**Question Bank of Mathematical Analysis**

**(More than 130 questions)**

**Chapter One: The Real numbers**

**Q1:** State and prove Archimedes Principle Theorem.

**Q2:** Ifand **,** then prove that there is such that .

**Q3:** Prove that between any two distinct real numbers there is a rational number.

**Q4:** If and , then prove that there is such that .

**Q5:** Prove that between any two distinct positive real numbers there is a rational number.

**Q6:** Prove that between any two distinct real numbers there is an infinite set of rational numbers.

**Q7:** Prove that between any two distinct real numbers there is an irrational number.

**Q8:** Prove that between any two distinct real numbers there is an infinite set of irrational numbers.

**Q9:** If and , then prove that .

**Q10:** If and , then prove that , provided that .

**Chapter Two: Sequences of Real numbers**

**Q1:** Prove that every convergent sequence is a bounded sequence.

**Q2:** Prove that every bounded monotone sequence is convergent.

**Q3:** Prove that every bounded non decreasing monotone sequence is convergent.

**Q4:** Prove that every bounded non increasing monotone sequence is convergent.

**Q5:** Prove or disprove that: Every convergent sequence is a monotone sequence.

**Q6:** Prove or disprove that: Every convergent sequence is a Cauchy sequence.

**Q7:** Prove or disprove that: Every Cauchy sequence is a convergent sequence.

**Q8:** Prove or disprove that: Every Cauchy sequence is bounded.

**Q9:** For any , prove that there is a Cauchy sequence of rational numbers converges to .

**Q10:** Prove that the sequence is bounded.

**Q11:** Prove that the sequence is monotone.

**Q12:** Give an example of a bounded sequence which is not convergent.

**Q13:** Give an example of a monotone sequence which is not convergent.

**Q14:** If is a sequence of closed intervals such that , , then prove that

 .

**Q15:** If is a sequence of closed intervals such that , , then prove that

 and if the sequence is convergent to 0, then prove that the intersection is

 consists of only one point, that is , where is the length of

**Q16:** state and prove Nested intervals Theorem.

**Q17:** If and are convergent sequences to and respectively, such that , and , then prove that is convergent to .

**Chapter Three: Metric spaces**

 **Q1:** Define a metric space. Let defined by Then prove that is a metric space.

**Q2:** Define a metric space. Let defined by Then prove that is a metric space.

**Q3:** Prove or disprove that: In every metric space , if a set is closed then .

**Q4:** Prove or disprove that: In every metric space , if for every set then is closed.

**Q5:** Prove or disprove that: In every metric space , a closed and bounded set is compact.

**Q6:** Prove that: In a metric space , every compact set is closed.

**Q7:** Prove or disprove that: The usual metric space is complete.

**Q8:** Consider the usual metric space , then answer the following:

 (i) Find (ii) Find (iii) Find (iv) Find (v) Is open? Why?

 (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

 **Q9:** Consider the usual metric space and the set of rational numbers, then answer the

 following: (i) Find (ii) Find (iii) Find (iv) Find (v) Is open? Why?

 (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

**Q10:** Consider the usual metric space and the set of irrational numbers, then answer the

 following: (i) Find (ii) Find (iii) Find (iv) Find (v) Is open?

 Why? (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

**Q11:** Consider the usual metric space and the set of integers , then answer the following:

 (i) Find (ii) Find (iii) Find (iv) Find (v) Is open? Why?

 (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

**Q12:** Consider the usual metric space and the set of natural numbers, then answer the

 following: (i) Find (ii) Find (iii) Find (iv) Find (v) Is open?

 Why? (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

**Q13:** Consider the usual metric space and the set of positive real numbers, answer the

 following: (i) Find (ii) Find (iii) Find (iv) Find (v) Is open?

 Why? (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is dense in itself? Why?

**Q14:** Consider the usual metric space and the set of negative integers , then answer the

 following: (i) Find (ii) Find (iii) Find (iv) Find (v) Is

 open? Why? (vi) Is closed? Why? (vii) Is discrete? Why? (viii) Is compact? Why?

 (ix) Is complete? Why? (x) Is dense in itself? Why?

**Q15:** Prove or disprove that: If a sequence convergent to a point in a metric space ,

 then the set is compact.

**Q16:** Prove that in a metric space every compact set is bounded.

**Q17:** Let be a metric space and . If is a finite set, then prove that is closed.

**Q18:** Prove and disprove that: In every metric space a compact set is closed and bounded.

**Q19:** Prove that in a metric space every compact set is finite.

**Q20:** Let be a metric space and . If is a sequence in convergent to , then

 prove that or .

