpet $Cp(n)$ is

$$\sigma\big(Cp(n)\big) = 2\left[\left(\frac{n(n+1)(n+2)}{3}\right) + \left(\frac{n(n-1)(n+1)}{3}\right) - n\right]$$

$$= \frac{2(n)(n+1)[n+2+(n-1)] - 6n}{3}$$

$$= \frac{2(n)(n+1)[2n+1] - 6n}{3}$$

$$= \frac{4n^3 + 6n^2 - 4n}{3}$$

$$\sigma\big(Cp(n)\big) = \frac{2n(2n^2 + 3n - 2)}{3}$$

Now the mean distance of the section carpet graph $Cp(n)$ is

$$\mu\big(Cp(n)\big) = \frac{\sigma\big(Cp(n)\big)}{2n(2n-1)}$$

$$\mu\big(Cp(n)\big) = \frac{2n(2n^2 + 3n - 2)}{(3)2n(2n-1)}$$

$$\mu\big(Cp(n)\big) = \frac{(2n^2 + 3n - 2)}{(3)(2n-1)}$$

$$\mu\big(Cp(n)\big) = \frac{(2\mathrm{n}-1)(n+2)}{(3)(2n-1)} \quad = \frac{n+2}{3}$$

Remark 3.1.3

The average distance of the carpet graph $Cp(2)$ is $\mu\big(Cp(2)\big) = \frac{4}{3}$

Proof. By Theorem 3.1.2, the average distance of the carpet graph $Cp(n)$ is $\mu\big(Cp(n)\big) = \frac{n+2}{3}$.

If $n = 2$,then we get the carpet graph $Cp(2)$ as given in Figure (4),

$\mu\big(Cp(2)\big) = \frac{2+2}{3} = \frac{4}{3}$.

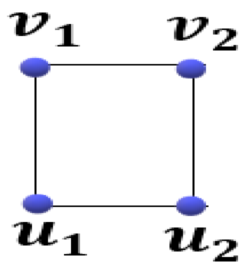

Figure (4) the graph of $Cp(2)$

Remark 3.1.4

The average distance of the carpet graph $Cp(3)$ is $\mu(Cp(3)) = \frac{5}{3}$.

Proof. By Theorem 3.1.2 the average distance of $Cp(n)$ is $\mu(Cp(n)) = \frac{n+2}{3}$, if $n = 3$, then we get the carpet graph $Cp(3)$ as given in Figure (5),

$\mu(Cp(3)) = \frac{3+2}{3} = \frac{5}{3}$.

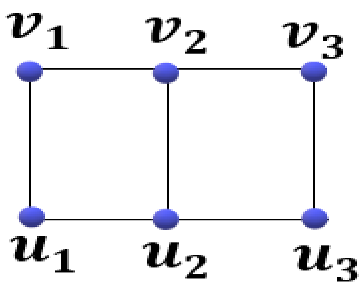

Figure (5) The graph of $Cp(3)$

Example 3.1.5 : In this example we find the average distance of $Cp(5)$ by using definitions ,transmission of a vertex, transmission of a graph ,and the average distance of a graph as follows :

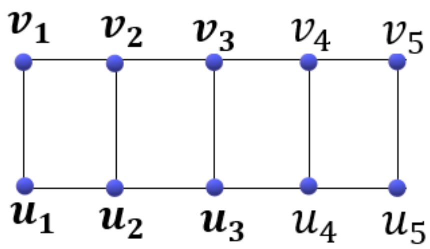

Figure (6) The graph of $Cp(5)$

$\sigma(v_1) = 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 5 = 25$

$\sigma(v_2) = 1 + 1 + 2 + 3 + 1 + 2 + 2 + 3 + 4 = 19$

$\sigma(v_3) = 1 + 2 + 1 + 2 + 1 + 2 + 3 + 2 + 3 = 17$

$\sigma(v_4) = 1 + 1 + 2 + 3 + 1 + 2 + 2 + 3 + 4 = 19$

$\sigma(v_5) = 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 5 = 25$

$\sigma(u_1) = 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 5 = 25$

$\sigma(u_2) = 1 + 1 + 2 + 3 + 1 + 2 + 2 + 3 + 4 = 19$

$\sigma(u_3) = 1 + 2 + 1 + 2 + 1 + 2 + 3 + 2 + 3 = 17$

$\sigma(u_4) = 1 + 1 + 2 + 3 + 1 + 2 + 2 + 3 + 4 = 19$

$\sigma(u_5) = 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 5 = 25$

The transmission of the carpet $Cp(5)$ is

$$\sigma\big(Cp(n)\big) = \sum_{i=1}^{5} \sigma(v_i) + \sum_{i=1}^{5} \sigma(u_i)$$

$$= \sigma(v_1) + \sigma(v_2) + \sigma(v_3) + \sigma(v_4) + \sigma(v_5) + \sigma(u_1) + \sigma(u_2) + \sigma(u_3) + \sigma(u_4) + \sigma(u_5)$$

$= 25 + 19 + 17 + 19 + 25 + 25 + 19 + 17 + 19 + 25$

$= 210$

Now the mean distance of the section carpet graph $Cp(5)$ is

$$\mu\big(Cp(5)\big) = \frac{\sigma\big(Cp(5)\big)}{2(5)[(2(5)-1)]} = \frac{210}{90} = \frac{3}{7}.$$

And by using the theorem 3.1.2 directly we can find average distance easily

$$\mu\big(Cp(n)\big) = \frac{n+2}{3} \;\; ; n = 5$$

$$\mu\big(Cp(n)\big) = \frac{5+2}{3} = \frac{7}{3}$$

3.2. Some Graphical properties of the special graph

3.2.1 Eulerian: The special graph is a not Eulerian graph, since the degree of vertices of $Cp(n)$ are odd.

3.2.2 Hamiltonian: The special graph is a Hamiltonian graph, since the cycle C_n passing through all vertices $Cp(n)$.

3.2.3 Eccentricity: The Eccentricity of the special graph is Ɛ $(Cp(n)) = n$.

3.2.4 Diameter: The Diameter of the special graph is

diam ($Cp(n)$)=max Ɛ ($Cp(n)$) =n.

3.2.5 Radius: The Radius of the special graph is

rad ($Cp(n)$)=min Ɛ ($Cp(n)$), if the number of vertices is odd so

rad $(Cp(n)) = \frac{n+1}{2}$, if the number of vertices is even so

rad $(Cp(n)) = \frac{n}{2} + 1$.

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پوختە:

لەم کارەدا تێکڕای دووری گرافێکی تایبەت بە ناوی (carpet graph) دەدۆزینەوە کە بە $Cp(n)$ هێمامان کردووە ،کە تێیدا دووری گرافێکە بریتییە لە :

$$\mu(Cp(n)) = \frac{2n^2+3n-2}{3(2n-1)} \quad \text{for } n, m > 1$$

لەگەل دیراسەکردنی هەندێک تایبەتمەندی ئەم گرافە تایبەتمان وەک دۆزینەوەی (نیوە تیرە ،تیرە ،نا ئۆیلەری،هامیڵتۆنی).