**Q21:** Prove and disprove that: In the discrete metric space , every compact set finite.

**Q22**: Let be a compact metric space. Prove that every infinite subset , has at least a

 cluster point.

**Q23**: Let be a convergent sequence to in a metric space . Then prove that the set

 is compact.

**Q24:** Prove or disprove that: Let be a compact metric space. Then every infinite subset

 , has at least a cluster point.

**Q25:** Prove or disprove that: Let be a convergent sequence to in a metric space .

 Then the set is compact.

**Q26:** Prove or disprove that: In a metric space , every compact set is bounded.

**Q27:** Consider the usual metric space and the set of integers , then answer the following:

 (i) Find (ii) Find (iii) Find (iv) Find (v) Is open? Why? (vi) Is

 closed? Why? (vii) Is discrete? Why? (viii) Is compact? Why? (ix) Is complete?

 Why? (x) Is dense in itself? Why?

**Chapter Four: Continuity**

**Q1:** Let defined by . Then test the continuity of on , where and

 are the usual and discrete metric functions on , respectively.

**Q2:** Prove that between any two metric spaces, a constant function is continuous.

**Q3:** Let defined by . Then test the continuity of on , where and are the discrete and usual metric functions on .

**Q4:** Let defined by Then test the continuity of on .

**Q5**: Let defined by Then test the continuity of on , where and are the usual and discrete metric spaces, respectively.

**Q6:** Let defined by Then test the continuity of on ,

 where and are the discrete and usual metric spaces.

**Q7:** Let defined by . Then test the continuity of on .

**Q8:** Let defined by Then test the continuity of on ,

 where and are the usual and discrete metric spaces.

**Q9**: Let defined by Then test the continuity of on ,

 where and are the usual and discrete metric spaces, respectively.

**Q10:** Show that the continuous function is not uniform on ).

**Q11:** Show that the continuous function is not uniform on .

**Q12:** Show that the continuous function defined by is not uniform.

**Q13:** Show that the continuous function is uniform on .

**Q14:** Show that the continuous function is not uniform on .

**Q15**: Show that the continuous function is uniform continuous on .

**Q16:** Show that the continuous function defined by is uniform.

**Chapter Five: Sequences of functions**

**Q1:** Is the sequence of functions defined on [ uniform convergent? Where .

**Q2:** Is the sequence of functions defined on [ uniform convergent? Why?

**Q3:** Is the sequence of functions defined on uniform convergent? Where

**Q4:** Definepointwise convergence of a sequence of functions. Prove and disprove that: the

 sequence of functions is uniform convergent on , where .

**Q5**: Let a sequence of functions converges uniformly to on . If is bounded

 on , for all , then prove that is bounded on .

**Q6**: Prove or disprove that: The sequence of functions is uniform convergence

 on .

**Q7:** Let a sequence of functions converges uniformly to on . If is

 continuous on , for all , then prove that is continuous on .

**Q8:** Is the sequence of functions defined on uniform convergent? Where

**Chapter Six: Riemann integrals**

**Q1:** If is a bounded Riemann integrable function on and is a real constant, then prove

 that is Riemann integrable and .

**Q2:** If is a bounded Riemann integrable function on and is a positive real constant, then

 prove that is Riemann integrable and .

**Q3:** If is a bounded Riemann integrable function on , then prove that is Riemann

 integrable on and .

**Q4**: If is a bounded function on and for all , there is a partition of

 such that , then prove that is Riemann integrable on .

**Q5**: If a bounded function is R-integrable on closed intervals and , then prove that

 is R-integrable on .

**Q6:** If is a bounded Riemann integrable function on , then prove that for all , there is

 a partition of such that .

**Q7**: Prove or disprove that: A bounded function is R-integrable on , if and only if for all

 , there is a partition of such that .

**Q8**: If a bounded function is R-integrable on closed intervals and , then prove that is R-integrable on .

**Q9:** By using the definition of Riemann integral, show that the function is Riemann

 integrable on and find its integral value on .

**Q10:** Define a negligible set. Prove that the set is negligible.

**Q11:** By using the definition of Riemann integral, show that the function is Riemann

 integrable on and find its integral value on .

**Q12:** Let be a Riemann integrable function on . Then prove or disprove the that the

 function is Riemann integrable on .

**Q13:** Prove or disprove that: Let and be Riemann integrable functions on . Then the

 function is Riemann integrable on ].

**Q14:** Let be a function defined by Then answer the following:

 (i) Is Riemann integrable on ? Justify your answer.

(ii) Is Riemann integrable on ? Explain your answer.

**Q15:** Let be a function defined by Then show that is not Riemann integrable on .

**Q16:** If is a continuous function on a closed interval , then prove that is a

 Riemann integrable function on .

**Q17:** Prove that every continuous function on a closed interval , is R- integrable.

**Q18:** If is an R- integrable function on and , then prove that is R-integrable on and moreover .

**Q19:** Let , then prove that is negligible.

**Q20:** Prove or disprove that: If defined by , whenever , for

 every and , then is R-integrable on and .

**Q21:** Let , for all and If possible find

 , and .

**Q22:** Prove or disprove that: If is RS – integrable function with respect to a nondecreasing

 function on closed intervals and , then is RS- integrable with respect to on

 .

**Q23:** Let and be Riemann integrable functions on a . Then prove the function is

 Riemann integrable on and

**Q24:** Define a negligible set. Prove that the union of two negligible sets is negligible.

**Q25:** Let be a function defined by Is Riemann integrable on ? Justify your answer.

**Q26:** If is a monotone function on a closed interval , then prove that is a Riemann integrable function on .

**Q27:** Show by an example that if is RS – integrable function with respect to a nondecreasing

 function on closed intervals and , then is not RS- integrable with respect to

 on ].

**Q28:** Let and be Riemann integrable functions on a . Then prove the function is Riemann integrable on .

**Q29:** Define a negligible set. Prove that the union of two negligible sets is negligible.

**Chapter Seven: The Differentiation**

**Q1:** By using the definition of differentiation, prove that the function is

 differentiable at every and find its derivative at (using sequences) .

**Q2:** By using the definition of differentiation, prove that the function is differentiable

 at every and find its derivative at (using sequences) .

**Q3:** By using the definition of differentiation, prove that the function is differentiable

 at every and find its derivative at (using sequences) .

**Q4:** By using the definition of differentiation, prove that the function is differentiable

 at every , and find its derivative at (using sequences) .

**Q5:** By using the definition of differentiation, prove that the function is differentiable

 at every and find its derivative at (using sequences) .

**Q6:** If is a differentiable function on , then prove that there is a sequence of continuous functions converges to .

**Q7:** If is a differentiable function on an open interval , and is continuous on , then

 Prove that , for every .

**Q8:** If is a differentiable function on , then prove that there is a sequence of continuous functions converges to on .

**Q9:** Let be an open interval and be a continuous function . If defined by

 then prove that is a differentiable on and .

**Q10:** By using the definition of differentiation, prove that the function is differentiable at every and find its derivative at (using sequences) .

**Q11:** Let and be continuous functions on and differentiable functions on . If , then prove that such that .

**Q12:** Prove or disprove that: If is a differentiable function on an open interval , and is

 continuous on , that , for every .

**Chapter Eight: Measure Theory**

**Q1:** If and are bounded open intervals such that , then prove that

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**Q2:** If and are bounded open intervals such that , and is bounded, then

 prove that .

**Q3:** Find and .

**Q4**: Define the outer measure of bounded sets. Show that .

**Q5:** Let and be bounded measurable sets in . Then prove that is a measurable set.

**Q6:** Prove that the set of positive integers is a measurable set and find its measure.

**Q7:** Define the outer measure of bounded sets. Show that .

**Q8:** If is a bounded measurable set, prove that for all , there is a bounded open set ,

 such that .

**Q9:** Define a bounded measurable set. Show that , where is the set of Positive real numbers.

**Q10:** If is a bounded subset of and for all , there is a bounded open set ,

 such that , then prove that is a measurable set.

**Chapter Nine: Lebesgue Theory of integration**

**Q1:** Let be a function defined by Is Lebesgue

 integrable on ? If so, find .

**Q2:** Let be a function defined by Is Lebesgue integrable on ? If so, find its integral value on .

**Q3:** Let be a function defined by Is Lebesgue integrable on ? If so, find .

**Chapter Ten: Integrable functions and measurable functions**

**Q1:** Define a measurable set. Let g defined by g Then prove that is a

 measurable function on .

**Q2:** Define Lebesgue partition of bounded measurable sets. Let g defined by

 g Then prove that is a measurable function on